Very High Q Measurements on a Fused Silica Monolithic Pendulum for Use in Enhanced Gravity Wave Detectors

G. Cagnoli,[2,3] L. Gammaitoni,[1,2,*] J. Hough,[3,†] J. Kovalik,[1,‡] S. McIntosh,[3] M. Punturo,[1] and S. Rowan[3,4]
[1]Istituto Nazionale di Fisica Nucleare, Sezione di Perugia, VIRGO Project, I-06100 Perugia, Italy
[2]Dipartimento di Fisica, Universitá di Perugia, I-06100 Perugia, Italy
[3]GEO600 Project, Department of Physics and Astronomy, University of Glasgow, G12 8QQ Glasgow, United Kingdom
[4]Department of Applied Physics, Ginzton Laboratory, Stanford University, Stanford, California 94305-4085
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We present for the first time the results of very high Q factor measurements for a 2.8 kg fused silica mass suspended by two fused quartz fibers attached by a novel technique for joining fused silica or quartz. The Q for the pendulum mode at 0.93 Hz was $(2.3 \pm 0.2) \times 10^7$, the highest value demonstrated to date for a mass of this size. By employing such a new suspension system the sensitivity of the gravitational wave detectors currently under construction can be increased up to 1 order of magnitude.

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A gravitational wave interferometer, such as GEO600 [1], LIGO [2], TAMA [3], or VIRGO [4], must detect very small displacements of free test masses which are hung as mechanical pendulums. Once seismic noise is filtered, the dominant noise source between 10 and 50 Hz (a region where a number of candidate sources is expected) is likely to be the thermal noise of the pendulum motion. One way to reduce this noise source is to construct the pendulum with as high a mechanical Q as possible in order to concentrate the thermal motion in a narrow band around the pendulum resonant frequency.

In designing a high Q pendulum, one must take into account the ratio of energy stored in the pendulum to the energy dissipated in one cycle of the pendulum oscillation. Since most of the energy is stored in the gravitational field which is lossless, while all of the energy dissipated is in the wires, the highest pendulum, Q_p, occurs when the wires are as thin as possible. The quality factor Q_p of an individual pendulum depends upon the internal friction of the suspension wires and for a pendulum suspended by two wires (or a single loop of wire) is given by the formula [5]

$$Q_p = l\sqrt{\frac{2Mg}{EI}}\,\frac{1}{\phi_w}, \qquad (1)$$

where l is the length of the pendulum wires, M is the suspended mass, E is the Young's modulus of the suspension wire material, $I = \pi d^4/64$ is the moment of inertia of the cross section of a cylindrical wire with diameter d, and the *loss angle* ϕ_w accounts for the internal friction of the specific material.

Thus, the optimum wire is one that is made from a material with high breaking stress and low internal friction. The loss angle for different wire materials has already been measured, and both fused silica (synthetic fused silicon dioxide) and fused quartz (natural fused silicon dioxide) show inherent losses that are about 2 orders of magnitude lower than metals [6–9]. For these reasons, fused silica suspensions are currently under investigation for constructing high sensitivity gravity wave interferometers.

In constructing a fused silica suspension system a number of technical difficulties have to be taken care of. One of the most relevant problems is related to the dependence of the breaking strength on the surface properties of the silica sample. Some studies [10,11] suggest that if the surface of a thin fused silica/quartz fiber is damage-free, it can have a breaking stress better than that of carbon steel. Another relevant difficulty arises from the clamping technique. It has been shown [12] that the best way to reduce clamp losses is to use a high clamping torque. Such a technique, which proved successful with carbon steel wires, cannot be applied in a satisfactory way to fused silica wires due to their low resistance to mechanical squeezing. In order to overcome such difficulty we studied a new design based on a new technique—hydroxide catalysis bonding—for joining together fused silica or quartz elements [13]. The pendulum mass was a cylindrical piece of fused silica 63.5 mm radius and 101.6 mm thick with two flats of width 25 mm polished along the length of its cylindrical sides. The flats were on opposite sides of a diameter of the mass (see Figs. 1–3). In order to create an attachment point for the suspension fibers, an "ear" of fused quartz was fixed on each flat as shown in Fig. 3 using the technique of hydroxide-catalysis bonding.

