Quantum Theory of the Smectic Metal State in Stripe Phases

V. J. Emery,[1] E. Fradkin,[2] S. A. Kivelson,[3,4] and T. C. Lubensky[5]
[1]Brookhaven National Laboratory, Upton, New York 11973-5000
[2]Department of Physics, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801-3080
[3]Department of Physics, U.C.L.A., Los Angeles, California 90095
[4]Department of Physics, Stanford University, Stanford, California 94305
[5]Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19104
(Received 10 January 2000)

We present a theory of the electron smectic fixed point of the stripe phases of doped layered Mott insulators. We show that in the presence of a spin gap three phases generally arise: (a) a smectic superconductor, (b) an insulating stripe crystal, and (c) a smectic metal. The latter phase is a stable two-dimensional anisotropic non-Fermi liquid. In the absence of a spin gap there is also a more conventional Fermi-liquid-like phase. The smectic superconductor and smectic metal phases (or glassy versions thereof) may have already been seen in Nd-doped $La_{2-x}Sr_xCuO_4$.

PACS numbers: 71.10.Hf, 71.27.+a, 74.20.Mn

In the past few years very strong experimental evidence has been found for static or dynamic charge inhomogeneity in several strongly correlated electronic systems, in particular, in high-temperature superconductors [1], manganites [2], and quantum Hall systems [3]. In d dimensions, the charge degrees of freedom of a doped Mott insulator are confined to an array of self-organized ($d-1$)-dimensional structures. In $d = 2$ these structures are linear and are known as stripes. Stripe phases may be insulating or conducting. We have recently proposed that quite generally the quantum mechanical ground states, and the thermodynamic phases which emerge from them, can, on the basis of broken symmetries, be characterized as electronic liquid crystal states [4]. Specifically, a conducting stripe ordered phase is an electronic smectic state [5], while a state with only orientational stripe order (such as is presumably observed in quantum Hall systems [3]) is an electronic nematic state [5,6].

Here, we use a perturbative renormalization group analysis, which is asymptotically exact in the limit of weak interstripe coupling, to reexamine the stability of the electronic phases of a stripe ordered system in $d = 2$ and $T \to 0$. The results are summarized in Figs. 1 and 2. In addition to an insulating stripe crystal phase, a variant of a Wigner crystal, we prove that there exist stable smectic phases: (1) an anisotropic smectic metal (non-Fermi-liquid) state, which is a new phase of matter; (2) a stripe ordered smectic superconductor. We consider the cases of both spin-gap and spin-1/2 electrons.

One-dimensional correlated electron systems are Luttinger liquids, [7] which are quintessential scale-invariant non-Fermi liquids with correlation functions exhibiting power-law behavior, typically with anomalous exponents. Interest in arrays of Luttinger liquids has recently been restimulated following a proposal by Anderson [8] that the fermionic excitations of a Luttinger liquid are confined [7] and consequently that interchain transport is incoherent. However perturbative studies of the effects of interchain couplings at the decoupled Luttinger liquid fixed point have invariably concluded that such systems always order at low temperatures, or cross over to a higher-dimensional Fermi-liquid state, i.e., that the Luttinger behavior is restricted to a high-energy crossover regime [7,9]. In the important case in which the interactions within a chain are repulsive, the most divergent susceptibility within a single chain, especially when there is a spin gap, is associated with $2k_F$ or $4k_F$ charge-density wave (CDW) fluctuations, i.e., the decoupled Luttinger fixed point is typically unstable to two-dimensional crystallization [4,5,7]. There is however a loophole in this argument. The decoupled Luttinger fixed point is not the most general scale-invariant theory compatible with the symmetries of an electron smectic. In particular, the *long wavelength* density-density and/or current-current interactions between neighboring Luttinger liquids are marginal operators, and should be included in the fixed point Hamiltonian [Eq. (2)], which we call the generalized smectic non-Fermi-liquid fixed point. Our principal results follow from a straightforward analysis of the perturbative stability of this fixed point. To the best of our knowledge, the model presented here is the first explicit example of a system with stable non-Fermi-liquid behavior (albeit very anisotropic) in more than one dimension and which exhibits "confinement of coherence" [10]. Sliding phases, which are classical analogs of the smectic metal state [4] in 3D stacks of coupled 2D planes with XY, crystalline, or smectic order, have, however, been investigated [11,12].

The low-energy Luttinger liquid behavior of an isolated system of spinless interacting fermions is described by the fixed point Hamiltonian of a bosonic phase field [7], $\phi(x,\tau)$, whose dynamics is governed by the Lagrangian density (in imaginary time τ)

$$\mathcal{L} = \frac{w}{2}\left[\frac{1}{v}\left(\frac{\partial\phi}{\partial\tau}\right)^2 + v\left(\frac{\partial\phi}{\partial x}\right)^2\right], \qquad (1)$$

 0031-9007/00/85(10)/2160(4)$15.00

where w (the inverse of the conventional Luttinger parameter K) and the velocity of the excitations v are nonuniversal functions of the coupling constants and depend on microscopic details. For repulsive interactions we expect $w \geq 1$ and, for weak interactions w and v are determined by the backward and forward scattering amplitudes g_2 and g_4 [7]. Physical observables, such as the long wavelength components of the charge-density fluctuations j_0 and the charge current j_1, are given by the bosonization formula $j_\mu = \frac{1}{\sqrt{\pi}} \epsilon_{\mu\nu} \partial^\nu \phi$ where $\epsilon_{\mu\nu}$ is the Levi-Civita tensor. If both spin and charge are dynamical degrees of freedom, there are two Luttinger parameters (K_c, K_s), and two velocities (v_c, v_s).

