

## New Microwave Background Constraints on the Cosmic Matter Budget: Trouble for Nucleosynthesis?

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We compute the joint constraints on ten cosmological parameters from the latest cosmic microwave background measurements. The lack of a significant second acoustic peak in the new BOOMERANG and MAXIMA data favors models with more baryons than big bang nucleosynthesis predicts, almost independently of what prior information is included. The simplest flat inflation models with purely scalar scale-invariant fluctuations prefer a baryon density  $0.022 < h^2\Omega_b < 0.040$  and a total nonbaryonic (hot + cold) dark matter density  $0.14 < h^2\Omega_{\text{dm}} < 0.32$  at 95% confidence and allow reionization no earlier than  $z \sim 30$ .

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One of the main challenges in modern cosmology is to refine and test the standard model of structure formation by precision measurements of its free parameters. The cosmic matter budget involves at least the four parameters  $\Omega_b$ ,  $\Omega_{\text{cdm}}$ ,  $\Omega_\nu$ , and  $\Omega_\Lambda$ , which give the percentages of critical density corresponding to baryons, cold dark matter, massive neutrinos, and vacuum energy, respectively. A “budget deficit”  $\Omega_k \equiv 1 - \Omega_b - \Omega_{\text{cdm}} - \Omega_\nu - \Omega_\Lambda$  manifests itself as spatial curvature. The description of the initial seed fluctuations predicted by inflation requires at least four parameters, the amplitudes  $A_s$  and  $A_t$  and slopes  $n_s$  and  $n_t$  of scalar and tensor fluctuations, respectively. Finally, the optical depth parameter  $\tau$  quantifies when the first stars or quasars reionized the Universe and the Hubble parameter  $h$  gives its current expansion rate.

During the past year or so, a number of papers [1–8] have used the measured cosmic microwave background (CMB) fluctuations to constrain subsets of these parameters. CMB data have improved dramatically since fluctuations were first detected [9]. The measurement of a first acoustic peak at the degree scale [10], suggesting that the Universe is flat ( $\Omega_k = 0$ ), has now been beautifully confirmed and improved by using the ground-breaking high fidelity maps of the BOOMERANG [11] and MAXIMA [12] experiments. As can be seen in Fig. 1, perhaps the most important new information from BOOMERANG and MAXIMA is their accurate measurements of the angular power spectrum  $C_\ell$  on even smaller scales, out to multipole  $\ell \sim 600$ –800. The striking lack of a significant second acoustic peak places strong constraints on the cosmological parameters [8,11,13], making a new full-fledged analysis of all the CMB data very timely.

In this Letter, we jointly constrain the following ten cosmological parameters:  $\tau$ ,  $\Omega_k$ ,  $\Omega_\Lambda$ ,  $n_s$ ,  $n_t$ ,  $A_s$ , the tensor-to-scalar ratio  $r \equiv A_t/A_s$ , and the physical matter densities  $\omega_b \equiv h^2\Omega_b$ ,  $\omega_{\text{cdm}} \equiv h^2\Omega_{\text{cdm}}$ , and  $\omega_\nu \equiv h^2\Omega_\nu$ . The identity  $h = \sqrt{(\omega_{\text{cdm}} + \omega_b + \omega_\nu)/(1 - \Omega_k - \Omega_\Lambda)}$

fixes the Hubble parameter. We use the ten-dimensional grid method described in [5]. In essence, this utilizes a technique for accelerating the the CMBFAST package [14] by a factor around  $10^3$  to compute theoretical power spectra on a grid in the ten-dimensional parameter space, fitting these models to the data and then using cubic interpolation of the resulting ten-dimensional likelihood function to marginalize it down to constraints on individual or pairs of parameters. We use the 87 data points shown in Fig. 1, combining the 65 tabulated in [5] with the 12 new BOOMERANG points [11] and the 10 new MAXIMA points [12].