The top hanging support of the pendulum was a fused quartz cylinder (ring) 135 mm in diameter with a wall thickness of 5 mm that was clamped to a large steel structure by an aluminum plate. Two small fused quartz (synthetic silicon dioxide) cylindrical rods of diameter 5 mm and length 10 mm were then welded onto this ring using an oxyhydrogen torch.

The fused quartz fibers were pulled from 5 mm rod stock in an oxyhydrogen flame. The diameter of the fibers was approximately 430 μm with a length of 300 mm. The tops of the fibers had approximately 5 mm of the original rod left on them and these were welded to the cylindrical rods on the top support ring. The rods on the bottoms of the

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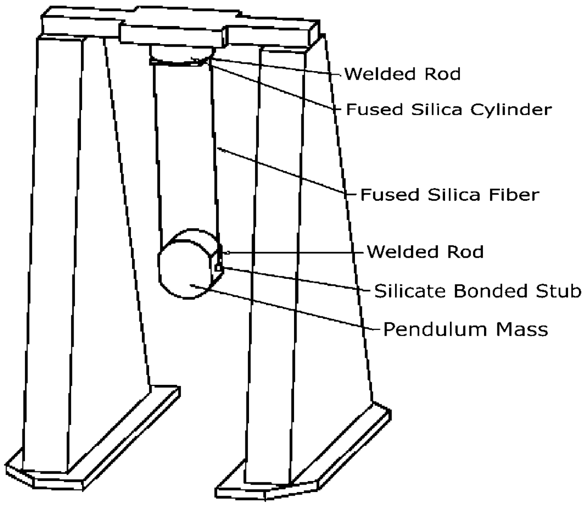

FIG. 1. Sketch of experimental setup.

fibers were cut off leaving a section of expanding diameter, of approximately 2 mm length in each case, and these "necks" were welded to the ears on the pendulum mass.

Previous results indicate that the points where the fibers are connected both at the top support and at the contact with the mass must be solid and as lossless as possible [12–17]. In this case welds were used to fuse the quartz of the expanded fiber ends with the quartz of the support pieces. One part that was untested was the loss characteristics at the pendulum frequency of the hydroxide catalysis bonds used to attach the fused quartz ears to the pendulum mass.

There are several extrinsic loss mechanisms that may affect the measurements of the quality factor of a pendulum. These include residual gas damping and the recoil losses of the structure that supports the pendulum. The limiting Q of the pendulum set by the presence of residual gas in the system may be calculated from [18]

$$Q_{\text{gas}} = \frac{4M\omega_0}{\pi r_c^2 P_T}\sqrt{\frac{\pi k_b T}{8\mu}}, \qquad (2)$$

where M is the mass of the pendulum, r_c is the radius of the cylindrical test mass face, P_T is the residual gas pressure, k_b is Boltzmann's constant, T is temperature, and μ is the mass of a molecule of the residual gas in the measurement system. The actual experiment was carried out in a vacuum of 5×10^{-6} torr which, assuming a residual gas of predominantly nitrogen, would give a limiting Q to the pendulum under test of 1.4×10^9. The pendulum was hung from a stiff steel "A" frame structure that is designed to have low recoil at the pendulum resonance frequency of 0.93 Hz. The limiting Q due to the recoil losses can be shown to be given by

$$Q_r = \frac{kl}{Mg\theta}, \qquad (3)$$

where k is the spring constant of the structure (at the pendulum frequency) and θ is the phase angle between the force applied to the structure and its displacement. The measured values for these parameters are $k = (5.0 \pm 0.1) \times 10^7$ N/m and $\theta = 0.25° \pm 0.01°$. This sets an upper limit to the measurable Q (for a 2.8 kg mass and 300 mm length) of $Q_r = (1.2 \pm 0.3) \times 10^8$.

