Explicit SO(10) Supersymmetric Grand Unified Model for the Higgs and Yukawa Sectors

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A complete set of fermion and Higgs superfields is introduced with well-defined SO(10) properties and U(1) $\times$ Z_2 $\times$ Z_2 family charges from which the Higgs and Yukawa superpotentials are constructed. The structures derived for the four Dirac fermion and right-handed Majorana neutrino mass matrices coincide with those previously obtained from an effective operator approach. Ten mass matrix input parameters accurately yield the twenty masses and mixings of the quarks and leptons with the bimaximal atmospheric and solar neutrino vacuum solutions favored in this simplest version.

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In a series of recent papers [1–4] the authors have shown how fermion mass matrices can be constructed in an SO(10) supersymmetric grand unified framework by use of a minimal Higgs structure which solves the doublet-triplet splitting problem [5]. The construction was carried out in an effective operator approach with phenomenological input, including the Georgi-Jarlskog relations [6]. Here we show how one can introduce a set of matter and Higgs SO(10) superfields with U(1) $\times$ Z_2 $\times$ Z_2 family charges from which the derived Higgs and Yukawa superpotentials uniquely give the structure of the fermion mass matrices previously obtained. The quark and lepton mass and mixing data are reproduced remarkably well with the solar neutrino vacuum solution preferred, provided the up quark mass is not zero at the grand unified theory (GUT) scale—otherwise the small angle Mikheyev-Smirnov-Wolfenstein (MSW) solution [7] is obtained. The right-handed Majorana neutrino matrix arises from a Higgs field which couples pairs of superheavy conjugate neutrino singlets.

We begin with a listing in Table I of the Higgs and matter superfields in the proposed model along with their family charges. As demonstrated in [5], in order to do all the symmetry breaking, one **45** adjoint Higgs with its vacuum expectation value (VEV) pointing in the B-L direction, a pair of $\mathbf{16} + \overline{\mathbf{16}}$ spinor Higgs, plus a pair of **10** vector Higgs and several Higgs singlets are required. In order to complete the construction of the Dirac mass matrices, four more vector Higgs and four additional singlets are needed. Finally, one Higgs singlet is introduced to generate the right-handed Majorana mass matrix.

TABLE I. Higgs superfields in the proposed model.

	Higgs fields needed to solve the 2-3 problem:
$\mathbf{45}_{B\text{-}L}$:	$A(0)^{+-}$
$\mathbf{16}$:	$C(\frac{3}{2})^{-+}, C'(\frac{3}{2} - p)^{++}$
$\overline{\mathbf{16}}$:	$\overline{C}(-\frac{3}{2})^{++}, \overline{C}'(-\frac{3}{2} - p)^{-+}$
$\mathbf{10}$:	$T_1(1)^{++}, T_2(-1)^{+-}$
$\mathbf{1}$:	$X(0)^{++}, P(p)^{+-}, Z_1(p)^{++}, Z_2(p)^{++}$
	Additional Higgs fields for the mass matrices:
$\mathbf{10}$:	$T_0(1 + p)^{+-}, T_0'(1 + 2p)^{+-}$, $\overline{T}_0(-3 + p)^{-+}, \overline{T}_0'(-1 - 3p)^{-+}$
$\mathbf{1}$:	$Y(2)^{-+}, Y'(2)^{++}, S(2 - 2p)^{--}, S'(2 - 3p)^{--}$, $V_M(4 + 2p)^{++}$

From the Higgs SO(10) and family assignments, it is then possible to write down explicitly the full Higgs superpotential, where we have written it as the sum of five terms:

$$\begin{aligned}
W_{\rm Higgs} &= W_A + W_{CA} + W_{2/3} + W_{H_D} + W_R\,,\\
W_A &= trA^4/M + M_A trA^2,\\
W_{CA} &= X(\overline{C}C)^2/M_C^2 + F(X) + \overline{C}'(PA/M_1 + Z_1)C\\
&\quad + \overline{C}(PA/M_2 + Z_2)C',\\
W_{2/3} &= T_1AT_2 + Y'T_2^2\,,\\
W_{H_D} &= T_1\overline{C}\overline{C}Y'/M + \overline{T}_0CC' + \overline{T}_0(T_0S + T_0'S')\,,\\
W_R &= \overline{T}_0\overline{T}_0'V_M\,.
\end{aligned} \tag{1}$$

