Parabolic Plasma Sheath Potentials and their Implications for the Charge on Levitated Dust Particles

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(Received 23 March 2000)

Analysis of several numerical plasma sheath models, as well as data from several previously reported experiments, is shown to indicate that the sheath potential function may often be very closely approximated with a parabola. We also demonstrate that once this potential function is suitably determined the charge on isolated dust particles levitated in the plasma sheath may be calculated directly from their equilibrium heights.

PACS numbers: 52.25.Zb, 52.25.Vy, 52.40.Hf

Plasmas are commonly contaminated by dust. For example, ionospheric dust complicates interpretation of over-the-horizon radar returns [1], while dust from volcanic emissions and other sources forms rings around several planets in the solar system [2]. While exact yield numbers are closely held industrial secrets, it is known that dust contamination in semiconductor processing plasmas causes a large portion of the product to be discarded as waste [3]. Dust purposely grown in plasmas can also be used in a variety of positive ways. For example, the deposition of dust during the manufacture of nanostructured silicon can accelerate the crystallization process [4], lower the number of crystal defects [5], and lead to the production of more efficient solar cells [6]. The charge on dust suspended in these plasmas is a critical but not well-understood parameter for eliminating or efficiently using the dust.

The subject of the plasma sheath is also of wide interest. As the ions required for etching of semiconductors receive their energy from the fields within the sheath, knowledge of the spatial variation of these fields is essential for the intelligent design of manufacturing processes. Other disciplines requiring a detailed understanding of the plasma sheath range from basic astronomical research [7] to applications mitigating the effects of the charging of artificial satellites [8].

The purpose of this Letter is twofold. First, we discuss a very simple, highly accurate approximation of the potential function in the plasma sheath. This approximation is well supported by existing numerical and experimental results. It can be determined from simple experimental measurements, instead of requiring the time-consuming modification of a complex numerical model from the literature to fit experimental conditions. We then discuss a simple method for determining the magnitude of the charge on dust suspended in the plasma, based on our novel generalization concerning the nature of that potential function.

In terrestrial plasmas, dust typically settles not in the plasma itself, but in the plasma sheath, where a balance of forces exists between, primarily, gravity and electrostatic repulsion [9]. Calculation of the charge residing on dust particles suspended in the plasma sheath has been the subject of much recent research. Several experimental techniques have been used in attempts to understand this phenomenon better. The most prevalent technique uses resonant oscillations of the dust, driven by various methods, and employs various assumptions about the electrical nature of the sheath to calculate the charge [10–15]. However, many of these techniques significantly disturb the sheath, fail to account for significant intergrain forces, or both. Another technique involves observing collisions between dust particles, deriving the interaction potential, and thence the charge [16,17]. Analysis of the speed of sound in plasma crystals, dust that settles into stable, latticelike structures under specific plasma parameters, e.g., [18], has also led to another charge calculation proposal [19].

Much of the experimental work is motivated by the lack of a coherent theory of dust charging in the plasma sheath. At least two sheath-charging theories exist in the literature [9,20], but both rely on the discredited [21] orbital motion limited charging theory, e.g., [22]. Development of such a sheath-charging theory is highly desirable, as it would help to put this body of experimental work on stronger footing.

The majority of the cited experimental work relies on an *a priori* assumption that the electric field in the sheath varies linearly, or the electric potential varies parabolically, with the height above the electrode. The first purpose of this Letter is to demonstrate that experimental evidence and numerical simulations available in the literature strongly support this assumption. The second purpose of this Letter is to show that, once the parabolic nature of the electric potential has been suitably determined, there exists a relatively simple way of calculating the charge on dust suspended in the sheath.

Numerical support for the parabolic sheath potential has existed, unrecognized, in the literature for some time. Detailed analysis of several models has shown that, to an accuracy of better than 5% (often better than 1%) across the entire sheath, the potentials calculated by a number of accepted plasma/sheath and sheath models [9,23–29] may be modeled by simple parabolas. A novel model, similar to [9] and [29], was developed for this analysis. The chief improvements incorporated into this model were to

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the boundary conditions, where the log-cosine form of the plasma potential from [26] was matched to the numerical solution to allow a smooth ion velocity profile, and to the collisionality assumptions, where the new model allows for velocity-dependent variations in the ion-neutral collision cross section. Any further description of this model is beyond the scope of this Letter and is planned for a future publication. Suffice it to say that its results were in agreement with the parabolic generalizations noted above.