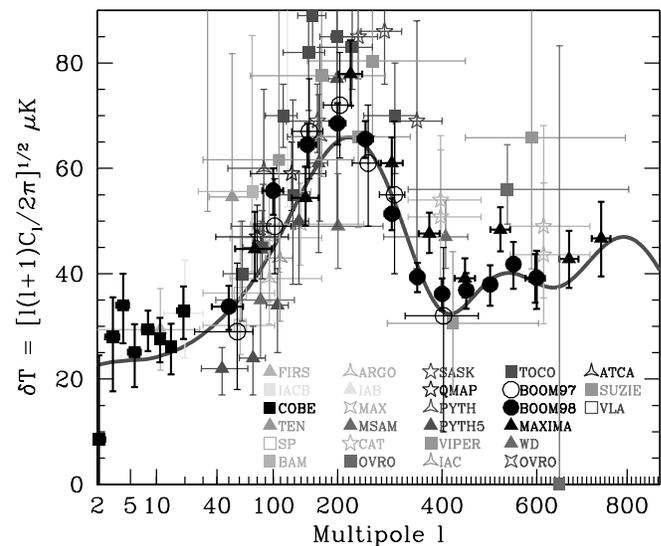


FIG. 1. The 87 band power measurements used. The curve shows the simple inflationary model with  $\tau = \Omega_k = \Omega_\nu = r = 0$ ,  $\Omega_\Lambda = 0.43$ ,  $h^2\Omega_{\text{cdm}} = 0.20$ ,  $h^2\Omega_b = 0.03$ ,  $n_s = 1$ ,  $h = 0.63$ . Note that although we include the calibration uncertainties in our analysis, they are not reflected by the plotted error bars.

TABLE I. Maximum-likelihood values and 95% confidence limits. The “inflation prior” for each parameter is indicated in boxes in Fig. 3.  $\Omega_{\text{dm}} \equiv \Omega_{\text{cdm}} + \Omega_{\nu}$ . A dash indicates that no limit was found, with the likelihood still above  $e^{-2}$  at the edge of our grid. Extrapolation would suggest a limit  $n_s \lesssim 1.75$ .

Quantity	Ten free parameters			Inflation prior		
	Min.	Best	Max.	Min.	Best	Max.
$\tau$	0.0	0.0	0.33	0.0	0.0	0.28
$h^2\Omega_b$	0.017	0.05	...	0.022	0.03	0.040
$h^2\Omega_{\text{dm}}$	0.02	0.08	...	0.14	0.20	0.32
$\Omega_{\Lambda}$	...	0.2	0.80	-0.16	0.43	0.65
$\Omega_k$	...	-0.6	0.13	-0.13	0	0.10
$n_s$	0.8	1.5	...	0.84	1.0	1.17

Our 95% confidence limits on the best constrained parameters are summarized in Table I. Figure 2 shows that CMB alone suggests that the Universe is either flat (near the diagonal line  $\Omega_m + \Omega_{\Lambda} = 1$ , where  $\Omega_m \equiv \Omega_b + \Omega_{\text{cdm}} + \Omega_{\nu}$ ) or closed (upper right). These constraints come largely from the location of the first peak, which is well known to move to the right if the curvature  $\Omega_k$  is increased [16]. Very closed models work only because the first acoustic peak can also be moved to the right by increasing the tilt  $n_s$  or decreasing the matter density and bringing the large-scale Cosmic Background Explorer (COBE) signal back up with tensor fluctuations (gravity waves) [17,18]. Galaxy clustering constraints disfavor such strong blue-tilting, and Figs. 2 and 3 show that closing this loophole by barring gravity waves ( $r = 0$ ) favors curvature near zero and  $n_s$  near unity. This is a striking success for the oldest and simplest inflation models, which make the three predictions  $r \approx 0$ ,  $\Omega_k \approx 0$ , and  $n_s \approx 1$  [19,20]. Another important success for inflation is that the first peak is so narrow—if the data had revealed the type of broad peak expected in many topological defect scenar-

