Experimental Evidence of Dynamical Localization and Delocalization in a Quasiperiodic Driven System

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This paper presents the first experimental evidence of the transition from dynamical localization to delocalization under the influence of a quasiperiodic driving on a quantum system. A quantum kicked rotator is realized by placing cold atoms in a pulsed, far-detuned, standing wave. If the standing wave is periodically pulsed, one observes the suppression of the classical chaotic diffusion, i.e., dynamical localization. If the standing wave is pulsed quasiperiodically, dynamical localization is observed or not, depending on the driving frequencies being commensurable or incommensurable. One can thus study the transition from the localized to the delocalized case as a function of the effective dimensionality of the system.

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Dynamical localization (DL) is a specifically quantum phenomenon taking place in time-periodic systems whose corresponding classical dynamics displays chaotic diffusion. While in the classical limit, because of the diffusion process, the system spreads indefinitely in the phase space, the quantum system follows the classical diffusive dynamics for a short time only but after some *localization time* freezes its evolution with no further increase of the average energy.

This behavior, attributed to quantum interferences among the diffusive paths which for long times are in the average completely destructive, was numerically observed at the end of the 1970's on the one-dimensional kicked rotator exposed to periodic kicks [1], a paradigmatic simple system whose classical dynamics can be reduced to iterations of the Chirikov's standard map.

The possibility of observing DL with a system constituted of cold atoms placed in a far-detuned standing wave was theoretically suggested in 1992 [2] and experimentally observed in 1994 [3]. A crucial question is whether DL is robust versus perturbations of the system. Indeed, as it strongly relies on quantum interferences, it is expected to be rather fragile. As a matter of fact, it has been experimentally shown that DL can be partly or totally destroyed by decoherence (i.e., coupling of the system to external degrees of freedom; in the present context spontaneous emission plays such a role) and noise, that is, deviation from strict periodicity [4,5].

Moreover, there is a relevant connection of DL with the Anderson localization taking place in disordered systems. Indeed, the periodically kicked rotator problem can be mapped on a one-dimensional Anderson model, that is a model of a particle moving along a one-dimensional chain of sites [6], with coupling between neighbors and diagonal disorder, i.e., pseudorandom values of the potential energies on each site. In 1D, although the classical motion is diffusive, the quantum eigenstates are all localized. Because of the similarity of the Anderson and kicked top models, Anderson localization and DL are similar quantum phenomena: the first one taking place along space coordinates, the second one along the time coordinate with localization in the momentum domain. It is well known that Anderson localization is strongly dependent on the number of spatial degrees of freedom. Similarly, DL is expected to be highly sensitive to the number of temporal degrees of freedom, that is on the frequency spectrum of the external driving.

We consider here the interesting simple case where the external driving is not periodic but quasiperiodic, with two independent frequencies. Theoretical arguments and numerical simulations [7] suggest that the situation is similar to the 2D Anderson model, and that the localization time should become so large that it might be impossible to observe DL experimentally. The goal of this paper is to study *experimentally* such a situation of increased dimensionality and to test the theoretical predictions.

We realized a quantum kicked rotator with a primary series of kicks of frequency f_1 to which a secondary series of kicks (frequency f_2) can be added, with $f_2/f_1 = r$. A physical experiment cannot be sensitive to the rational or irrational character of a number; one might thus consider only rational values of this frequency ratio. For a given rational value of $r = p/q$ (irreducible fraction), the periodicity of the system cannot have any physical effect before at least q kicks of the primary series. Thus, periodicity effects like DL cannot show up unless the number of primary kicks is large compared to q. As the number of primary kicks increases, one expects to find more and more "rational" values of p/q for which DL is effectively observed. Working with a two-frequency quantum rotator thus allows one to go from the 1D to the 2D case by choosing irreducible frequency ratios corresponding to larger and larger q [7]. Experimental results on the two-frequency microwave ionization of Rydberg atoms have indirectly shown the importance of rational or irrational values in quantum transport properties [8].

The atomic quantum kicked rotator is realized by placing cold atoms (cesium in our case) of mass M in a far-detuned, pulsed standing wave of intensity I_0, wave number k_L, and detuning Δ with respect to the closest atomic transition (the cesium $D2$ line at 852 nm). If the detuning is large enough, the dominant interaction between atoms and the laser light is the light potential which is proportional to the intensity. One then obtains a Hamiltonian of the form

$$H = \frac{p^2}{2M} - V_0 \cos(2k_L x) f(t), \tag{1}$$

where $f(t)$ is a function of period T, and $V_0 = \hbar\Omega^2/8\Delta$, where Ω is the resonant Rabi frequency. V_0 is proportional to the light intensity.