The models cited above represent a range of plasma parameters, from collisionless to transitional to fully collisional, and encompass both dc and rf plasma generation techniques; the parabolic nature of the sheath potential appears to be a general result for this wide variety of plasma parameters in many gases. Figure 1 shows the sheath potentials calculated for these models, their corresponding parabolic fits, and the small deviations of the fits from the numerical calculation. Although the definitions of the sheath edge vary from model to model, in general, the parabolic fits are extremely good (to better than 2%) across a very substantial fraction of the sheath.

Convincing experimental evidence for a parabolic sheath electric potential and the equivalent form of a linear electric field also exist. Analysis of the trajectories of isolated, micron-sized dust particles injected into the plasma sheath [31] showed them to be extremely well modeled by a damped harmonic oscillator for amplitudes approaching 20% of the sheath width. Figure 2 shows a typical dust particle trajectory, along with the fit to damped harmonic theory. Additional data and analysis have previously been presented in that study. The theoretical fit breaks down for larger amplitude oscillations, most probably due to the

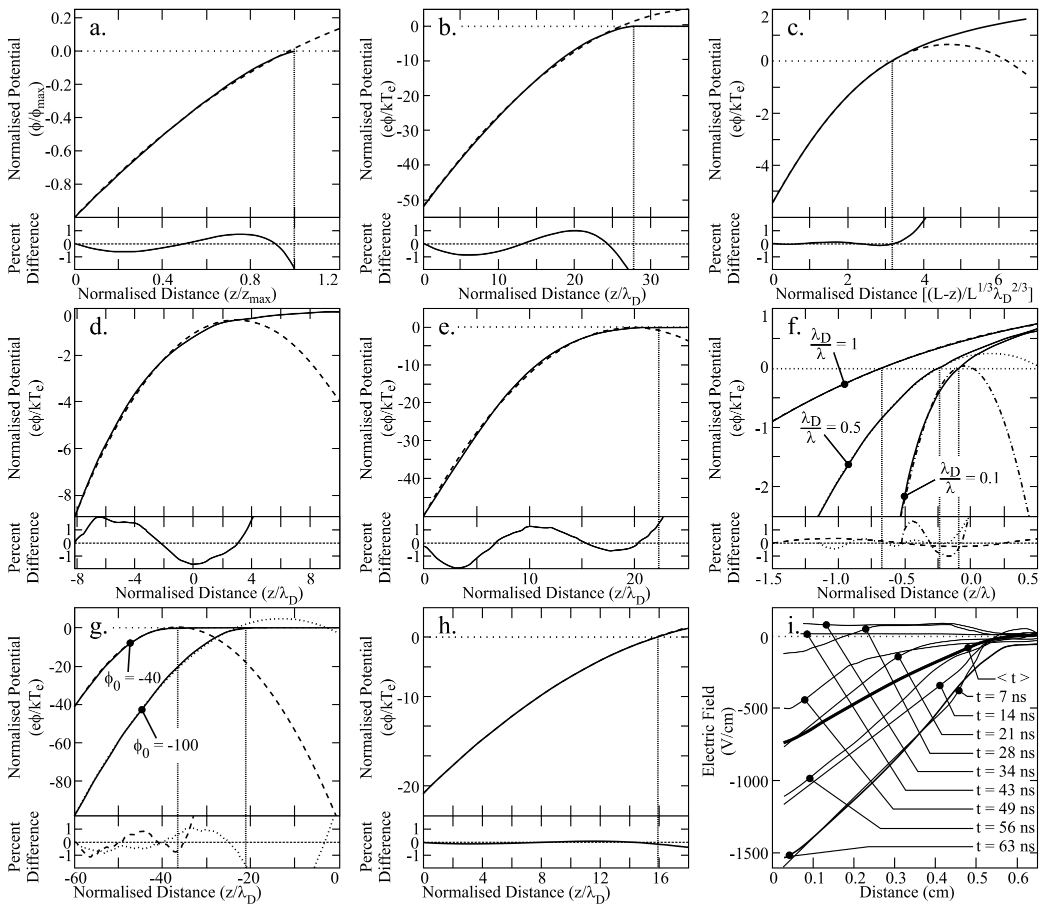

FIG. 1. Approximations to sheath models and an experiment from the literature. The upper portion of each frame shows the numerically derived sheath potentials from the indicated models (full curves; time-averaged potential for the rf models) and their parabolic approximations (dotted/dashed/dash-dotted curves). The lower portion shows the percent difference between the parabolic approximation and the numerical results. Where applicable, the sheath edge is indicated by a vertical dotted line. The models analyzed (and the applicable collisionality, generating voltage type, and gas) are as follows: (a) Child-Langmuir [23,24] [collisionless dc, no specific gas (nsg)]; (b) Bohm [25] (collisionless dc, nsg); (c) Blank [26] (fully collisional dc, nsg); (d) Franklin and Ockendon [27] (collisionless dc, nsg); (e) Nitter [9] (collisional rf, argon); (f) Riemann [28] (transitional dc, nsg); (g) Valentini [29] (collisionless rf, argon); (h) a model produced for the present analysis (transitional rf, argon); (i) experimental results from Czarnetzki *et al.* [30] (collisional rf, hydrogen; instantaneous curves from the cited reference; time-averaged plot and linear fits by the authors of this Letter. The linear correlation coefficient for the fit is 0.9990 for a fit from the left to 0.51 cm, increasing to 0.999 90 if the fit is limited to only 0.28 cm). Figure symbology—e: elementary charge; k: Boltzmann's constant; L: plasma scale length; T_e: electron temperature; z: linear distance; ϕ: potential; λ: ion-neutral mean free path; λ_D: Debye length; t: time; $\langle t \rangle$: time-averaged value.

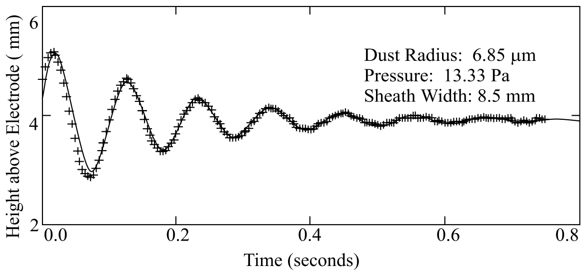

