Vortex Precession in Bose-Einstein Condensates: Observations with Filled and Empty Cores

B. P. Anderson,* P. C. Haljan, C. E. Wieman, and E. A. Cornell*

JILA, National Institute of Standards and Technology and Department of Physics, University of Colorado, Boulder, Colorado 80309-0440

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We have observed and characterized the dynamics of singly quantized vortices in dilute-gas Bose-Einstein condensates. Our condensates are produced in a superposition of two internal states of ^{87}Rb, with one state supporting a vortex and the other filling the vortex core. Subsequently, the state filling the core can be partially or completely removed, reducing the radius of the core by as much as a factor of 13, all the way down to its bare value of the healing length. The corresponding superfluid rotation rates, evaluated at the core radius, vary by a factor of 150, but the precession frequency of the vortex core about the condensate axis changes by only a factor of 2.

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The dynamics of quantized vortices in superfluid helium and superconductors have been fascinating and important research areas in low-temperature physics [1,2]. Even at zero temperature, vortex motion within a superfluid is intricately related to the quantization of current around the vortex core. Besides these superfluid systems, studies of the dynamics of optical vortices have also become an active area of research [3]. More recently, demonstrations of the creation of quantized vortices in dilute-gas Bose-Einstein condensates (BEC) [4,5] have emphasized the similarities between the condensed matter, optical, and dilute-gas quantum systems. Because of the observational capabilities and the techniques available to manipulate the quantum wave function of the condensates, dilute-gas BEC experiments provide a unique approach to studies of quantized vortices and their dynamics. This paper reports direct observations and measurements of singly quantized vortex core precession in a BEC.

Numerous theoretical papers have explored the expected stability and behavior of vortices in BEC [6–17]. One interesting expected effect is vortex core precession about the condensate axis [7,9–16]. Radial motion of the core within the condensate can also occur, and may be understood as being due to energy dissipation and damping processes. Core precession may be described in terms of a Magnus effect—a familiar concept in fluid dynamics and superfluidity [1]. An applied force on a rotating cylinder in a fluid leads to cylinder drift (due to pressure imbalances at the cylinder surface) that is orthogonal to the force. Analogously, a net force on a vortex core in a superfluid results in core motion perpendicular to both the vortex quantization axis and the force. In the condensate vortex case, these forces can be due to density gradients within the condensate, for example, or the drag due to thermal atoms. The density-gradient force may be thought of as one component of an effective buoyancy: just as a bubble in a fluid feels a force antiparallel to the local pressure gradient, a vortex core in a condensate will feel a force towards lower condensate densities. The total effective buoyancy, however, is due less to displaced mass (the "bubble") than it is to dynamical effects of the velocity-field asymmetry, a consequence of a radially offset core. Typically, the total buoyancy force is away from the condensate center, and the net effect is an azimuthal precession of the core via the Magnus effect. Drag due to the motionless (on average) thermal atoms opposes core precession, causing the core to spiral outwards towards the condensate surface. In the absence of this drag (for temperature ~ 0), radial drift of the core may be negligible.

Our techniques for creating and imaging a vortex in a coupled two-component condensate are described in Refs. [4,18]. The two components are the $|F = 1, m_F = -1\rangle$ and $|F = 2, m_F = 1\rangle$ internal ground states of ^{87}Rb, henceforth labeled as states $|1\rangle$ and $|2\rangle$, respectively. We start with a condensate of 10^6 $|2\rangle$ atoms, confined in a spherical potential with oscillator frequency 7.8 Hz. A near-resonant microwave field causes some of the $|2\rangle$ atoms to convert to $|1\rangle$ atoms. The presence of a rotating, off-resonant laser beam spatially modulates the amplitude and phase of the conversion. The net result is a conversion of about half of the sample into an annular ring of $|1\rangle$ atoms with a continuous quantum phase winding from 0 to 2π about the circumference—a singly quantized vortex. The balance of the sample remains in the nonrotating $|2\rangle$ state and fills the vortex core. With resonant light pressure we can selectively remove as much of the core material as we desire. In the limit of complete removal, we are left with a single-component, bare vortex state.

In this bare-core limit, the core radius is on the order of the condensate healing length $\xi = (8\pi n_0 a)^{-1/2}$, where n_0 is the peak condensate density and a is the scattering length. For our conditions, $\xi = 0.65$ μm, well under our imaging resolution limit. The bare core can be observed after ballistic expansion [5] of the condensate, but this is a destructive measurement. On the other hand, if we leave some of the $|2\rangle$-state atoms filling the core, the pressure

of the filling material opens up the radius of the $|1\rangle$ vortex core to the point where we can resolve the core in a time series of nondestructive phase-contrast images.