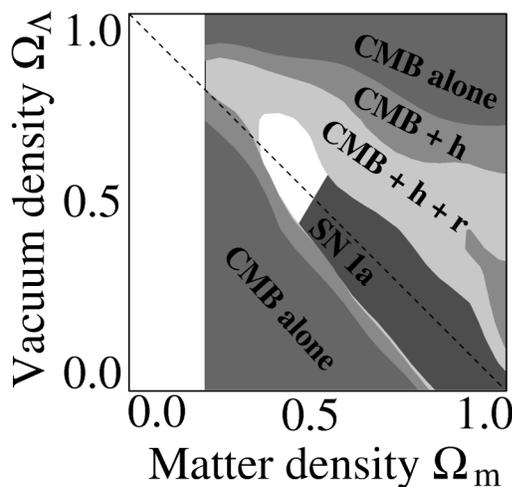


FIG. 2. The regions in the  $(\Omega_m, \Omega_{\Lambda})$  plane that are ruled out at 95% are shown using (starting from the outside) no priors, the prior that  $0.5 < h < 0.8$  (95%), and the additional constraint  $r = 0$ . The SN 1a constraints are from White [15].

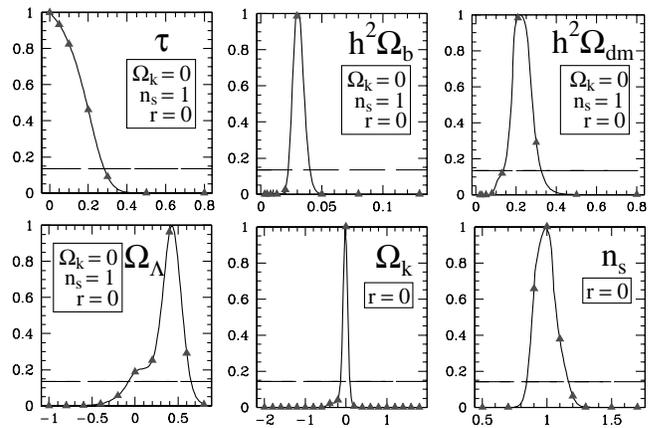


FIG. 3. Marginalized likelihoods assuming that  $0.5 < h < 0.8$  (95%) and the inflationary priors specified in the boxes.  $2 - \sigma$  limits are roughly where the curves drop beneath the dashed lines.

ios, *none* of the models in our grid would have provided an acceptable fit. Because of these tantalizing hints that “back to basics” inflation is correct, Table I and Fig. 3 include results assuming this inflation prior  $r = \Omega_k = 0$ ,  $n_s = 1$ .

The constraints in Table I are seen to be much more interesting than those before BOOMERANG and MAXIMA [5], thanks to new information on the scale of the second peak and beyond. Cold dark matter and neutrinos have indistinguishable effects on the CMB except for very light neutrinos (small  $\omega_{\nu}$ ), and the current data still lack the precision to detect this subtle difference. The predicted height ratio of the first two peaks therefore depends essentially on only three parameters [8,13,21]:  $n_s$ ,  $\omega_b$ , and  $\omega_{\text{dm}}$ , where the total dark matter density  $\omega_{\text{dm}} \equiv \omega_{\text{cdm}} + \omega_{\nu}$ . Let us focus on the constraints on these parameters. Increasing  $\omega_b$  tends to boost the odd-numbered peaks (1, 3, etc.) at the expense of the even ones (2, 4, etc.) [16], whereas

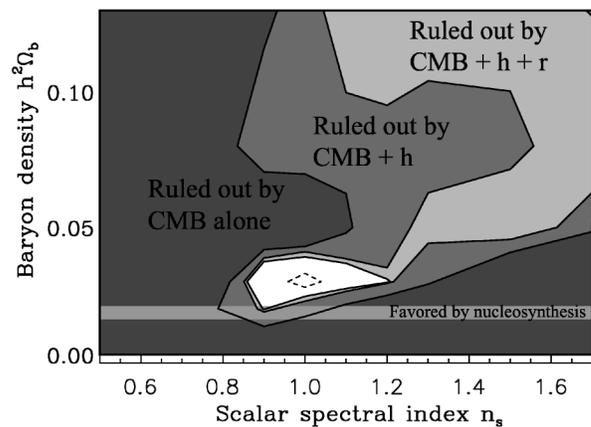


FIG. 4. The regions in the  $(n_s, \omega_b)$  plane that are ruled out at 95% are shown using (starting from the outside) no priors, the prior  $0.5 < h < 0.8$  (95%), the additional constraint  $r = 0$ , and the additional constraint  $\tau = 0$  (dashed line). The horizontal band shows the nucleosynthesis constraints  $\omega_b = 0.019 \pm 0.0024$ .