In the limit where the width of the peaks in $f(t)$ is negligible compared to T (i.e., each peak approaches a delta function), rescaling variables [2] allow one to reduce this Hamiltonian to the standard form corresponding to the quantum rotator:

$$\mathcal{H}_1 = \frac{P^2}{2} - K\cos\theta \sum_n \delta(\tau - n), \tag{2}$$

where K is the so-called *stochasticity parameter* and where the new conjugate variables obey the quantum commutation rule $[\theta, P] = i\hbar_{\mathrm{eff}}$ with $\hbar_{\mathrm{eff}} = 4\hbar k_L T/M$ the *effective Planck constant*. The classical limit ($\hbar_{\mathrm{eff}} \to 0$) of such a system becomes (weakly) chaotic for $K \approx 1$ and fully chaotic for $K \approx 10$. When a second series of pulses is applied, the reduced Hamiltonian becomes

$$\mathcal{H}_2 = \frac{P^2}{2} - \cos\theta\Big\{K_1 \sum_n \delta(\tau - n) + K_2 \sum_n \delta[\tau - (n + \phi/2\pi)/r]\Big\}, \tag{3}$$

with $K_1 = K_2$ in our experiment. In the above equation ϕ is the phase of the second series of pulses with respect to the first series. The classical dynamics of this system is essentially identical to the periodic kicked rotator: for $K_1 = K_2 \approx 10$, it is a chaotic diffusion.

Our realization of the kicked rotator (Fig. 1) is similar to that of Ref. [5]. Cold cesium atoms issued from a magneto-optical trap (MOT) are placed in a far-detuned, pulsed standing wave. The measurement of momentum distribution is accomplished in our setup by velocity-selective Raman stimulated transitions between the $F_g = 3$ and $F_g = 4$ hyperfine ground-state sublevels [9]. Generation of the Raman beams is based on direct current

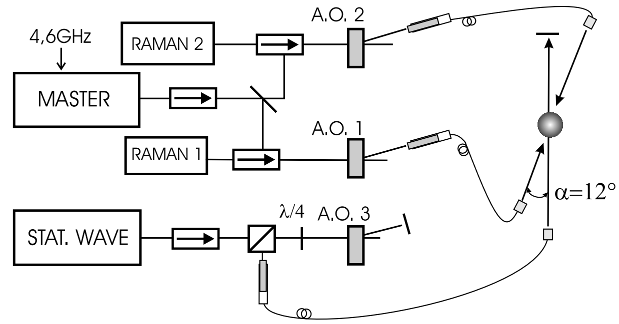

FIG. 1. Experimental setup. A master diode laser modulated at 4.6 GHz is used to inject two power slave Raman lasers producing phase-coherent, 9.2 GHz frequency-split, Raman beams. A power monomode diode laser is used to generate the stationary wave that can be pulsed through an acousto-optical modulator (mounted in double passing). Both the Raman and stationary wave beams are horizontal, making an angle of 12°.

modulation at 4.6 GHz of a diode laser, detuned by 200 GHz with respect to the atomic transition. The two symmetric first-order optical sidebands are then used to inject two diode lasers that produce 150 mW beams with a 9.2 GHz beat note of subhertz spectral width [10].

Cesium atoms are first optically pumped into the $F_g = 3$ hyperfine sublevel. A Raman pulse of detuning δ_R brings the atoms in the velocity class $v = \delta_R/(2k_R)$ (k_R is the wave number of the Raman beams) back to the $F_g = 4$ hyperfine sublevel. A probe beam resonant with the transition from the sublevel $F_g = 4$ is frequency modulated, and its absorption signal detected by a lock-in amplifier, yielding a signal proportional to the population of the $F_g = 4$ level.

Stray magnetic fields are harmful for the Raman velocity measurement. 3D magnetoresistive probes are placed at the eight corners of the MOT cell. Their signal is electronically interpolated and generates a feedback signal to three mutually orthogonal Helmholtz coil pairs [11]. We measured a residual magnetic field below 250 μG and an effective compensation bandwidth of 500 Hz. The $\hbar k_L/2$ momentum resolution then obtained is largely sufficient for this experiment and is much better than that obtained by time-of-flight methods [5].

