Observation of Resonance Trapping in an Open Microwave Cavity

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The coupling of a quantum mechanical system to open decay channels has been theoretically studied in numerous works, mainly in the context of nuclear physics but also in atomic, molecular, and mesoscopic physics. Theory predicts that with increasing coupling strength to the channels the resonance widths of all states should first increase but finally decrease again for most of the states. In this Letter, the first direct experimental verification of this effect, known as resonance trapping, is presented. In the experiment a microwave Sinai cavity with an attached waveguide with variable slit width was used.

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For more than ten years, interference phenomena in open quantum systems have been studied theoretically in the framework of different models. Common to all these studies is the appearance of different time scales as soon as the resonance states start to overlap (see [1] and the recent papers [2] with references therein). Some of the states align with the decay channels and become short lived, while the remaining ones decouple a great deal from the continuum and become long lived (trapped). Because of this phenomenon, the number of relevant states will, in the short-time scale, be reduced while the system as a whole becomes dynamically stabilized. The phenomenologically introduced doorway states in nuclear physics provide an example for the alignment of the short-lived states with the channels [3]. Calculations for microwave resonators showed that the trapped resonance states can be identified in the time-delay function and that short-lived collective modes are formed at large openings of the resonator [4]. Resonance narrowing is inherent also in the Fano formalism [5]. Similar effects have been found in the linewidths in a semiconductor microcavity with variable strength of normal-mode coupling [6]. In spite of the many theoretical studies, the effect of resonance trapping has not yet been verified unambigously in an experiment. For a clear experimental demonstration of the trapping effect, the coupling strength to the decay channels should be tunable, which was not possible in all above mentioned experiments.

The mechanism of resonance trapping can be illustrated best on the basis of a schematical model. In an open quantum system the resonance states are allowed to decay. The Hamilton operator is non-Hermitian,

$$\mathcal{H} = \mathcal{H}_0 - i\alpha VV^\dagger. \tag{1}$$

Here $\mathcal{H}_0$ describes the N discrete states of the closed quantum system, which is coupled to K decay channels by the $N \times K$ matrix V. $\mathcal{H}_0$ and $VV^\dagger$ are Hermitian and α is a real parameter for the total coupling strength between the closed system and the channels. The complex eigenvalues $\mathcal{E}_R = E_R - \frac{i}{2}\Gamma_R$ of $\mathcal{H}$ give the energy positions E_R and widths Γ_R of the resonance states. Studies on the basis of this model were performed by different groups with different assumptions on the properties of $\mathcal{H}_0$ and V; see, e.g., [7–9]. The Hamiltonian $\mathcal{H}_0$ and the elements of V can be given by different statistical ensembles describing chaotic or regular motion. In all cases, the results show clearly the formation of different time scales due to the anti-Hermitian part of $\mathcal{H}$. At a certain critical value of the coupling parameter α, the widths of $N - K$ states start to decrease with increasing α and approach zero with $\alpha \to \infty$ while the widths of K states increase.

The widths of the states of realistic systems show a more complicated behavior than described in the statistical model. The widths of the long-lived states of molecules saturate with increasing coupling strength to the continuum, but do not approach zero [10]. A saturation of the widths of the trapped states occurs, however, also in the calculations with the schematical model when it is improved by considering different coupling strengths for the different decay channels [11]. This improvement, as well as the introduction of a complex coupling parameter instead of the real α, are justified since they follow from formally rewriting the Schrödinger equation in the function space of both discrete *and* scattering states [12]. Similar results are obtained for nuclei [12] and for open microwave cavities [4]. On the basis of this result it is possible to clarify another problem discussed for resonance states in molecules at high level density [10]: The fundamental quantum mechanical relation between the average width of the states and the average lifetime is violated when all resonance states are considered in the averaging procedure. The mechanism of resonance trapping makes, however, the averaging procedure meaningful only for either the long-lived or the short-lived states. Performing the average over the trapped states only, the relation between the average width and the average lifetime is recovered.

While $\Gamma_R \to 0$ with $\alpha \to \infty$ is a necessary condition for trapped states in the schematical model with the Hamiltonian (1), the calculations for realistic systems show that the

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widths of the trapped states may decrease, stop to increase, and even slowly increase with increasing coupling strength to the continuum. This behavior may lead to such an interesting effect as population trapping resulting from the coupling of two states of an atom by means of a strong laser field [13]. The interplay between the direct interaction of two states and the real and imaginary parts of their interaction via the continuum causes unexpected and sometimes contraintuitive effects. In any case, the local processes between individual resonances are of central importance for the properties of the system.