increasing  $\omega_{\text{dm}}$  suppresses all peaks [22]. The low second peak can therefore be fit by either decreasing the tilt  $n_s$  or by increasing the baryon density  $\omega_b$  [8,13] compared to the usually assumed values  $n_s \approx 1$ ,  $\omega_b \approx 0.02$ . As illustrated in Fig. 4, this conclusion is essentially independent of what priors are assumed. However, reducing  $n_s$  below 0.9 is seen to make things worse again, as the first peak becomes too low relative to the COBE normalization.

In short, there are two very simple ways of explaining the lack of a prominent second acoustic peak: more baryons or a red-tilted spectrum [8,13]. However, as we will now discuss, both of these solutions have problems of their own.

Figure 5 shows that when more baryons are added, more dark matter is needed to keep the first peak height constant. When the tilt  $n_s$  is fixed by the inflation prior, the constraints on the remaining two parameters  $\omega_{\text{dm}}$  and  $\omega_b$  are seen to become quite tight. Intriguingly, the preferred baryon fraction is of the same order as preferred by big bang nucleosynthesis, but nonetheless higher than the tight nucleosynthesis error bars [23,24]  $\omega_b = 0.019 \pm 0.0024$  allow. Even if the nucleosynthesis error bars have somehow been underestimated so that  $\omega_b \geq 0.023$  as required by the CMB data plus simple inflation is allowed, this solution may conflict with other astrophysical constraints. For instance, x-ray observations of clusters of galaxies can be used to determine the ratio of baryons to dark matter [25,26], and  $\omega_b = 0.03$  can be reconciled with these observations only by having  $\Omega_m \geq 0.7$  which would conflict with the supernova 1a results and other estimates of the dark matter density [27].

On the other hand, the tilt solution is no panacea either. In a class of popular inflationary models known as power law inflation, the amplitude of the tensor component is approximately related to the tilt of the scalar spectrum,  $r \sim 7(1 - n_s)$  [28]. If we choose to fit the data by lowering the tilt to  $n_s = 0.9$ , this would raise the COBE normalization by 70%. Models that match the COBE normalization therefore make the first peak too low by a factor of 1.7 in power, which is ruled out by the data. In other words, imposing  $r \sim 7(1 - n_s)$  (which we have *not* done

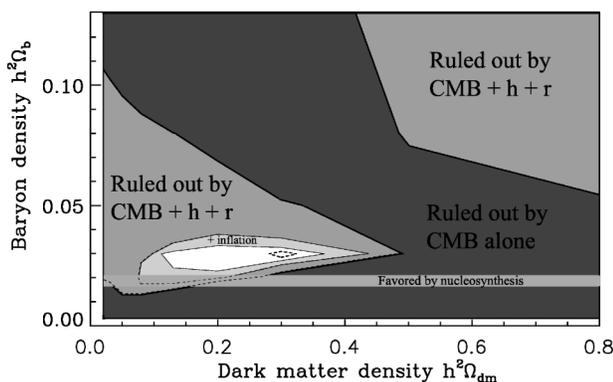


FIG. 5. Similar to Fig. 4, but for the  $(\omega_{\text{dm}}, \omega_b)$  plane.

in our analysis) would exclude  $\omega_b$  as low as 0.02. Thus the simple tilt solution does not work for all inflation models.