Filled-core dynamics.—We first discuss vortex dynamics in two-component condensates, where 10%–50% of the atoms were in the $|2\rangle$ fluid filling the $|1\rangle$ vortex core. We took successive images of the $|1\rangle$ atoms in the magnetic trap, with up to 10 images of each vortex. The vortex core is visible as a dark spot in a bright $|1\rangle$ distribution, as shown in Fig. 1(a). Instabilities in our vortex creation process usually resulted in the creation of off-center vortex cores, allowing us to observe precession of the cores. We observed precession out to $\sim$2 s, after which the $|2\rangle$ fluid had decayed to the point that the vortex core was too small to be observed in the trapped condensate.

The recorded profile of each trapped condensate was fit with a smooth Thomas-Fermi distribution. Each vortex core profile was fit with a Gaussian distribution to determine its radius and position within the condensate. From the fits, we determined the overall radius R_t of the trapped condensate (typically 28 μm), the HWHM radius r of the filled vortex core, and the displacement d_t and angle θ_t of the core center with respect to the condensate center. Core angles and radii for the images in Fig. 1(a) are shown in Figs. 1(c) and 1(d). The vortex core is seen precessing in a clockwise direction, which is the same direction as the vortex fluid flow around the core. The angular precession frequency was determined from the time dependence of θ_t [Fig. 1(c)]. This and other similar data sets showed no reproducible radial motion of the core over the times and parameters examined. However, consistent decrease in the size of the core was observed, which we interpret as being due to known decay of the $|2\rangle$ fluid through inelastic atomic collisional processes.

For each data set, we determined a mean core radius and displacement. The data cover a range of core radii ($r = 7\xi$ to 13ξ), displacements ($d_t = 0.17R_t$ to $0.48R_t$), and percentage of atoms in the core (10% to 50%). Except for a few "rogue vortices" (discussed below), the measured precession frequencies are clustered around 1.4 Hz, as shown in Fig. 2, precessing in the same direction as the fluid rotation. The data [Fig. 2(a)] suggest a slight increase in frequency for cores farther from the condensate center. We also see [Fig. 2(b)] a slight decrease in precession frequency for larger cores. These measurements are in qualitative agreement with two-dimensional numerical simulations for two-component condensates [16].

As indicated in Fig. 2, a few vortex cores exhibited precession opposite to that of the fluid flow (negative frequencies). The quality of the corresponding vortex images was routinely lower than for the positive-frequency precession points, with vortices looking more like crescents and "D" shaped objects rather than like the images of Fig. 1. We only found such occurrences with our two-component (in-trap) measurements. We speculate that this "inverse precession" may be due to rare events in which total angular momentum may be distributed in a complicated way among both internal states. Such cases might arise due to position instabilities of the rotating laser beam during the vortex creation process. Recent theoretical attention has addressed the possibly related situation of nonsymmetric configurations of vortices in two-component condensates [17].

Bare-core dynamics.—To examine the dynamics of bare vortices, our procedure consisted of taking a nondestructive phase-contrast picture of the partially filled $|1\rangle$ vortex

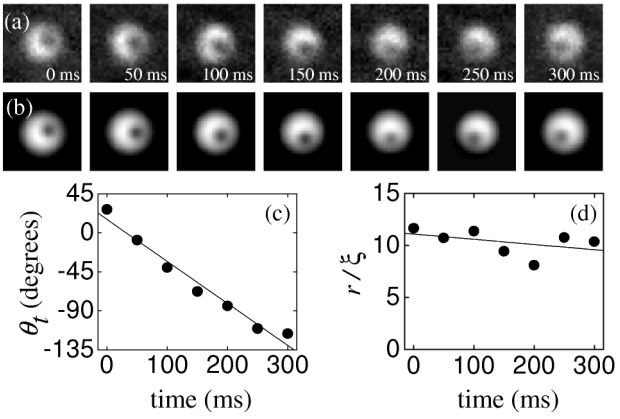

FIG. 1. (a) Seven successive images of a condensate with a vortex and (b) their corresponding fits. The 75 μm-square nondestructive images were taken at the times listed, referenced to the first image. The vortex core is visible as the dark region within the bright condensate image. (c) The azimuthal angle of the core is determined for each image, and is plotted vs time held in the trap. A linear fit to the data indicates a precession frequency of 1.3(1) Hz for this data set. (d) Core radius r in units of healing length ξ. The line shown is a linear fit to the data.