The Higgs singlets are all assumed to develop VEV's at the GUT scale. W_A fixes $\langle A\rangle$ through the $F_A = 0$ condition where one solution is $\langle A\rangle \propto B - L$, the Dimopoulos-Wilczek solution [8]. W_{CA} gives a GUT-scale VEV to $\overline{C}$ and C by the $F_X = 0$ condition and also couples the adjoint A to the spinors C, $\overline{C}$, C', and $\overline{C}'$ without destabilizing the Dimopoulos-Wilczek solution or giving Goldstone modes. $W_{2/3}$ gives the doublet-triplet splitting by the Dimopoulos-Wilczek mechanism. W_{H_D} mixes the $(1, 2, -1/2)$ doublet in T_1 with those in C' (by $F_{\overline{C}} = 0$), and in T_0 and T_0' (by $F_{\overline{T}_0} = 0$).

To fill out the model, we specify the SO(10) $\times$ U(1) $\times$ Z_2 $\times$ Z_2 quantum numbers of the various matter fields in Table II. We require three chiral spinor fields $\mathbf{16}_i$, one for each light family, two vectorlike pairs of $\mathbf{16} + \overline{\mathbf{16}}$ spinors which can get superheavy, a pair of superheavy **10** fields in the vector representation, and three pairs of superheavy $\mathbf{1} - \mathbf{1}^c$ fermion singlets.

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TABLE II. Matter superfields in the proposed model.

$\mathbf{16}_1(-\frac{1}{2} - 2p)^{+-}$	$\mathbf{16}_2(-\frac{1}{2} + p)^{++}$	$\mathbf{16}_3(-\frac{1}{2})^{++}$
$\mathbf{16}(-\frac{1}{2} - p)^{-+}$	$\mathbf{16}'(-\frac{1}{2})^{-+}$	
$\overline{\mathbf{16}}(\frac{1}{2})^{+-}$	$\overline{\mathbf{16}}'(-\frac{3}{2} + 2p)^{+-}$	
$\mathbf{10}_1(-1 - p)^{-+}$	$\mathbf{10}_2(-1 + p)^{++}$	
$\mathbf{1}_1(2 + 2p)^{+-}$	$\mathbf{1}_2(2 - p)^{++}$	$\mathbf{1}_3(2)^{++}$
$\mathbf{1}_1^c(-2 - 2p)^{+-}$	$\mathbf{1}_2^c(-2)^{+-}$	$\mathbf{1}_3^c(-2 - p)^{++}$

In terms of these fermion fields and the Higgs fields previously introduced, one can then spell out all the terms in the Yukawa superpotential which follow from their SO(10) and U(1) $\times Z_2 \times Z_2$ assignments:

$$\begin{aligned} W_{\text{Yukawa}} = {} & \mathbf{16}_3 \cdot \mathbf{16}_3 \cdot T_1 + \mathbf{16}_2 \cdot \mathbf{16} \cdot T_1 \\ & + \mathbf{16}' \cdot \mathbf{16}' \cdot T_1 + \mathbf{16}_3 \cdot \mathbf{16}_1 \cdot T_0' \\ & + \mathbf{16}_2 \cdot \mathbf{16}_1 \cdot T_0 + \mathbf{16}_3 \cdot \overline{\mathbf{16}} \cdot A \\ & + \mathbf{16}_1 \cdot \overline{\mathbf{16}}' \cdot Y' + \mathbf{16} \cdot \overline{\mathbf{16}} \cdot P \\ & + \mathbf{16}' \cdot \overline{\mathbf{16}}' \cdot S + \mathbf{16}_3 \cdot \mathbf{10}_2 \cdot C' \\ & + \mathbf{16}_2 \cdot \mathbf{10}_1 \cdot C + \mathbf{10}_1 \cdot \mathbf{10}_2 \cdot Y \\ & + \mathbf{16}_3 \cdot \mathbf{1}_3 \cdot \overline{C} + \mathbf{16}_2 \cdot \mathbf{1}_2 \cdot \overline{C} \\ & + \mathbf{16}_1 \cdot \mathbf{1}_1 \cdot \overline{C} + \mathbf{1}_3 \cdot \mathbf{1}_3^c \cdot Z \\ & + \mathbf{1}_2 \cdot \mathbf{1}_2^c \cdot P + \mathbf{1}_1 \cdot \mathbf{1}_1^c \cdot X \\ & + \mathbf{1}_3^c \cdot \mathbf{1}_3^c \cdot V_M + \mathbf{1}_1^c \cdot \mathbf{1}_2^c \cdot V_M \,, \end{aligned} \tag{2}$$