Let us sketch the main ingredients of the trapping mechanism on the basis of calculations for two neighbored resonances. For $N = 2$, $K = 1$, and α replaced by the complex value $\alpha e^{i\beta}$, the Hamiltonian (1) reads

$$\mathcal{H}_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} - 2i\alpha e^{i\beta}\begin{pmatrix} \cos^2\varphi & \cos\varphi\sin\varphi \\ \cos\varphi\sin\varphi & \sin^2\varphi \end{pmatrix}, \tag{2}$$

where ϕ characterizes the relative coupling strength. In Fig. 1, the widths $\Gamma_R/2$ and eigenvalues of the two resonances are shown as a function of α for $\beta = \pi/18$ and $\varphi = \pi/5$. For small α, the widths of both states increase with α. Thereafter the two states attract each other in energy and their widths bifurcate: The width of one state starts to decrease with increasing α while the width of the other one increases more strongly. At still larger α, the broad resonance gets shifted towards lower energies due to $\beta \neq 0$.

The goal of this paper is to present an experimental verification of the effect of resonance trapping in a microwave cavity by considering the local interactions between individual resonances. The coupling of the discrete states of the cavity to an attached waveguide makes the system open.

The measuring technique is described elsewhere [14]. Here we note only that for flat cavities the electromagnetic spectrum is equivalent to the quantum mechanical spectrum of the corresponding system, as long as one does not surpass the frequency $\nu_{\max} = c/2h$, where h is the height of the cavity. The quantum mechanical energy E corresponds to the square of the wave number k. For the measurement we used a quartered Sinai billiard with an attached waveguide (see Fig. 2). The actual form of the cavity is of little importance since resonance trapping is expected to take place in chaotic as well as in regular billiards [4].

The frequency range of the waveguide is 8.2 to 12.5 GHz (i.e., E is between 2.95 and 6.85 cm^{-2}). Between the two thresholds at $E = 1.83\ \text{cm}^{-2}$ and 7.33 cm^{-2} only one mode can propagate through the waveguide. The microwaves are coupled into the system through an antenna at the end of the waveguide. They enter the billiard through a slit, the opening of which can be varied. The coupling matrix elements V, Eq. (1), are related to the width of the opening. The exact expression for this relation is not known for our situation with a limited number of resonances. We performed 102 measurements of the complex reflection coefficient $R(E)$ for different openings d from 3 to 23.2 mm in steps of 0.2 mm. Because of the finite length of the waveguide with reflecting end there are also broad channel resonances in the measured $R(E)$. This, together with the fact that a part of the incoming flux gets absorbed in the walls of the resonator, complicates the data analysis somewhat.

In Fig. 3a, a part of the measured spectrum $|R|$ for $d =$ 14 mm is shown. Because of the wall absorption, $|R| \leq 1$ and the resonances show up as dips in $|R|$. Around $E =$ 5.75 cm^{-2} a pair of closely lying resonances can be seen. It will be evident later (see Fig. 4) that resonance trapping takes place between these two resonances.

The method used in this Letter to obtain the energies E_R and widths Γ_R of the resonances is as follows: In Fig. 3b we plot the Argand diagram [real and imaginary part of $R(E)$ [15]] for $d = 14$ mm and 5.54 $\text{cm}^{-2} \leq E \leq 5.77\ \text{cm}^{-2}$. The narrow resonances show up as small circles superimposed on larger circles caused by the broader structures. In order to analyze the Argand diagram, we propose a centered time-delay analysis (CTDA). From the measured points in a small region around an energy E, we define a local circle segment with center point $C(E)$. We define the angle $\theta^c(E)$ as the angle of the complex number $R(E) - C(E)$. The point C

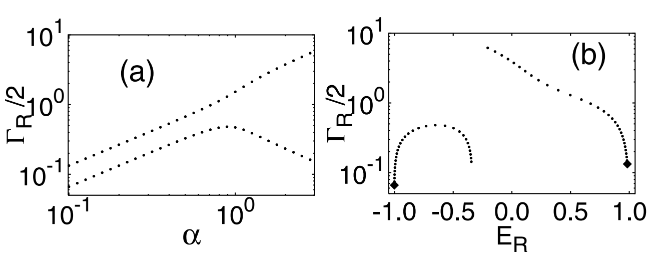

FIG. 1. Energy E_R and width $\Gamma_R/2$ for two resonances in the schematical model Eq. (2). (a) $\Gamma_R/2$ versus the coupling strength α. (b) Trajectories of $E_R - i\Gamma_R/2$ as a function of α. The latter plot is called the eigenvalue picture.