Could the apparent problem be a mere statistical fluke? It would certainly be premature to claim a rock-solid discrepancy between CMB and nucleosynthesis plus power law inflation. The  $\chi^2$  value for the best fit inflation model with  $\omega_b = 0.02$  is still statistically acceptable ( $\chi^2 \approx 81$  for 87 degrees of freedom reduced by about five effective parameters). However, serious discrepancies in peak heights tend to get statistically diluted by the swarm of points with large error bars at lower  $\ell$  that agree with most anything reasonable (indeed,  $\chi^2$  drops down to 71 for  $\omega_b = 0.03$  and as low as 68 without any priors), and the relative likelihood rises sharply with  $\omega_b$  regardless of what priors are imposed. To assess the sensitivity of the results to the choice of data, we therefore repeated our entire analysis for the following cases: (a) using all the data except MAXIMA and (b) using only COBE and the new BOOMERANG data. Omitting MAXIMA removed the “CMB only” and “CMB +  $h$ ” exclusion regions that are seen to protrude in from the left in Fig. 4. This is because the MAXIMA points place an upper limit on the height of (the left part of) the third peak, effectively giving an upper limit on the baryon density. Dropping MAXIMA also loosened the upper limit on  $\omega_{\text{dm}}$  somewhat and marginally weakened the bounds on  $\Omega_k$  and  $\Omega_\Lambda$ . The other constraints were essentially unaffected. Most importantly, the *lower* bound on  $\omega_b$  seen in Fig. 4 remained unchanged, since it comes from the low ratio of the second to first peak heights [21].

Although our inclusion of the 10% uncertainty in the BOOMERANG’s calibration (20% in power) was not very important in our full analysis, as the fitting procedure *de facto* calibrated BOOMERANG off of other experiments, this substantially degraded the results of our COBE + BOOMERANG analysis. We therefore repeated it three more times, without the calibration error but multiplying the BOOMERANG points by 0.9, 1.0, and 1.1, respectively. The results were quite similar to those using all the data, as expected from the experimental concordance seen in Fig. 1. However, most constraints got slightly tighter, consistent with the above-mentioned  $\chi^2$  dilution hypothesis. Rather than go away, the baryon problem became exacerbated: the 95% inflationary lower limit on  $\omega_b$  was tightened from 0.024 to 0.027 with  $\chi^2 = 12$  (with a total of 20 BOOMERANG + COBE points and four free parameters). In contrast, the tilt solution gave  $\chi^2 = 22$ , and higher still when the BOOMERANG normalization was raised or lowered by 10%. Most strikingly, in the COBE + BOOMERANG version of Fig. 4,  $\omega_b$  is not permitted to be low enough to agree with nucleosynthesis for *any* value of the tilt  $n_s$ , so the tilt solution may have worked using all the data merely because of the above-mentioned dilution effect.

Can the baryon problem be explained by inaccuracies in our numerical method? The correlations between the

BOOMERANG points (which we could not include since they have not yet been made public) are reportedly very small [11]. Although a range of approximations is involved as detailed in [5], for instance, in the likelihood calculation, it appears unlikely that such inaccuracies are large enough to have a major impact on the lower bound on  $\omega_b$ . Perhaps the best indication of this is that a number of independent analyses [21,29–31] have been made available since this paper was originally submitted, using a wide range of computational techniques, and they all favor baryon fractions in excess of the current nucleosynthesis prediction. More baryons also solve some older problems [32].

A number of other ways out have been proposed [8,13], ranging from mechanisms for delaying standard recombination to time variation of physical constants and severe misestimates of the BOOMERANG beamwidth. However, these explanations are all of a highly speculative nature. An excellent way to clear up this mystery will be to search for a third acoustic peak, which is boosted by more baryons but suppressed by most of the other proposed remedies.

Apart from the matter budget, Table I and Fig. 3 also show that the CMB data provide perhaps the first meaningful upper limit on  $\tau$ , the optical depth due to reionization (compare [5,33]). Since  $\tau \propto h\Omega_b z_{\text{ion}}^{3/2} \Omega_m^{-1/2}$  if the redshift of reionization  $z_{\text{ion}} \gg 1$ , our lower limit  $\omega_b > 0.024$  combined with our upper limit  $\tau < 0.35$  gives the constraint  $z_{\text{ion}} \lesssim 49h^{2/3}\Omega_m^{1/3}$ , or  $z_{\text{ion}} \lesssim 28$  for  $h = \Omega_\Lambda = 0.7$ . This is compatible with the range  $z_{\text{ion}} = 8\text{--}20$  favored by numerical simulations, but challenges more extreme models.