where the coupling parameters have been suppressed. To obtain the GUT scale structure for the fermion mass matrix elements, all but the three chiral spinor fields in the first line of Table II will be integrated out. The right-handed Majorana matrix elements will all be generated through the Majorana couplings of the V_M Higgs field with conjugate singlet fermions as given above.

With R parity conserved, $d = 4$ proton decay operators are forbidden. The $d = 5$ proton decay operators induced by colored-Higgsino exchange that are generally present in unified models are present here but are not dangerous. It can be shown that the family charge assignments prevent any new and dangerous proton decay operators from arising.

The procedure for deriving the Dirac mass matrices U, D, L, and N is the following. For each type of fermion f, where $f = u_L, u_L^c, d_L, d_L^c, \ell_L^-, \ell_L^+, \nu_L$, and ν_L^c, the superheavy mass matrix connecting the f to the SU(3) $\times$ SU(2) $\times$ U(1)-conjugate representation $\overline{f}$ is first found from Eq. (2) by setting the weak-scale VEV's and the intermediate-scale VEV, V_M, to zero. This will give three zero mass eigenstates for each type of f, corresponding to the three light families. Then the terms in Eq. (2) involving $\langle T_1 \rangle$, $\langle C' \rangle$, $\langle T_0 \rangle$, and $\langle T_0' \rangle$ give rise to the 3×3 Dirac mass matrices coupling u_L to u_L^c, etc. This procedure is spelled out explicitly in [9].

Under the assumption that the zero-mass states have their large components in the chiral representations $\mathbf{16}_1$, $\mathbf{16}_2$, and $\mathbf{16}_3$, and all the other components are small, the Dirac mass matrices obtained have precisely the structure previously found in our studies by means of an effective operator approach:

$$\begin{aligned} U &= \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & \epsilon/3 \\ 0 & -\epsilon/3 & 1 \end{pmatrix}, \\ D &= \begin{pmatrix} 0 & \delta & \delta' e^{i\phi} \\ \delta & 0 & \sigma + \epsilon/3 \\ \delta' e^{i\phi} & -\epsilon/3 & 1 \end{pmatrix}, \\ N &= \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & -\epsilon \\ 0 & \epsilon & 1 \end{pmatrix}, \\ L &= \begin{pmatrix} 0 & \delta & \delta' e^{i\phi} \\ \delta & 0 & -\epsilon \\ \delta' e^{i\phi} & \sigma + \epsilon & 1 \end{pmatrix}, \end{aligned} \tag{3}$$

with U and N in units of M_U and D and L in units of M_D. The matrix parameters are identified with the Yukawa couplings and Higgs couplings and VEV's as follows:

$$\begin{aligned} M_U &= (t_3)_{5(10)}, & M_D &= (t_3)_{\overline{5}(10)}\,, \\ \epsilon M_U &= |3(a_q/p)\,(t_2)_{5(10)}|, & \epsilon M_D &= |3(a_q/p)\,(t_2)_{\overline{5}(10)}|\,, \\ \eta M_U &= (y'/s'')^2 (t')_{5(10)}, & \sigma M_D &= -(c/y)\,(c')_{\overline{5}(16)}\,, \\ & & \delta M_D &= t_0 \overline{t}_0/s\,, \\ & & \delta' M_D &= (t_0' \overline{t}_0/s') e^{-i\phi}, \end{aligned} \tag{4}$$

where the subscripts on t_2, t_3, t', and c' refer to the SU(5)[SO(10)] representation content of the VEV's. The following shorthand notation has been introduced