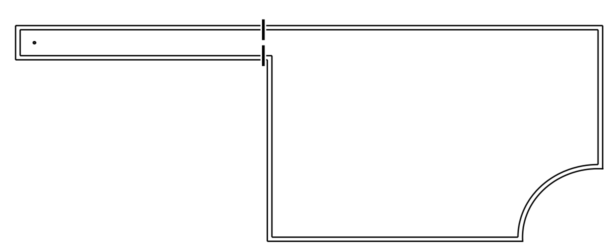

FIG. 2. Layout of the cavity (to scale): Quartered Sinai billiard (285 × 200 mm, radius 70 mm) with an attached waveguide (220 × 23.2 mm). The opening between waveguide and billiard can be varied in steps of 0.1 mm.

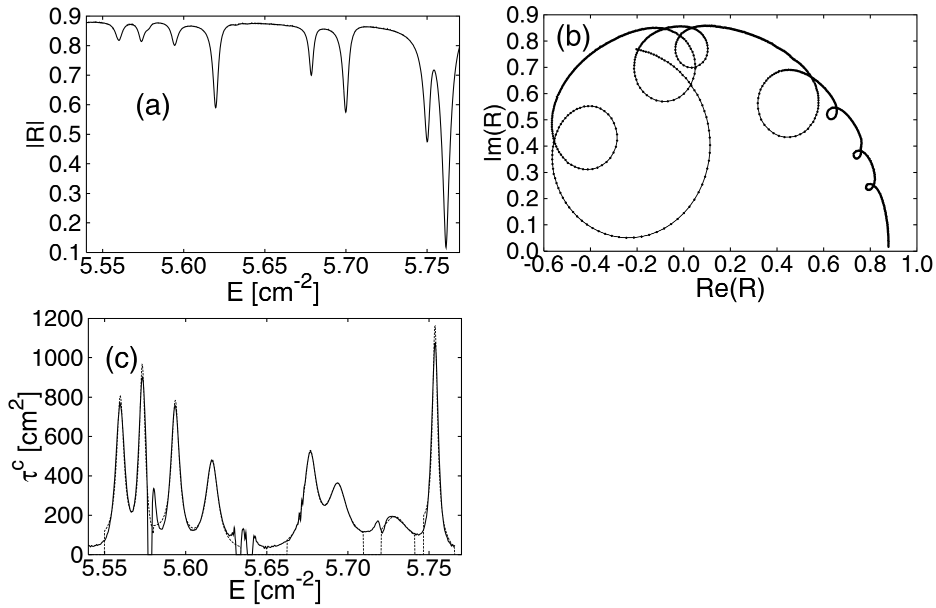

FIG. 3. (a) Measured $|R|$ as a function of $E = k^2$. (b) Complex R for $5.54 \leq E \leq 5.77$ cm^{-2}. (c) τ^c obtained directly from the measurement (full line) and the fitted τ^c (dashed line). In all three plots, the opening of the slit is $d = 14$ mm.

moves with E in such a way that $\theta^c(E)$ increases by 2π in the neighborhood of each resonance, even if the corresponding circle does not go around the origin. We define the corresponding "time delay" as $\tau^c(E) = d\theta^c(E)/dE$. For an isolated Breit-Wigner shaped resonance we have

$$\tau_w^R(E) = \Gamma_R/[(E - E_R)^2 + \Gamma_R^2/4]. \quad (3)$$

In Fig. 3c, the τ^c obtained directly from the measurement and the τ^c obtained from a fit to $\sum_R \tau_w^R(E)$, Eq. (3), are shown for $d = 14$ mm. The fitted curve agrees well with the measured values.

