Chiral Three-Nucleon Forces from p-wave Pion Production

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Production of p-wave pions in nucleon-nucleon collisions is studied according to an improved power counting that embodies the constraints of chiral symmetry. Contributions from the first two nonvanishing orders are calculated. We find reasonable convergence and agreement with data for a spin-triplet cross section in $pp \to pp\pi^0$, with no free parameters. Agreement with existing data for a spin-singlet cross section in $pp \to pn\pi^+$ constrains a short-range operator shown recently to contribute significantly to the three-nucleon potential.

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The use of (approximate) chiral symmetry of QCD to determine the form of the low-energy effective Lagrangian has proven to be a powerful aid to the understanding of strong interaction physics [1]. It has long been known [2–4] that the use of chiral symmetry for pion-nucleon (πN) scattering leads to a qualitative understanding of the pion-ranged part of the three-nucleon force, believed to produce important effects in nucleon-deuteron (Nd) scattering [5] and few-nucleon bound states [6]. Yet, discrepancies between theory and experiment (for A_y in Nd scattering [7] and for excited levels in bound states [8]) remain which have been widely attributed to unknown three-nucleon forces. A novel three-nucleon force, expected on the basis of power counting arguments, involves the exchange of a pion between one nucleon and two others interacting via short-ranged forces [3]. This force can indeed affect Nd scattering at a currently observable level, and thus potentially resolve the remaining discrepancies [9]. It depends on a pion-two-nucleon interaction of a form determined by chiral symmetry, but strength determined by parameters, d_i of Eq. (2), not fixed by symmetry. We argue here that the production of p-wave pions in nucleon-nucleon (NN) scattering offers a unique opportunity to determine d_i [10].

In the last few years, the various $NN \to NN\pi$ reactions have been studied both experimentally and theoretically [11], with a focus on near-threshold energies. The first high-quality data concerned the total cross section, and most theoretical analyses have concentrated on $\eta \lesssim 0.4$, a region dominated by the Ss state. (Final states are labeled by Ll with L and l being the relative angular momentum of the nucleon pair and the pion with respect to the two-nucleon center of mass, respectively; η is the maximum pion momentum in units of the pion mass, m_π). Many different mechanisms are expected at these kinematics: heavy meson exchanges [12], (off-shell) pion rescattering [13,14], excitations of baryon resonances [15], and pion emission from exchanged mesons [16].

The pion dynamics are largely controlled by chiral symmetry constraints, and the hope that the use of chiral perturbation theory (χPT) would yield insights led to the use of tree-level χPT to calculate the cross sections close to threshold [17–21]. A few calculations to one loop order, carried out using the heavy baryon formalism, have appeared [22]. According to Ref. [23], the large momenta of relevance in the pion production makes that particular formalism inappropriate for the present problem. However, the results of Ref. [22] can be viewed as a rough estimate of the actual loop contributions. Reference [17] emphasized that the diverse contributions to the Ss final states can be ordered in powers of $\sqrt{m_\pi/M_{\rm QCD}}$, where $M_{\rm QCD} \simeq 1$ GeV is the typical QCD mass scale. The implication of this relatively large parameter is that loop diagrams enter at next-to-leading order in s-wave pion production. Thus, a test of the convergence of the series is hindered. We shall show that such difficulties are not present for the case of π production in p waves ($\eta \sim 1$), because the production proceeds through leading-order operators better determined from other processes.

Our arguments rely on the use of symmetries. One may obtain the results of QCD by using the most general Lagrangian involving the low-energy degrees of freedom (pion $\boldsymbol{\pi}$, nucleon N, and delta isobar Δ) which has the same symmetries as QCD. These are approximate chiral symmetry, parity, and time-reversal invariance. Chiral symmetry plays a crucial role in low-energy processes because it demands that, in the chiral limit where the quark masses go to zero, the pion interactions contain derivatives, which are weak at small momenta, Q. Because the quark masses are small, any nonderivative pion interactions are also weak. Although the nucleon mass m_N is not small, it plays no dynamical role at low energies. The delta isobar can be excited, but its mass difference to the nucleon, $\delta m \equiv m_\Delta - m_N$, is not large. For processes in which $Q \sim m_\pi$ it is convenient to introduce the "chiral index" of an interaction $\Delta_\chi = d + \frac{f}{2} - 2$, where d is the number of small-scale factors, that is, derivatives, m_π, and δm; and f is the number of fermion field operators. Chiral symmetry implies that $\Delta_\chi \geq 0$ [24]. Our interaction Lagrangian is given, using an appropriate choice of fields, by the expressions [25,26]