$$\begin{gathered} t_3 = \lambda_{16_3 16_3 T_1} \langle T_1 \rangle, \qquad t_2 = \lambda_{16_2 16 T_1} \langle T_1 \rangle\,, \\ t' = \lambda_{16' 16' T_1} \langle T_1 \rangle, \qquad c' = \lambda_{16_3 10_2 C'} \langle C' \rangle, \\ c = \lambda_{16_2 10_1 C} \langle C \rangle, \qquad \overline{c}_i = \lambda_{16_i 1_i \overline{C}} \langle \overline{C} \rangle,\ i = 1,2,3\,, \\ p = \lambda_{16 \overline{16} P} \langle P \rangle, \qquad p_{22} = \lambda_{1_2 1_2^c P} \langle P \rangle\,, \\ a_q = \lambda_{16_3 \overline{16} A} \langle A \rangle_{B=1/3}, \qquad x = \lambda_{1_1 1_1^c X} \langle X \rangle, \\ y = \lambda_{10_1 10_2 Y} \langle Y \rangle, \qquad y' = \lambda_{16_1 \overline{16}' Y'} \langle Y' \rangle, \\ z = \lambda_{1_3 1_3^c Z} \langle Z \rangle, \qquad s = \lambda_{T_0 \overline{T}_0 S} \langle S \rangle\,, \\ s' = \lambda_{T_0' \overline{T}_0 S'} \langle S' \rangle, \qquad s'' = \lambda_{16' \overline{16}' S} \langle S \rangle\,, \\ t_0 = \lambda_{16_1 16_2 T_0}, \qquad t_0' = \lambda_{16_1 16_3 T_0'}\,, \\ \overline{t}_0 = \lambda_{CC' \overline{T}_0 \langle C \rangle \langle C' \rangle}\,. \end{gathered} \tag{5}$$

The parameter η is introduced to give a tiny nonzero mass to the up quark at the Λ_G scale. Its appearance in N will also play an important role in the determination of the type of solar neutrino solution. It should also appear in D and L but its effect is negligibly small there and of no consequence, so it is dropped. The only phase then appearing in the matrices is ϕ associated with δ', as other phases are unphysical and can be rotated away with the exception of that associated with ϵ. It turns out, however, that the best fits to the data prefer a real ϵ. Hence ϕ which can be identified with the complexity of the VEV of the

S' Higgs singlet is solely responsible for CP violation in the quark sector. The structures of the matrix elements given in Eqs. (3)–(5) can be understood in terms of simple Froggatt-Nielsen diagrams [10] given in [9].

Note that the 33 elements of the Dirac mass matrices are scaled by the VEV's of the $\mathbf{10}, T_1$. But the $F = 0$ conditions for the Higgs superpotential require that the pair of Higgs doublets which remain light down to the electroweak scale arise from $5(T_1), \overline{5}(T_1), \overline{5}(C')$, and, to a very small extent, from T_0 and T_0' terms, which are ignored here. In particular, we can write in terms of a mixing angle γ

$$H_U = 5(T_1), \qquad H_D = \overline{5}(T_1)\cos\gamma + \overline{5}(C')\sin\gamma, \tag{6}$$

whereas the orthogonal combination has become superheavy at the GUT scale. Thus the ratio of the 33 mass matrix elements found from Eqs. (4) and (6) is given in terms of the VEV's, v_u and v_d of H_U and H_D, respectively, by

$$M_U/M_D = v_u/(v_d \cos\gamma) \equiv \tan\beta/\cos\gamma. \tag{7}$$

Hence we find that the large M_U/M_D ratio required for the top to bottom quark masses can be achieved with a *moderate* $\tan\beta$ provided $\cos\gamma$ is small.