The usual fit of the complex $R(E)$ to a sum of Lorentzians was not possible because of the broad structures in $R(E)$ caused by the channel resonances. By means of the CTDA, however, any broad structures are automatically removed. It was also possible to do an automatic evaluation of the measured data, which was mandatory in view of the large amount of data. To test the CTDA method we performed an analysis of spectra created theoretically from $\mathcal{H}$, Eq. (1), with the wall absorption simulated by adding a constant to the diagonal of the imaginary part of $\mathcal{H}$. The comparison between the theoretical values and those extracted from the $R(E)$ showed a good agreement. A more detailed study of the CTDA method together with a comparison to possible other methods, e.g., the filter diagonalization method [16], will be published elsewhere.

The E_R and $\Gamma_R/2$ for four resonances obtained from the measured data with the CTDA method are presented in Fig. 4. The motion of the E_R and $\Gamma_R/2$ with d is clearly observable. At least three states start to decrease in width with increasing d; i.e., there are some evident cases of resonance trapping. Note that for each opening d of the slit the data have been fitted independently. The smoothness of the curves is thus a measure for the reliability by which the E_R and Γ_R are extracted from the measured data. Figure 4 even shows hierarchical trapping: One state first traps its

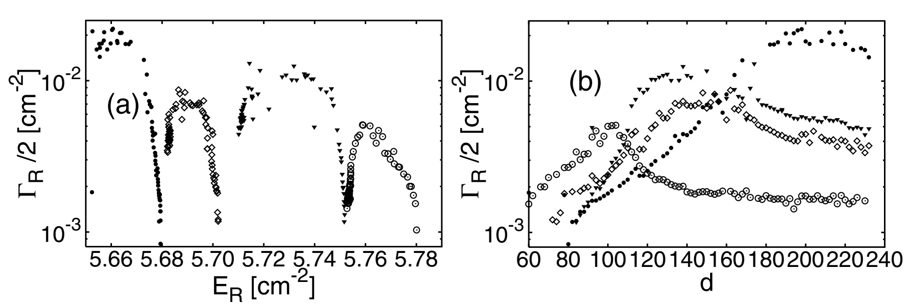

FIG. 4. Eigenvalue picture for four resonances (a) and the corresponding $\Gamma_R/2$ versus d (b). The different resonances are marked by different symbols. The widths of the resonances first increase as a function of the opening d of the slit but finally they decrease again. This demonstrates the effect of resonance trapping.

closest neighbor only to be trapped at stronger coupling by another state.

We followed the motion of all the 127 resonance poles $E_R - i/2\ \Gamma_R$ in the complex plane as a function of increasing d. The widths of 47 resonances decrease with d while those of 11 resonances stop to increase (within the accuracy of the CTDA). These two groups can unambigously be identified as trapped resonances, i.e., at least 46% of the resonances get trapped. For 16 of the resonances increasing further in width, it was possible to identify other resonances getting trapped by them. By this they acquired an extra contribution to their widths. In total, we have found that at least 58% of the resonances are strongly affected by trapping. These resonances are in the whole energy interval considered.

The estimation 58% of the resonances getting affected by resonance trapping is the lowest limit. According to theoretical studies of the diagonal elements of the effective Hamilton operator Eq. (1) (i.e., without taking resonance trapping into account) and studies for isolated resonances [4], one would expect a more or less uniform increase in width of all resonances. The experiment shows, however, that the widths of the remaining 42% increase nonuniformly in d. This indicates that some of these resonances gain (or lose) width at the cost (or in favor) of other ones. Thus practically all of the resonances are affected by the mechanism of resonance trapping.

In conclusion, we have demonstrated experimentally that resonance trapping takes place in a microwave cavity coupled to a waveguide. This is, to the best of our knowledge, the first unambigous experimental verification of the effect. This demonstration was possible by tracing the motion of the resonance poles as a function of the opening of the slit starting close to the real axis and following them into the region of overlapping resonances. Our experimental proof of resonance trapping does not depend on any model assumptions.

Theoretical studies have shown that the phenomenon of resonance trapping depends on neither the number of resonances nor the special shape of the microwave cavity. The effect appears also in various open many-body quantum systems. In any case, the decoupling of some states from the decay channels takes place. However, the experimental verification of resonance trapping is important not only for an understanding of the properties of open quantum systems with overlapping resonances. More important is, maybe, the necessity of a good knowledge of the detailed properties of open quantum systems for the design of mesoscopic systems. By tuning the coupling strength to the decay channels, the properties of the system can be controlled. Further experimental and theoretical studies of this interesting topic, including the conductance, have to be performed.

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