$$\mathcal{L}^{(0)}_{\rm int} = -\frac{1}{4f_\pi^2} N^\dagger \boldsymbol{\tau} \cdot (\boldsymbol{\pi} \times \dot{\boldsymbol{\pi}}) N + \frac{g_A}{2f_\pi} N^\dagger (\boldsymbol{\tau} \cdot \vec{\boldsymbol{\sigma}} \cdot \vec{\nabla}\boldsymbol{\pi}) N + \frac{h_A}{2f_\pi} [N^\dagger (\mathbf{T} \cdot \vec{S} \cdot \vec{\nabla}\boldsymbol{\pi})\Delta] + \cdots, \tag{1}$$

and [3]

$$\begin{aligned}\mathcal{L}^{(1)}_{\rm int} = {} & \frac{i}{8m_N f_\pi^2} N^\dagger \boldsymbol{\tau} \cdot (\boldsymbol{\pi} \times \vec{\nabla}\boldsymbol{\pi}) \cdot \vec{\nabla} N - \frac{c_3}{f_\pi^2} N^\dagger (\vec{\nabla}\boldsymbol{\pi})^2 N - N^\dagger \frac{\bar{c}_4}{2f_\pi^2} \vec{\sigma} \cdot \vec{\nabla}\boldsymbol{\pi} \times \vec{\nabla}\boldsymbol{\pi} \cdot \boldsymbol{\tau} N \\ & - \frac{i g_A}{4m_N f_\pi} N^\dagger \boldsymbol{\tau} \vec{\sigma} \cdot \dot{\pi} \vec{\nabla} N - \frac{h_A}{2m_N f_\pi} [i N^\dagger \mathbf{T} \cdot \dot{\pi} \vec{S} \cdot \vec{\nabla}\Delta] - \frac{d_1}{f_\pi} N^\dagger \boldsymbol{\tau} \cdot \vec{\sigma} \cdot \vec{\nabla}\boldsymbol{\pi} N N^\dagger N \\ & - \frac{d_2}{2f_\pi} \vec{\nabla}\boldsymbol{\pi} \times N^\dagger \vec{\sigma} \boldsymbol{\tau} N \cdot N^\dagger \vec{\sigma} \boldsymbol{\tau} N + \cdots,\end{aligned} \tag{2}$$

where $\bar{c}_4 = c_4 + \frac{1}{4m_N}$. The terms denoted by "$\cdots$" include Hermitian conjugates, s-wave πN scattering terms, and terms of higher powers in pion fields. Our principle aim is to determine the parameters $d_i = \mathcal{O}(1/f_\pi^2 M_{\rm QCD})$, which determine the desired three-nucleon force. The c_i have been determined from πN scattering at tree level ($c_3^{(\rm tree)} = -3.90$ GeV^{-1} and $c_4^{(\rm tree)} = 2.25$ GeV^{-1} [27]) as well as to one-loop order ($c_3^{(\rm loop)} = -5.29$ GeV^{-1} and $c_4^{(\rm loop)} = 3.63$ GeV^{-1} [28]). Since we treat the delta isobar explicitly, we need to subtract its contribution from these values of c_i [29]. This prescription leads to $c_3^{(\rm tree)} = -1.5$, $c_4^{(\rm tree)} = 1.1$ and $c_3^{(\rm loop)} = -2.95$, $c_4^{(\rm loop)} = 2.5$, all given in GeV^{-1}. As will be established below, up to next-to-leading order, the d_i, which support only $S \to Sp$ transitions, are the only undetermined parameters in p-wave pion production.

The next step is to extend the power counting of Ref. [17] to the region $\eta \sim 1$, where the outgoing pion has energy $\omega = \mathcal{O}(m_\pi)$ *and* momentum $|\vec{q}| = \mathcal{O}(m_\pi)$, and the two nucleons in the final state have momentum $|\vec{p}\,'| = \mathcal{O}(m_\pi)$ and total energy $p'^0 = \mathcal{O}(m_\pi^2/m_N)$. The unique difficulty of using χPT for pion production is that the entire pion energy is supplied by the relatively large momentum of the initial nucleons, $|\vec{p}| = \mathcal{O}(\sqrt{m_N m_\pi})$. Note that the nonrelativistic approximation holds, as $p^4/8m_N^3 \sim m_\pi^2/m_N \ll m_\pi \sim p^2/2m_N$.