Turning to the right-handed Majorana mass matrix, we use the zero mass left-handed conjugate states that were found implicitly above for the Dirac matrix N to form the basis for M_R. The right-handed Majorana neutrino matrix is then obtained from the last two terms in Eq. (2), and we find

$$M_R = \begin{pmatrix} 0 & A\epsilon^3 & 0 \\ A\epsilon^3 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Lambda_R, \tag{8}$$

where

$$\begin{aligned} \Lambda_R &= \lambda_{1_3^c 1_3^c V_M} \langle V_M \rangle (\overline{c}_3/z)^2, \\ A\epsilon^3 \Lambda_R &= \lambda_{1_1^c 1_2^c V_M} \langle V_M \rangle (\overline{c}_1/x)(\overline{c}_2/p_{22}). \end{aligned} \tag{9}$$

Note that the whole right-handed Majorana mass matrix has been generated in this simple model by the one Majorana VEV coupling superheavy conjugate fermion singlets. By means of the seesaw formula [11], one can then compute the light neutrino mass matrix

$$M_\nu = N^T M_R^{-1} N = \begin{pmatrix} 0 & 0 & -\frac{\eta}{A\epsilon^2} \\ 0 & \epsilon^2 & \epsilon \\ -\frac{\eta}{A\epsilon^2} & \epsilon & 1 \end{pmatrix} M_U^2/\Lambda_R. \tag{10}$$

We now address the predictions of the mass matrices. For this purpose it is convenient to replace the parameters δ and δ' by

$$t_L e^{i\theta} \equiv \frac{\delta - \sigma\delta' e^{i\phi}}{\sigma\epsilon/3}, \qquad t_R \equiv \frac{\delta\sqrt{\sigma^2 + 1}}{\sigma\epsilon/3}, \tag{11}$$

which are essentially the left-handed and right-handed Cabibbo angles. In terms of the dimensionless parameters ϵ, σ, t_L, t_R, $e^{i\theta}$, η, A, and M_U/M_D, we then find at the GUT scale

$$\begin{aligned}
m_t^0/m_b^0 &\cong (\sigma^2 + 1)^{-1/2} M_U/M_D, \qquad m_u^0/m_t^0 \cong \eta, \\
m_c^0/m_t^0 &\cong \tfrac{1}{9}\epsilon^2[1 - \tfrac{2}{9}\epsilon^2], \qquad m_b^0/m_\tau^0 \cong 1 - \tfrac{2}{3}\tfrac{\sigma}{\sigma^2+1}\epsilon, \\
m_s^0/m_b^0 &\cong \tfrac{1}{3}\epsilon\tfrac{\sigma}{\sigma^2+1}[1 + \tfrac{1}{3}\epsilon\tfrac{1-\sigma^2-\sigma\epsilon/3}{\sigma(\sigma^2+1)} + \tfrac{1}{2}(t_L^2 + t_R^2)], \\
m_d^0/m_s^0 &\cong t_L t_R[1 - \tfrac{1}{3}\epsilon\tfrac{\sigma^2+2}{\sigma(\sigma^2+1)} - (t_L^2 + t_R^2) \\
&\qquad + (t_L^4 + t_L^2 t_R^2 + t_R^4)], \\
m_\mu^0/m_\tau^0 &\cong \epsilon\tfrac{\sigma}{\sigma^2+1}[1 + \epsilon\tfrac{1-\sigma^2-\sigma\epsilon}{\sigma(\sigma^2+1)} + \tfrac{1}{18}(t_L^2 + t_R^2)], \\
m_e^0/m_\mu^0 &\cong \tfrac{1}{9}t_L t_R[1 - \epsilon\tfrac{\sigma^2+2}{\sigma(\sigma^2+1)} + \epsilon^2\tfrac{\sigma^4+9\sigma^2/2+3}{\sigma^2(\sigma^2+1)^2} \\
&\qquad - \tfrac{1}{9}(t_L^2 + t_R^2)], \\
V_{cb}^0 &\cong \tfrac{1}{3}\epsilon\tfrac{\sigma^2}{\sigma^2+1}[1 + \tfrac{2}{3}\epsilon\tfrac{1}{\sigma(\sigma^2+1)}], \\
V_{us}^0 &\cong t_L[1 - \tfrac{1}{2}t_L^2 - t_R^2 + t_R^4 + \tfrac{5}{2}t_L^2 t_R^2 + \tfrac{3}{8}t_L^4 \\
&\qquad - \tfrac{\epsilon}{3\sigma\sqrt{\sigma^2+1}}\tfrac{t_R}{t_L}e^{-i\theta}], \\
V_{ub}^0 &\cong \tfrac{1}{3}t_L\epsilon\tfrac{1}{\sigma^2+1}[\sqrt{\sigma^2 + 1}\,\tfrac{t_R}{t_L}e^{-i\theta}(1 - \tfrac{1}{3}\epsilon\tfrac{\sigma}{\sigma^2+1}) \\
&\qquad - (1 - \tfrac{2}{3}\epsilon\tfrac{\sigma}{\sigma^2+1})], \\
m_2^0/m_3^0 &\cong (\tfrac{\eta}{A\epsilon\sqrt{1+\epsilon^2}})[1 + \tfrac{\eta}{A\epsilon^3\sqrt{1+\epsilon^2}}], \\
m_1^0/m_3^0 &\cong (\tfrac{\eta}{A\epsilon\sqrt{1+\epsilon^2}})[1 - \tfrac{\eta}{2A\epsilon^3\sqrt{1+\epsilon^2}}], \\
U_{\mu 3}^0 &\cong -\tfrac{1}{\sqrt{\sigma^2+1}}(\sigma - \epsilon\tfrac{\sigma^2}{\sigma^2+1}), \\
U_{e2}^0 &\cong -\tfrac{1}{\sqrt{2}}[1 - \tfrac{\epsilon}{3\sigma}t_L e^{i\theta} + \tfrac{1}{3\sqrt{\sigma^2+1}}(1 + \epsilon\sigma)t_R], \\
U_{e3}^0 &\cong \frac{1}{3\sqrt{\sigma^2 + 1}}(\sigma - \epsilon)t_R - \tfrac{\eta}{A\epsilon^2}.
\end{aligned} \tag{12}$$