FIG. 2. Typical dust oscillation data [31]. The solid line is the damped harmonic theoretical curve fit to the data. Typical position measurement errors are on the order of 10 μm; these error bars are not visible at this scale.

increasing significance of charge fluctuation on the dust particle as it is exposed to different local potentials during its oscillations; however, the fits are extremely good over much larger amplitudes than would be expected from the standard small-amplitude approximation. It is known that sheath behavior in rf discharges is different in some molecular gases, e.g., hydrogen [32]. Our analysis of relevant measurements show, however, that the time-averaged electric field is still highly linear over very large portions of the sheath [30], as shown in the final frame of Fig. 1.

These data show that the dust must oscillate in a parabolic potential energy well. The only significant forces are gravity and electrostatic repulsion [31], so the electrical potential energy is the difference between the total potential and the gravitational potential. As the total potential energy has been shown to be parabolic and the gravitational potential energy is linear, the electrical potential energy—and hence, the electric potential—must be parabolic as well. Thus, the experimental and numerical evidence both strongly support the assumption that the sheath electric potential is a parabolic function of height above the electrode.

Now that the general parabolic nature of the sheath potential has been demonstrated, it remains to determine the boundary conditions for the parabola in order to get specific sheath potential functions. The boundary conditions we feel to be convenient were the spatial location of the sheath edge, potential difference between the sheath edge and electrode, as well as the electric field at the sheath edge. The sheath edge location may be determined either visually or via probe measurements, the sheath-edge-to-electrode potential difference may be determined by probe measurements, and the sheath edge electric field may be calculated from one of several proposed in the literature [26,28]. These boundary conditions together with the assumed parabolic nature of the sheath serve to specify the sheath potential function.

The second purpose of this Letter is to show that, once the parabolic nature of the electric potential has been suitably determined, there exists a relatively simple way of calculating the charge on isolated dust suspended in the sheath. To do this one simply measures the mass of the dust and the height at which it suspends and then employs the force-balance equation (gravitational force equals electrostatic force) at the dust equilibrium suspension height,

$$Q_d(r_d) = \frac{m_d(r_d)g}{E(z_{\mathrm{eq}}(r_d))}. \qquad (1)$$

Here, Q_d is the equilibrium dust charge, r_d is the dust radius, $E(z)$ is the electric field, z_{eq} is the equilibrium suspension height, m_d is the dust mass, and g is the acceleration due to gravity. As the electric field is just the negative derivative of the sheath potential, the value of the dust charge may readily be obtained.