A power diode laser is detuned by 7 GHz with respect to the cesium $D2$ line at 852 nm. An acousto-optical modulator is used to generate arbitrary series of pulses. The modulated beam is then transported by an optical fiber to the neighborhood of the MOT apparatus. The standing wave, obtained by backreflection of this beam, has a waist of 0.6 mm and a typical power of 50 mW in each direction. It is modulated with two series of pulses: the primary pulses of fixed frequency $f_1 = 36$ kHz are 500 ns long, corresponding to a stochasticity parameter $K_1 = 10$ and to an effective Planck constant $\hbar_{\mathrm{eff}} = 2.9$. The pulse shape is rectangular with a rise and a fall time of the order of 50 ns. The secondary pulses have the same duration and the same intensity, but their frequency f_2 and phase ϕ

can be adjusted at will. A typical experiment is done with 50 primary pulses. In order to avoid pulse superposition effects between the two series, the phase ϕ is fixed to an arbitrary nonzero value.

In an experimental run, cesium atoms are first cooled and trapped by the MOT. A Sisyphus-molasses phase further reduces the temperature to about 3.3 μK. The MOT beams and the magnetic field gradient are turned off and a pulse of a repumper beam transfers the atoms from the $F_g = 4$ to the $F_g = 3$ hyperfine sublevel. The standing wave is then turned on. When the standing wave excitation ends, the Raman sequence described above is used to detect the population of a velocity class. The whole sequence then starts over with a different value of the Raman detuning to probe a new velocity class. The pulse sequence is produced by two synthesizers at frequencies f_1 and f_2 with a fixed phase relation. We show in Fig. 2 the initial momentum distribution (just before the kicks are applied) and the final distributions (after interaction with the standing wave) for $f_2/f_1 = 1.000$ and $f_2/f_1 = 1.083$ and a phase of $\phi = 180°$. The initial distribution is a Gaussian with a typical full width at half maximum (FWHM) of $10\hbar k_L$. Both final distributions show a clear broadening with respect to the initial one. For the "resonant" case ($f_2/f_1 = 1$) [trace (b)], the distribution presents a characteristic exponential shape $P(p) \simeq \exp(-|p|/L)$, with a localization length (along the momentum axis) $L \approx 8.5\hbar k_L$, which is a signature of the dynamical localization. This is not surprising as for $f_1 = f_2$ the system is strictly time periodic and thus should present dynamical localization. The measured localization length agrees fairly well with theoretical estimates. Trace (c) corresponds to a nonresonant truly quasiperiodic case, where the ratio $f_2/f_1 = 1.083$ is sufficiently far from any simple rational number. The momentum distribution presents a broader and more complex shape. We have performed numerical simulations of the system, as described by Eq. (3): we have solved "exactly" the Schrödinger equation using a method similar to the one described in [12]. The resulting momentum distribution is averaged over the measured initial momentum distribution of the atoms and over the inhomogeneous laser intensity. We have used $K_1 = K_2 = 10$ at the center of the laser beam, in accordance with the value deduced from the laser power, detuning, and geometrical properties. The only adjustable parameter is the ratio of *effective* sizes of the standing wave and the Raman beams. Because of the nonlinearity of the processes, this ratio (which is 2) is different from ratio of the waists (4.8). For $f_1 = f_2 = 36$ kHz, we obtain a dynamically localized (exponential) distribution with a localization length which agrees with the experimentally observed one (at the 10% level). For $f_2/f_1 = 1.083$, the result of the simulation, shown in the figure, agrees very well with the experimental data.

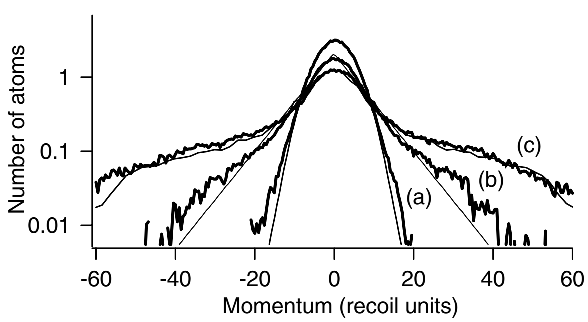

FIG. 2. Typical momentum distributions (in logarithmic scale) corresponding to curves (a) initial distribution produced by the MOT, corresponding to a temperature of 3.3 μK; the fitting curve (thin line) is a Gaussian. (b) Momentum distribution obtained after the interaction of the atoms with two series of kicks having $f_2 = f_1 = 36$ kHz and a relative phase $\phi = 180°$; it displays the exponential shape characteristic of dynamical localization for a time-periodic quantum system; the fitting curve is exponential (thin line). (c) Momentum distribution after interaction with two series of kicks having $f_2/f_1 = 1.0833$ and the initial relative phase $\phi = 180°$. The distribution is broader, indicating the destruction of dynamical localization in a quasiperiodic driven quantum system; the fitting curve is a numerical simulation (thin line); for details, see text. The recoil momentum is $\hbar k_L$.