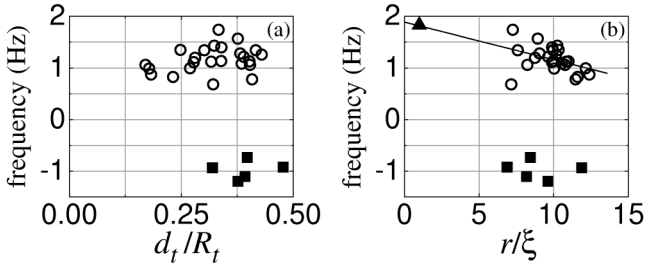

FIG. 2. Compiled data for filled vortex core precession, with each data point extracted from a series (as in Fig. 1) of nondestructive images of a single vortex. Precession frequency is plotted vs (a) core displacement d_t in units of condensate radius R_t, and (b) core radius r in units of healing length ξ. Circles correspond to positive frequencies and filled squares to negative frequencies. (Positive frequency is defined as core precession having the same handedness as the vortex angular momentum.) The triangle at $r = \xi$ shows for reference the average measured precession frequency of many bare vortices [see text and Fig. 3(b)]. A line is drawn as a guide to the trend in frequency vs core size.

distribution [Fig. 3(a) inset], as previously discussed, followed by complete removal of the core filling [19]. We then held the bare vortex in the trap for a variable hold time t_h, after which the condensate was released from the trap. We took a final near-resonance phase-contrast image [20] of the atomic distribution [Fig. 3(a)] after the condensate had ballistically expanded by a factor of ~3.5 [21] and the empty core had expanded [22] to a fit radius of ~9 μm.

Displacements d_t and angular positions θ_t of the cores for the in-trap images were extracted as described before. The images of the expanded clouds were fit with identical distributions, and the Thomas-Fermi radius R_e of the expanded cloud and the vortex core displacement d_e and angle θ_e were obtained for each image. For each pair of images, we determined the angular difference $\Delta\theta_{et} \equiv \theta_e - \theta_t$ between the cores in the expanded and in-trap images. We also determined the core displacement ratio d_e/d_t, an indicator of the radial motion of the core during the hold time t_h.

From the measurements of $\Delta\theta_{et}$ at different hold times t_h [Fig. 3(b)], we find a bare-core precession frequency of 1.8(1) Hz, slightly faster than the precession of filled cores and consistent with the trend shown in Fig. 2(b) for filled cores. To emphasize that our measurements of filled and empty cores are different limits in a continuum of filling material, we indicate the measured bare-core precession frequency in Fig. 2(b) with a point at $r = \xi$.

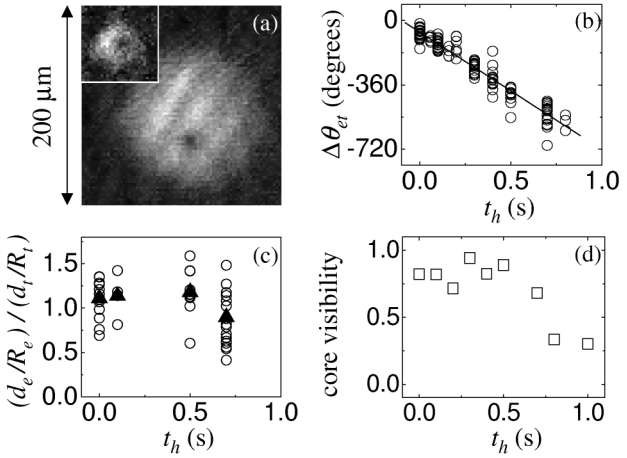

FIG. 3. (a) Ballistic expansion image of a vortex after all $|2\rangle$ atoms have been removed. The dark spot is the bare vortex core. Inset—the corresponding, preceding in-trap nondestructive image of the partially filled core. (b) Angular differences $\Delta\theta_{et}$ between vortex cores from the in-trap and expansion images, plotted against hold time t_h in the magnetic trap. The line is a fit through the data, indicating a bare-core precession frequency of 1.8(1) Hz. (c) Radial core motion is determined by $(d_e/R_e)/(d_t/R_t)$, the ratio of the fractional core displacements from the expansion and in-trap images of each data set. The data are shown as open circles, with the average of all data at each given hold time plotted as a filled triangle. (d) Core visibility of an expanded vortex, defined as the conditional probability for observing a vortex in an expanded image given the observation of a vortex in the corresponding, preexpansion in-trap image. Visibility drops dramatically for hold times $t_h > 1$ s.