In conclusion, the new BOOMERANG results look like a triumph for the simplest possible inflationary model but for one rather large fly in the ointment: the lack of a significant second acoustic peak suggests that we may need to abandon either a popular version of inflation, the current nucleosynthesis constraints, or some even more cherished assumption. In answering one question, BOOMERANG has raised another. Its answer is likely to lie in the third peak, and the race to reach it has now begun.

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- [1] C. H. Lineweaver, *Science* **284**, 1503 (2000).
- [2] S. Dodelson and L. Knox, *Phys. Rev. Lett.* **84**, 3523 (2000).
- [3] A. Melchiorri *et al.*, *Astrophys. J.* **536**, L63 (2000).
- [4] O. Lahav *et al.*, astro-ph/9912105.
- [5] M. Tegmark and M. Zaldarriaga, astro-ph/0002091.
- [6] G. P. Efstathiou, astro-ph/0002249.
- [7] M. Le Dour, M. Douspis, J. G. Bartlett, and A. Blanchard, astro-ph/0004282.
- [8] M. White, D. Scott, and E. Pierpaoli, astro-ph/0004385.
- [9] G. F. Smoot, *Astrophys. J.* **396**, L1 (1992).
- [10] A. D. Miller, *Astrophys. J.* **524**, L1 (1999).
- [11] P. de Bernardis *et al.*, *Nature (London)* **404**, 995 (2000).
- [12] S. Hanany *et al.*, astro-ph/0005123.
- [13] W. Hu, *Nature (London)* **404**, 939 (2000).
- [14] U. Seljak and M. Zaldarriaga, *Astrophys. J.* **469**, 437 (1996).
- [15] M. White, *Astrophys. J.* **506**, 495 (1998).
- [16] W. Hu, N. Sugiyama, and J. Silk, *Nature (London)* **386**, 37 (1997).
- [17] M. Tegmark, *Astrophys. J. Lett.* **514**, L69 (1999).
- [18] A. Melchiorri, M. V. Sazhin, V. V. Shulga, and N. Vittorio, *Astrophys. J.* **518**, 562 (1999).
- [19] P. J. Steinhardt, astro-ph/9502024.
- [20] M. S. Turner, astro-ph/9704062.
- [21] W. Hu, M. Fukugita, M. Zaldarriaga, and M. Tegmark, astro-ph/0006436.
- [22] See the CMB movies at [www.hep.upenn.edu/~max](http://www.hep.upenn.edu/~max) or [www.ias.edu/~whu](http://www.ias.edu/~whu).
- [23] S. Burles and D. Tytler, astro-ph/9803071.
- [24] S. Burles, K. M. Nollett, J. N. Truran, and M. S. Turner, *Phys. Rev. Lett.* **82**, 476 (1999).
- [25] A. E. Evrard, *Mon. Not. R. Astron. Soc.* **292**, 289 (1997).
- [26] J. Mohr, Z. Haiman, and G. Holder, astro-ph/0004244.
- [27] N. Bahcall, J. P. Ostriker, S. Perlmutter, and P. J. Steinhardt, *Science* **284**, 1481 (1999).
- [28] A. R. Liddle and D. H. Lyth, *Phys. Lett. B* **291**, 391 (1992).
- [29] A. E. Lange *et al.*, astro-ph/0005004.
- [30] A. Balbi *et al.*, astro-ph/0005124.
- [31] J. Lesgourgues and M. Peloso, astro-ph/0004412.
- [32] M. White, P. T. P. Viana, A. R. Liddle, and D. Scott, *Mon. Not. R. Astron. Soc.* **283**, 107 (1996).
- [33] L. M. Griffiths, D. Barbosa, and A. R. Liddle, *Mon. Not. R. Astron. Soc.* **308**, 854 (1999).