The scales of momenta and energy are not the same, so it is simpler to count powers of the small scales in time-ordered perturbation theory. Equivalently, one first integrates over the time component of loop momenta in covariant diagrams. In this case, an intermediate state is associated with an energy denominator $1/E$, a loop with a $Q^3/(4\pi)^2$, a spatial (time) derivative with Q (E), and a virtual pion vertex with $1/E^{1/2}$ from wave function normalization. For N, $E \sim Q^2/m_N$, for Δ, $E \sim Q^2/m_N + \delta m$, and, for $\boldsymbol{\pi}$, $E \sim \sqrt{Q^2 + m_\pi^2}$.

Final-state interactions are those which occur after the emission of the real pion. In this case, the nucleons have typical $Q \sim m_\pi$. The energies of intermediate states containing a $\boldsymbol{\pi}$ or Δ can be $E \sim m_\pi$, but otherwise $E \sim m_\pi^2/m_N$. The sum of "irreducible" subdiagrams where all energies are $\mathcal{O}(m_\pi)$ is by definition the NN potential, which is then amenable to a χPT expansion. The sum of "reducible" subdiagrams produces the final-state wave function $|\psi_f\rangle$.

In contrast, all intermediate states occurring before the radiation of the real pion are characterized by loop momenta $\sim\sqrt{m_N m_\pi}$. For these kinematics, we find that any additional loop requires at least (i) one more interaction—pion exchange or shorter range—with an associated factor no larger than $1/f_\pi^2$; (ii) a volume integral with an associated factor of $(\sqrt{m_\pi m_N})^3/(4\pi)^2$; and (iii) an additional time slice. If the additional time slice cuts a pion line, a factor of $1/\sqrt{m_\pi m_N}$ comes in, and the overall extra loop factor is at least $\frac{1}{f_\pi^2}[(\sqrt{m_\pi m_N})^3/(4\pi)^2](1/\sqrt{m_\pi m_N}) = \frac{m_\pi}{m_N}$, that is, a suppression by two powers of the expansion parameter. If the additional time slice does not cut a pion line, a factor of $1/m_\pi$ appears, and there is a relative enhancement of $\sqrt{m_N/m_\pi}$. Integrals over two-nucleon states typically also have enhancements by factors of π from the unitarity cut. Thus we resum those diagrams that differ by the addition of interactions between the initial nucleons, and the effects are contained in an initial state wave function $|\psi_i\rangle$.

These considerations yield a pion production amplitude $T = \langle\psi_f|K|\psi_i\rangle$. Both the kernel K and $|\psi_{i,f}\rangle$ can be obtained from the chiral expansion, but the currently available $|\psi_i\rangle$ do not yield an accurate fit to the measured NN scattering phase shifts. Therefore we use a phenomenological coupled-channel (NN, $N\Delta$, $\Delta\Delta$) model [30] fitted to NN scattering.

According to these counting rules, loop contributions enter at next-to-leading order for s-wave pion production. However, the situation is much better for p-wave production. The leading contributions are displayed in Fig. 1. At lowest order [$\mathcal{O}(1)$, apart from overall factors] there are contributions from the direct production off the nucleon and off the delta, where all vertices are from $\mathcal{L}^{(0)}$ [Figs. 1(i) and 1(ii)]. At next-to-leading nonvanishing order [$\mathcal{O}(m_\pi/m_N)$], there are four types of contributions. First, there is a recoil correction to the direct production. Second, there are rescattering diagrams that proceed through the seagull vertices in $\mathcal{L}^{(1)}$ proportional to $1/4f_\pi^2$ (Galilean correction to the Weinberg-Tomozawa term), c_3, and c_4 [Fig. 1(iii)]. Third, there is a rescattering through the Weinberg-Tomozawa term, where the primary production vertex is proportional to the external pion momentum.