From Fig. 2(b) it is apparent that the structure and content of the vortex core have a relatively modest effect on precession frequency. One can calculate, for instance, the fluid rotation rate ν_r at the inner core radius. The value of ν_r is given by the quantized azimuthal superfluid velocity evaluated at the radius of the core, divided by the circumferential length at that radius. For bare vortices, ν_r is about 260 Hz, while for the largest filled cores of Fig. 2(b) (for which nearly half of the sample mass is composed of core filling), ν_r is only about 1.7 Hz. Thus between vortices whose inner-radius fluid rotation rates vary by a factor of 150, we see only a factor of 2 difference in precession frequency.

The slower precession of filled cores can be understood in terms of our buoyancy picture. Because of its slightly smaller scattering length, $|2\rangle$ fluid has negative buoyancy with respect to $|1\rangle$ fluid, and consequently tends to sink inward towards the center of the condensate [23]. With increasing amounts of $|2\rangle$ material in the core, the inward force on the core begins to counteract the outward buoyancy of the vortex velocity field, resulting in a reduced precession velocity. It is predicted that with a filling material of sufficiently negative buoyancy in the core, the core precession may stop or even precess in a direction opposite to the direction of the fluid flow [16], but our data do not reach this regime.

Various theoretical techniques involving two- and three-dimensional numerical and analytical analyses have been explored to calculate the precession frequency of a vortex core within a condensate. We briefly compare those most readily applied to our physical parameters, assuming a spherical, single-component condensate with 3×10^5 atoms ($R_t = 22$ μm) in a nonrotating trap. Where relevant, we assume a core displacement of $d_t = 0.35R_t$. A two-dimensional hydrodynamic image vortex analysis [24] has been analytically explored in the homogeneous gas [11] and two-dimensional harmonic confinement [14] limits. The latter of these predicts a bare-core precession frequency of ~0.8 Hz. Svidzinsky and Fetter's three-dimensional [12] solution to the Gross-Pitaevskii equation predicts a precession frequency of 1.58 Hz. Jackson *et al.* [10] have obtained results in close agreement with this analytical solution using a numerical solution to the Gross-Pitaevskii equation. Finally, a path-integrals technique by Tempere and Devreese [13] predicts a 1.24 Hz precession, and a two-dimensional simulation by McGee and Holland [16] using a steepest-descents technique predicts a precession frequency of 1.2 Hz.

Measurements of d_e/d_t for different hold times t_h show the radial motion of the bare cores and are a probe of energy dissipation of the vortex states. The plot of Fig. 3(c) displays no trend of the core towards the condensate surface during t_h, indicating that thermal damping is negligible on the 1 s time scale [25]. However, we notice a

sharply decreasing visibility of expanded bare vortices for hold times greater than $t_h \gtrsim 1$ s, as indicated in Fig. 3(d). The absence of radial core motion suggests that decreased visibility is due to imaging limitations rather than true decay. One hypothesis is that the vortex core may be tilting away from the imaging axis [26], suppressing contrast in optical depth below our signal-to-noise threshold. Such a situation may arise if the trap is not perfectly spherical, and if the vortex is not aligned along a principle axis of the trap. Evidence in support of this hypothesis [27] will be presented in a future paper [28].

Through a combination of destructive and nondestructive imaging techniques we have obtained measurements of vortex dynamics in bare- and filled-core vortices in dilute-gas BEC. Vortex precession frequencies show only modest dependence on the radius and content of the vortex core. Further measurements of vortex dynamics in condensates may reveal the rate of loss of angular momentum at finite temperatures, an indication of energy dissipation. Such measurements may suggest interpretations for "persistence of current" in condensates, further strengthening ties between BEC and superfluidity. In order to pursue these goals, we plan to extend the studies reported here to investigate higher-order dynamical behavior and to characterize the dissipative effects of finite temperatures.

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*Quantum Physics Division, National Institute of Standards and Technology.

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[19] The core-filling material is removed slowly enough (100 ms) that the vortex core has time to shrink adiabatically to its final bare size, but rapidly enough that the initial location of the bare core is well correlated with the location of the original filled core.
[20] We use phase-contrast imaging for both the in-trap and expansion images to simplify imaging procedures. When imaging the expanded atom cloud, we obtain good signal when the probe is detuned three linewidths from resonance.
[21] Because of our low trapping frequencies (7.8 Hz), expansions are correspondingly slow. We get larger final spatial distributions by giving the atoms a 6 ms preliminary "squeeze"at high spring constant, followed by a 50 ms expansion period.
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