Note that the Georgi-Jarlskog relations [6], $m_s^0 \cong \frac{1}{3}m_\mu^0$ and $m_d^0 \cong 3m_e^0$, emerge as required by design. The quark and charged lepton data are best fit at the low scale (see below) by assigning the following values to the model parameters: $\epsilon = 0.145$, $\sigma = 1.78$, $t_L = 0.236$, $t_R = 0.205$, $\theta = 34^\circ$ (corresponding to $\delta = 0.0086$, $\delta' = 0.0079$, $\phi = 54^\circ$), $\eta = 8 \times 10^{-6}$, and $M_U/M_D \simeq 113$.

As noted earlier, in order to obtain the simple mass matrices in Eq. (3), we had to assume that the zero-mass states have their large components in the chiral representations $\mathbf{16}_1$, $\mathbf{16}_2$, and $\mathbf{16}_3$. The conditions on the state normalization factors are all satisfied provided the following ratios are much less than unity:

$$(a/p)^2, (y'/s'')^2, (c/y)^2, (\overline{c}_2/x)^2, (\overline{c}_2/p_{22})^2, (\overline{c}_3/z)^2 \ll 1. \tag{13}$$

With the numerical choice of parameters given above and near equality of the various Higgs couplings, we find $(a/p)^2 \simeq 0.02$ and $(y'/s'')^2 \sim 6 \times 10^{-6}$, so the first two conditions are easily satisfied. Requiring that $(c/y)^2 \ll 1$ and with the expression for σ obtained from Eqs. (4), we find

$$\begin{aligned} \tan\gamma &\equiv \tfrac{\langle \overline{5}(C') \rangle}{\langle \overline{5}(T_1) \rangle} \gg \sigma, \\ \tan\beta &\simeq \sqrt{\sigma^2 + 1}\,(\cos\gamma) m_t^0/m_b^0 \ll m_t^0/m_b^0 \end{aligned} \tag{14}$$

in terms of the $T_1 - C'$ mixing angle, γ, in Eq. (6). With $c/y \cong 0.1$, for example, $\tan\gamma \simeq 18$ which implies

$\tan\beta \simeq 6$, a very reasonable midrange value allowed by experiment. The others can also be satisfied [9].