The experimental results of [31], which show the dust charge as a function of dust radius determined by analysis of damped oscillations, may be compared with the force-balance method discussed above. These experiments were performed at two pressures: 6.67 Pa ($kT_e = 3.7$ eV and $n_0 = 1.7 \times 10^{15}$ m^{-3}, $z_{\mathrm{sh}} = 9.4$ mm) and 13.33 Pa ($kT_e = 3.9$ eV and $n_0 = 2.4 \times 10^{15}$ m^{-3}, $z_{\mathrm{sh}} = 8.5$ mm). Here, kT_e and n_0 are the electron temperature and electron density of the center of the discharge determined by Langmuir probe measurements and z_{sh} is the visually determined sheath width. In all cases, the dust was monodispersive melamine formaldehyde ($\rho = 1514$ kg/m^3) suspended in an argon plasma, the electrode spacing was 40.0 mm, the rf frequency was 13.5 MHz, and the rf amplitude was 96.4 V. The derivation of the parabolic sheath functions from these experimental conditions is presented in detail in the cited reference. It must be noted that the boundary conditions for the electric field at the sheath edge, E_s, used in [31] were merely the ones found to be the most convincing at the time that paper was written (i.e., $E_s = kT_e/e\lambda$, where e is the elementary charge, and λ is the ion mean free path); should better boundary conditions become available, they may be used with existing equilibrium height data to calculate improved values for the dust charge.

A plot of the dust charge calculated from Eq. (1) is shown in Fig. 3. The solid trend line was calculated using the same equation and a fit to the experimental equilibrium heights. Also shown are data from the damped oscillation method of [31]. It is important to note that the charge results are not entirely independent of theory; E comes from the parabolic sheath model justified above. As can be seen in this figure, simple measurement of the equilibrium heights of the dust particles can provide charge values in good agreement with previous measurements and theories. The inherent simplicity of this new method, employing only one dust trajectory parameter and one derivative of the assumed potential, naturally results in smaller measurement errors than previous methods. We also note that the force-balance charge curves from Eq. (1) are remarkably similar for the two different pressures, supporting the limited dependency of the dust charge on pressure noted by other researchers [11]. The charges predicted by these curves also agree well with those from other methods [10–19], considering differences in experimental parameters.

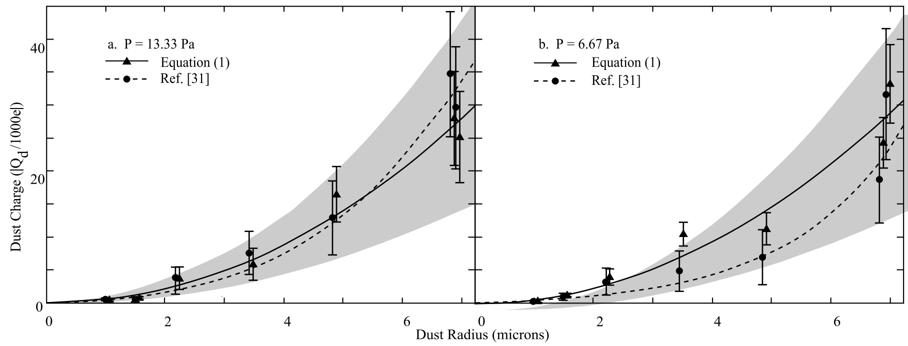

FIG. 3. Variation of the dust charge as a function of dust radius. This figure shows plots of normalized dust charge for (a) 13.33 Pa and (b) 6.67 Pa. The dashed lines with circles represent the charge curve from the method of [31]. The solid line with triangles is a plot of the charge from Eq. (1), as discussed in the text. Shading indicates the region between 1.5 and 0.5 times the fit line from Eq. (1). The mean values for the ratios of the charges obtained from these two methods are 1.1 and 1.5 for 13.33 Pa and 6.67 Pa, respectively, indicating their good agreement. The data points have been offset from each other for clarity; the actual horizontal positions are halfway between the corresponding vertical error bars. These plots are for melamine formaldehyde suspended in an argon plasma.

In summary, experimental and numerical evidence both strongly support the existence of a parabolic electric potential profile in the plasma sheath in many cases. Boundary conditions specifying the exact sheath potential function are readily available from experimental measurements and theory from the literature. Once the potential function is known, the charge on isolated dust particles may readily be calculated from knowledge of the dust mass and equilibrium suspension height.

The authors express their appreciation to the United Kingdom Engineering and Physical Sciences Research Council (EPSRC) for their financial and material support of this project and to S. C. Dudley for helpful comments. One of us (E. B. T.) also thanks the United States Air Force for supporting his research.

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