The motion of the pendulum was detected by using a light emitting diode to cast the shadow of one of the suspension fibers on a split photodiode. The differential signal from the photodiode was amplified and filtered.

To allow excitation of the pendulum mode, a conducting plate was placed next to the pendulum mass and an offset voltage of typically a kilovolt applied to the plate. Then, the amplified signal from the photodiode was phase shifted and added to the high voltage on the exciting plate. This produced a positive feedback that excited the pendulum

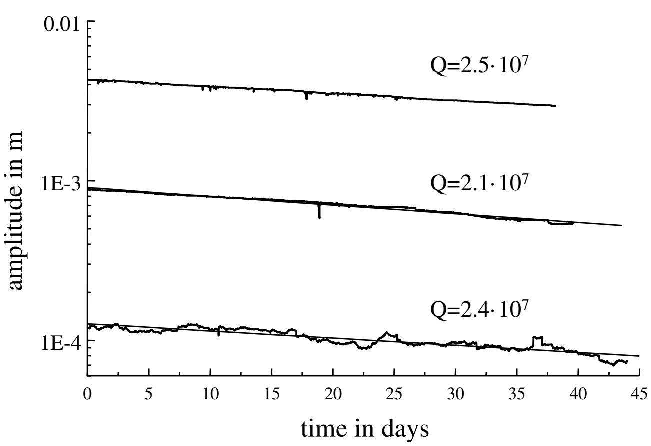

FIG. 2. Amplitude of pendulum motion vs time for three different amplitudes.

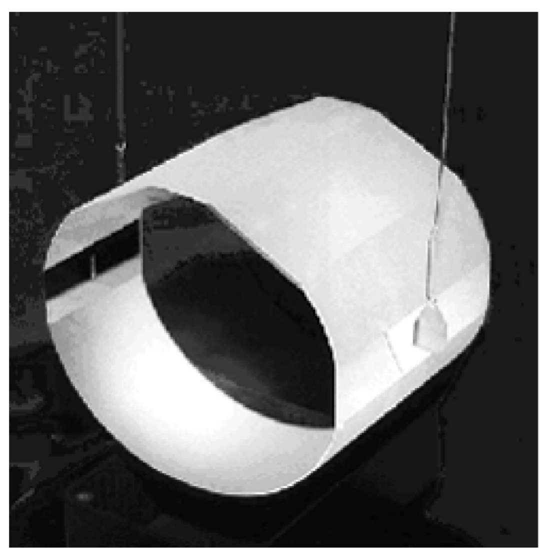

FIG. 3. Suspended mass.

motion to an appropriate level. For the measurements of the suspension wire violin modes, one of the wires was excited by means of an electrostatic actuator, a copper plate placed a few millimeters from the fiber.

In both cases, the free decay of the amplitude of the oscillation was recorded when the driving signal was switched off. The slope γ of the exponential function which describes the decay is related to the Q_n of the mode with frequency f_n by

$$Q_n = \frac{\pi f_n}{\gamma}. \quad (4)$$

The measurements for the lowest three violin modes of a wire gave the following result: $f_1 = 342.73 \pm 0.01$, $Q = (5.67 \pm 0.05) \times 10^6$; $f_2 = 629.88 \pm 0.01$, $Q = (1.65 \pm 0.05) \times 10^6$; $f_3 = 920.06 \pm 0.01$, $Q = (1.51 \pm 0.05) \times 10^6$. It should be noted that the frequencies of the modes are not exactly harmonically related due, we believe, to the presence of the small rods and quartz ring at the top of the system. The frequencies are consistent with an average fiber diameter of approximately 430 μm. If it is assumed that the loss in the violin modes takes place at the ends of the fiber and that the diameter at the point of bending is approximately 650 μm, Q values of approximately 5×10^7 are expected if the intrinsic material loss of the fused quartz between 100 Hz and 1 kHz has a typical value of approximately 4×10^{-7} as measured in our laboratories. The expected Q has been calculated from [19]

$$Q_n^{-1} = 2\phi\sqrt{\frac{2EI}{Mgl^2}}\left(1 + \frac{2}{(n\pi)^2}\sqrt{\frac{2EI}{Mgl^2}}\right), \quad (5)$$

assuming $n = 1$ (first violin mode). The measured Q's are somewhat lower. This is most likely to be due to coupling to other mechanical resonances in the structure.