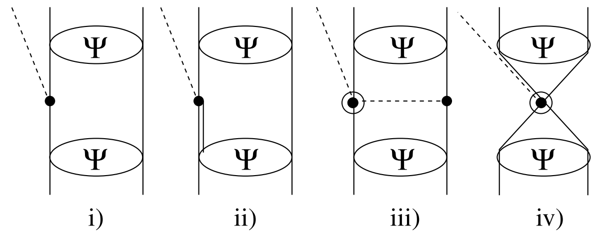

FIG. 1. Lowest-order contributions to p-wave production. Diagrams at $\mathcal{O}(1)$ are (i) and (ii), and of $\mathcal{O}(m_\pi/m_N)$ are (iii) and (iv). A solid (dashed) line denotes a nucleon (pion), and a double line a Δ. Interactions from $\mathcal{L}^{(0)}$ ($\mathcal{L}^{(1)}$) are denoted by a dot (circled dot). Diagrams with a Δ in the final state are also included.

Fourth, there are short-range $\pi(N^\dagger N)^2$ interactions proportional to d_1 and d_2 [Fig. 1(iv)]. Diagram 1(iv) and most of the rescattering diagrams contribute to charged-pion production only.

With our theory in place, we consider the available pion production database. This has been enriched recently by very accurate determinations of spin observables at $0.5 \lesssim \eta \lesssim 1$ for $pp \rightarrow pp\pi^0$ [31], $pp \rightarrow pn\pi^+$ [32], and $pp \rightarrow d\pi^+$ [33]. It is useful to describe the total cross section in terms of components ${}^{2S+1}\sigma_m$, where S is the initial NN spin with projection m along the direction of the incoming momentum. The ${}^{2S+1}\sigma_m$ can be expressed as linear combinations of the total cross section and the double polarization observables $\Delta\sigma_T$ and $\Delta\sigma_L$ [31].

We now use the feature of Eqs. (2) that d_i, the only undetermined parameters at $\mathcal{O}(m_\pi/M_N)$, support only $S \rightarrow Sp$ transitions. Therefore, we may test convergence for p-wave production, by using an observable in which the lowest contributing π partial wave is p and the initial and final nucleons are not both in S states. Such an observable exists, namely, the ${}^3\sigma_1$ cross section in neutral-pion production with Pp as the lowest partial waves contributing. While the ratios between double polarization observables and the total cross section, $\Delta\sigma_T/\sigma_{\text{tot}}$ and $\Delta\sigma_L/\sigma_{\text{tot}}$, have recently been accurately measured at IUCF [31], the total cross section is known to a much lesser accuracy (see the compilation of Ref. [34]). To determine the error of the total cross section, we simply take the total spread of the data as the error band. We defer a more detailed analysis until it can benefit from the soon-to-be-available [35] much better data.

In p-wave production the lowest-order loop contributions enter one order higher, at $\mathcal{O}[(m_\pi/m_N)^{3/2}]$, than the rescattering terms of Fig. 1, and are ignored. Besides the coupling constants of the pion to the baryon fields, the only parameter that enters is c_3. We will use both values given above to get an estimate of loop effects on the final result.

The *predictions* of χPT are compared to the data in Fig. 2. Up to values of $\eta \simeq 0.7$, the data is well described. Deviations at higher energies might be due to higher partial waves entering, and/or to higher-order p-wave contributions. In any case, we see that subleading corrections are smaller than leading contributions throughout the range $\eta \lesssim 1$.

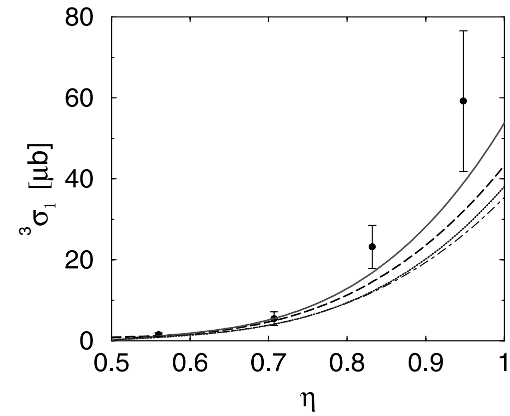

FIG. 2. Chiral perturbation theory predictions for ${}^3\sigma_1$ in the reaction $pp \rightarrow pp\pi^0$. Lowest order (long-dashed line), lowest order plus recoil contribution (dot-dashed line), and next-to-leading order using $c_3^{(\text{loop})}$ (solid line) and $c_3^{(\text{tree})}$ (dotted line) are shown. Data are from Refs. [31,34].