In [9] we have evolved the results in Eqs. (12) down to the low scales with a value for $\tan\beta = 5$, $\Lambda_G = 2 \times 10^{16}$ GeV, $\Lambda_{\rm SUSY} = m_t(m_t)$, $\alpha_s(M_Z) = 0.118$, $\alpha(M_Z) = 1/127.9$, and $\sin^2\theta_W = 0.2315$. With the quantities $m_t(m_t) = 165$ GeV, $m_\tau = 1.777$ GeV, $m_\mu = 105.7$ MeV, $m_e = 0.511$ MeV, $m_u = 4.5$ MeV, $V_{us} = 0.220$, $V_{cb} = 0.0395$, and $\delta_{CP} = 64^\circ$ used to determine the input parameters, $M_U \simeq 113$ GeV, $M_D \simeq 1$ GeV, and σ, ϵ, t_L, t_R, θ, and η given earlier, the following values are obtained compared with experiment [12] in parentheses:

$$\begin{aligned} m_c(m_c) &= 1.23 \text{ GeV}, \qquad (1.27 \pm 0.1 \text{ GeV}), \\ m_b(m_b) &= 4.25 \text{ GeV}, \qquad (4.26 \pm 0.11 \text{ GeV}), \\ m_s(1 \text{ GeV}) &= 148 \text{ MeV}, \qquad (175 \pm 50 \text{ MeV}), \\ m_d(1 \text{ GeV}) &= 7.9 \text{ MeV}, \qquad (8.9 \pm 2.6 \text{ MeV}), \\ |V_{ub}/V_{cb}| &= 0.080, \qquad (0.090 \pm 0.008), \end{aligned} \tag{15}$$

where finite SUSY loop corrections for m_b and m_s have been scaled to give $m_b(m_b) \simeq 4.25$ GeV for $\tan\beta = 5$.

The effective light neutrino mass matrix of Eq. (10) leads to bimaximal mixing with a large angle solution for atmospheric neutrino oscillations [13] and the "just-so" vacuum solution [14] involving two pseudo-Dirac neutrinos, if we set $\Lambda_R = 2.4 \times 10^{14}$ GeV and $A = 0.05$. We then find

$$\begin{gathered} m_3 = 54.3 \text{ MeV}, \quad m_2 = 59.6\ \mu\text{eV}, \quad m_1 = 56.5\ \mu\text{eV}, \\ U_{e2} = 0.733, \quad U_{e3} = 0.047, \quad U_{\mu 3} = -0.818, \\ \delta'_{CP} = -0.2^\circ, \\ \Delta m^2_{23} = 3.0 \times 10^{-3} \text{ eV}^2, \quad \sin^2 2\theta_{\rm atm} = 0.89, \\ \Delta m^2_{12} = 3.6 \times 10^{-10} \text{ eV}^2, \quad \sin^2 2\theta_{\rm solar} = 0.99, \\ \Delta m^2_{13} = 3.0 \times 10^{-3} \text{ eV}^2, \quad \sin^2 2\theta_{\rm reac} = 0.009. \end{gathered} \tag{16}$$

The effective scale of the right-handed Majorana mass contribution occurs 2 orders of magnitude lower than the SUSY GUT scale of $\Lambda_G = 1.2 \times 10^{16}$ GeV. The effective two-component reactor mixing angle given above should be observable at a future neutrino factory, whereas the present limit from the CHOOZ experiment [15] is approximately 0.1 for the above Δm^2_{23}. In principle, the parameter A appearing in M_R can also be complex, but we find that in no case does the leptonic CP-violating phase, δ'_{CP} exceed 10° in magnitude. Hence the model predicts leptonic CP violation will be unobservable.

The vacuum solar solution depends critically on the appearance of the parameter η in the matrix N, corresponding to the nonzero η entry in U which gives the up quark a mass at the GUT scale. Should we set $\eta = 0$, only the small-angle MSW solution [7] would be obtained for the solar neutrino oscillations. The large angle MSW solution is disfavored by the larger hierarchy, i.e., very small A value, required in M_R.

In summary, we have constructed an explicit SO(10) supersymmetric grand unified model for the Higgs and Yukawa superpotentials which reproduces the fermion mass matrices previously obtained in an effective operator approach. All the quark and lepton mass and mixing data are fit remarkably well with a $\tan\beta$ in the range of 5–10 with matrix parameters which are also quite reasonable.

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