The fact that the broad contribution is significantly larger than the resonant distribution—together with the fact that the classical diffusion constant is practically identical in the two cases—shows that diffusion has persisted during a longer time in the nonresonant case. Furthermore, the fact that the distribution is not exponential strongly suggests that we did not reach DL and that diffusion should persist for longer times. A simple and useful method to detect the presence of DL is to probe only the zero-velocity class: as DL corresponds to a thinner distribution, it also corresponds to a higher zero-velocity signal in the localized case than in the diffusive case. In other words, the zero-velocity signal contains essentially the same information as, e.g., the total average energy but is much easier to measure. This allows us to sweep the frequency f_2 of the secondary kick, keeping all other parameters (f_1, ϕ, K_1, and K_2) fixed, and search for the values of the frequency ratio presenting localization. The result is shown in Fig. 3. One clearly sees peaks at the simple rational values of $r = f_2/f_1$. Each peak is associated with an increased number of zero-velocity atoms, that is, an increased degree of localization. The most prominent peaks are associated with integer values of r, a rather natural result. Smaller peaks are associated with half-integer values of r, even smaller ones with $r = p/3$ rational numbers, etc. All these features are reproduced very well by the numerical simulation (performed as described above, with no adjustable parameter) shown in the inset of Fig. 3. The fact that the simulation displays exactly the same behavior proves that it is not due to an experimental artifact. Classical numerical calculations performed with the same parameters do not show any kind of localization, neither in the rational nor in the irrational case. The peaks are thus a purely quantum feature.

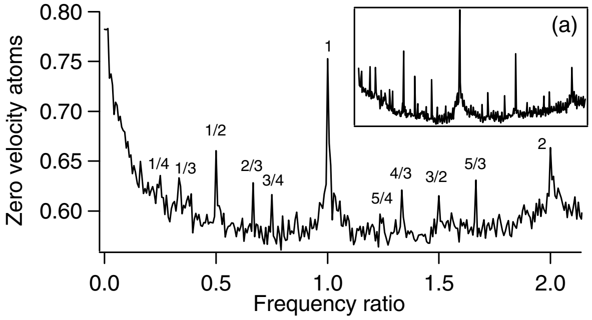

FIG. 3. The population of zero-velocity atoms (probed with the Raman signal) as a function of the frequency ratio $r = f_2/f_1$ (with $f_1 = 36$ kHz) and phase $\phi = 52°$. The increase of the zero-velocity signal is a signature of dynamical localization. Dynamical localization for commensurate frequencies, and simple rational r values, is clearly seen. For incommensurate frequencies, as in Fig. 2, no dynamical localization is visible. The inset (a) shows the corresponding curve obtained by numerical simulation (see text), reproducing very well the features of the experimental curve.

We have also checked that the observed behavior does not sensitively change when f_1 is varied. This rules out the possible role of the so-called quantum resonances where the dynamics is dominated by the quasidegeneracy between unperturbed Floquet eigenstates. The observed width of the 1:1 resonance is about 300 Hz, in good agreement with the numerical calculation. A detailed study of its width will be presented in the near future.

In conclusion, we have shown that, in the presence of a quasiperiodic driving with two base frequencies, the kicked rotator does not show any "short time" dynamical localization except when the ratio of the frequencies is close to a rational number. In the latter case, the system is time periodic and displays clear evidence of dynamical localization. This conclusion is drawn from experiments performed with both 50 and 100 primary kicks, whereas the localization time is of the order of 15 kicks. Longer kick sequences are impossible because of the free fall of the atoms under gravity, but numerical simulations show the same behavior up to few thousands kicks. Although it is currently impossible, experimentally or numerically, to decide if the DL is effectively suppressed by the secondary kicks or if it corresponds to a much longer localization time, the results presented here clearly evidence a dramatic change in the behavior of the system due to a secondary irrational frequency. Furthermore, the destruction of DL by a secondary frequency is found to be a very sensitive phenomenon.

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