In order to determine that seismic noise was not affecting the measured values of the pendulum mode Q, results were taken for three different values of initial excitation of the pendulum mode. At the lowest initial amplitude of about 0.1 mm, the ringdown of the motion does not give a strictly exponential decay and appears to have been influenced by seismic noise. At the next level of initial amplitude of about 1 mm, the curve is closer to an exponential decay indicating a pendulum Q of $(2.1 \pm 0.1) \times 10^7$. Finally, at the highest excitation of a few mm, the curve follows exactly that of an exponential decay and corresponds to the pendulum mode having a Q of $(2.5 \times 0.1) \times 10^7$. Thus it seems quite reasonable to interpret these results as suggesting that the pendulum has a Q of $(2.3 \pm 0.2) \times 10^7$.

The theoretically expected pendulum Q depends upon the suspension fiber thickness, the loss of fused quartz, including thermoelastic damping, at the pendulum frequency, and the recoil of the supporting structure. Using a typical loss angle $\phi = 4 \times 10^{-7}$, a fiber thickness of 650 μm at the bending point and the values for recoil loss detailed in the previous section would produce an expected pendulum Q of approximately 5×10^7. The measured value is clearly below this; thus there is some excess loss in the system. This could be due to grinding between the top fused quartz ring and the metal structure it is clamped against.

Finally, the bifilar pendulum mode (yaw motion or rotation about the vertical line through the center of the pendulum mass) was also measured. It can be shown that the Q of the bifilar mode is given by

$$Q^{-1} = \frac{\pi G r^4}{Mga^2}\phi_G + \frac{1}{l}\sqrt{\frac{\pi E r^4}{2Mg}}\phi_w, \quad (6)$$

where G is the shear modulus of the fiber material and is given by

$$G = \frac{1}{2}\frac{E}{1 + \nu}, \quad (7)$$

a is half the distance between the suspension fibers, ϕ_G is the mechanical loss factor associated with the shear modulus, ν is the Poisson's ratio of the material of the suspension fibers, and all other symbols are as previously defined. Assuming the material loss angle in shear is the same as the intrinsic material loss angle for bending (approximately 4×10^{-7}) and correcting the bending loss to include the effects of thermoelastic damping, the expected Q is given by

$$Q^{-1} = 3.7 \times 10^{-9} + 3.2 \times 10^{-8}; \quad (8)$$

thus the expected Q is 2.8×10^7, with the loss associated with the fibers bending dominating this value. However, the measured Q of this mode was $(1.31 \pm 0.05) \times 10^6$ at 1.61 Hz. Clearly in practice there is a higher level of excess loss for the bifilar mode than for the pendulum mode. This lends weight to the supposition that some grinding was taking place at the top fused quartz ring. Recoil of the structure for this mode is assumed to be negligible.

In conclusion, the measurements of mechanical loss reported here are for a 2.8 kg test mass suspended on a pair of fused quartz fibers. We believe this is the heaviest test mass to date to be hung on fused quartz and tested in this way. Results are very encouraging even if not at the theoretical limit that one expects from the inherent material losses of fused quartz and thus fused quartz or fused silica is a promising material for the suspensions of test masses in gravity wave detectors.

It should be noted that the quartz fibers for this pendulum are stressed to a conservative level of approximately 100 MPa, whereas recent laboratory results in both Perugia and Glasgow [11] have demonstrated breaking stresses of up to several GPa. The use of thinner fibers should allow higher pendulum Q's to be attained in the future once other loss effects such as grinding are eliminated.

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*Electronic address: gammaitoni@perugia.infn.it
†Electronic address: J.Hough@physics.gla.ac.uk
‡Electronic address: kovalik@ligo-la.caltech.edu

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