We next consider the amplitude for the ${}^1S_0 \rightarrow ({}^3S_1 - {}^3D_1)p$ transition, denoted a_0, which has recently been extracted from the reaction $pp \rightarrow pn\pi^+$ [36]. The loop corrections are again expected to be small, but the number of rescattering diagrams is larger, since isospin-odd operators [the recoil correction to the Weinberg-Tomozawa term as well as the c_4 term of Eq. (2)] enter. The striking feature of a_0 is that interactions proportional to the d_i's also contribute. Because there seems to be reasonable convergence in the p waves, we assume that they can be reliably computed and that we can attribute any deviation between theory and experiment to the effects of the terms involving the coefficients d_i. The contact interactions enter as the linear combination $d_1 + 4d_2$. Thus there is one unknown parameter to be fixed by the data. On the basis of dimensional analysis, we expect $d \equiv \frac{1}{5}(d_1 + 4d_2) = \frac{\delta}{f_\pi^2 M_{\text{QCD}}}$ with $\delta = \mathcal{O}(1)$.

Our result for a_0 is shown in Fig. 3. We find a destructive interference between direct nucleon and delta contributions that makes a_0 small and more sensitive to subleading terms. For the c_i parameters, we employ the values extracted from the tree-level fit to πN scattering ($c_i^{(\text{tree})}$). We use dipole form factors; to make contact with Ref. [9], we employ cutoff parameters $\Lambda = 1$ GeV for diagrams containing pion exchange and $\Lambda = m_\omega$ for the contact interactions. The result for $\delta = 0$ is not in disagreement with data, whereas a value of $\delta = 1$ leads to a serious disagreement with experiment. In Ref. [9], $\delta = -0.2$ was shown to yield an important contribution to A_y in Nd scattering at energies of a few MeV. Using $\delta = -0.2$ here is also consistent with the pion production data.

In contrast to ${}^3\sigma_1$, the result for a_0 is quite sensitive to the cutoff parameter used in the rescattering contribution, because the momentum range scanned by the c_4 term is

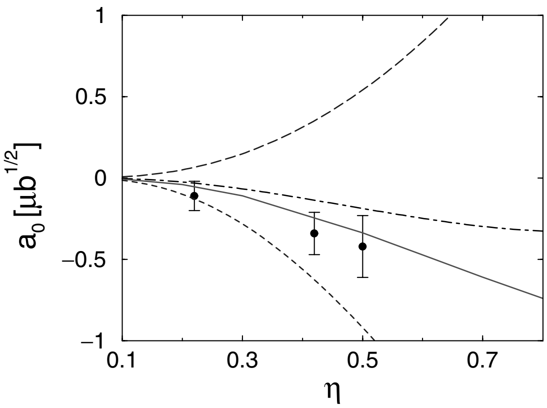

FIG. 3. a_0 of $pp \rightarrow np\pi^+$ in chiral perturbation theory. The different lines correspond to values of the parameter related to the three-nucleon force: $\delta = 1$ (long-dashed line). $\delta = 0$ (dot-dashed line), $\delta = -0.2$ (solid line), and $\delta = -1$ (short-dashed line). Data are from Ref. [36].

quite large. For example, our results for a_0 can vary up to a factor of 2 if the corresponding cutoff parameter is increased to 2 GeV. The cutoff sensitivity is not a serious difficulty because it also occurs in calculations of three-nucleon forces. From the viewpoint of an effective field theory, this can be simply understood: the large momentum pieces of the loop integrals involved in the evaluation of the c_4 contribution can be absorbed by a counterterm, namely, d_2. Thus, the cutoff dependence of c_4 directly translates into a scale dependence of d_2. A reasonable phenomenological estimate should follow from using the same cutoff and parameter set in both calculations. On the experimental side, it is clear from Fig. 3 that a reduction of the uncertainty in the data would allow a stronger constraint on δ. We find this a strong motivation to the continuation of the existing program on pion production.

We have shown that there is convergence in p-wave pion production, and that data on this reaction can be used to extract information about the three-nucleon force. It is clear that more accurate data would be very useful. In particular, the parameter d could be extracted and the calculation of Ref. [9] repeated to predict three-nucleon observables. We find it very gratifying that chiral symmetry provides a direct connection between pion production at energies ~350 MeV (IUCF) and Nd scattering at energies ~10 MeV (Madison, TUNL).

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