The one-dimensional correlated electron fluids in the stripe phases of high-temperature superconductors are coupled to an active environment, and so are expected to have gapped spin excitations [13]. As such they are best described as Luttinger liquids in the Luther-Emery regime [14] whose low-energy physics is described by a single Luttinger liquid for charge. The same is true of the stripe states of the spin-polarized two-dimensional electron gas (2DEG) in magnetic fields [5].

Now consider a system with N stripes, each labeled by an integer $a = 1, \ldots, N$. We will consider first the phase in which there is a spin gap. Here, the spin fluctuations are effectively frozen out at low energies. Nevertheless each stripe a has 2 degrees of freedom [4]: a transverse displacement field which describes the local dynamics of the configuration of each stripe, and the phase field ϕ_a for the charge fluctuations on each stripe. The action of the generalized Luttinger liquid which describes the smectic charged fluid of the stripe state is obtained by integrating out the displacement fields. These fluctuations give rise to a finite renormalization of the Luttinger parameter and velocity of each stripe. More importantly, the shape fluctuations, combined with the long wavelength interstripe Coulomb interactions, induce interstripe density-density and current-current interactions, leading to an imaginary time Lagrangian density of the form

$$\mathcal{L}_{\text{smectic}} = \frac{1}{2} \sum_{a,a',\mu} j_\mu^a(x) \tilde{W}_\mu(a - a') j_\mu^{a'}(x) . \quad (2)$$

These operators are marginal, i.e., have scaling dimension 2, and preserve the smectic symmetry $\phi_a \to \phi_a + \alpha_a$ (where α_a is constant on each stripe) of the decoupled Luttinger fluids. Whenever this symmetry is exact, the charge-density wave order parameters of the individual stripes do not lock with each other, and the charge-density profiles on each stripe can slide relative to each other without an energy cost. In other words, there is no rigidity to shear deformations of the charge configuration on nearby stripes. This is the smectic metal phase [4].

The fixed point action for a generic smectic metal phase thus has the form (in Fourier space)

$$\begin{aligned} S &= \sum_Q \frac{1}{2} \{W_0(Q)\omega^2 + W_1(Q)k^2\} |\phi(Q)|^2 \\ &= \sum_Q \frac{1}{2} \left\{ \frac{\omega^2}{W_1(Q)} + \frac{k^2}{W_0(Q)} \right\} |\theta(Q)|^2, \end{aligned} \quad (3)$$

where $Q = (\omega, k, k_\perp)$, and θ is the field dual to ϕ. Here k is the momentum along the stripe and $k_\perp$ perpendicular to the stripes. The kernels $W_0(Q)$ and $W_1(Q)$ are analytic functions of Q whose form depends on microscopic details, e.g., at weak coupling they are functions of the interstripe Fourier transforms of the forward and backward scattering amplitudes $g_2(k_\perp)$ and $g_4(k_\perp)$, respectively. Thus, the smectic fixed point is characterized by effective (inverse) Luttinger and velocity functions, $w(k_\perp) = \sqrt{W_0(k_\perp) W_1(k_\perp)}$ and $v(k_\perp) = \sqrt{W_1(k_\perp)/W_0(k_\perp)}$, and, like a 1D Luttinger liquid, by power-law decay of correlations functions [9].

In the presence of a spin gap, single electron tunneling is irrelevant [13], and the only potentially relevant interactions involving pairs of stripes a, a' are singlet pair (Josephson) tunneling, and the coupling between the CDW order parameters. These interactions have the form $\mathcal{H}_{\text{int}} = \sum_n (\mathcal{H}_{\text{SC}}^n + \mathcal{H}_{\text{CDW}}^n)$ for $a' - a = n$, where

$$\begin{aligned} \mathcal{H}_{\text{SC}}^n &= \left(\frac{\Lambda}{2\pi}\right)^2 \sum_a \mathcal{J}_n \cos[\sqrt{2\pi}(\theta_a - \theta_{a+n})], \\ \mathcal{H}_{\text{CDW}}^n &= \left(\frac{\Lambda}{2\pi}\right)^2 \sum_a \mathcal{V}_n \cos[\sqrt{2\pi}(\phi_a - \phi_{a+n})]. \end{aligned} \quad (4)$$

Here $\mathcal{J}_n$ are the interstripe Josephson couplings (SC), $\mathcal{V}_n$ are the $2k_F$ components of the interstripe density-density (CDW) interactions, and Λ is an ultraviolet cutoff, $\Lambda \sim 1/a$ where a is a lattice constant. A straightforward calculation yields the scaling dimensions $\Delta_{1,n} \equiv \Delta_{\text{SC},n}$ and $\Delta_{-1,n} \equiv \Delta_{\text{CDW},n}$ of $\mathcal{H}_{\text{SC}}^n$ and $\mathcal{H}_{\text{CDW}}^n$:

$$\Delta_{\pm 1,n} = \int_{-\pi}^{\pi} \frac{dk_\perp}{2\pi} [\kappa(k_\perp)]^{\pm 1} (1 - \cos n k_\perp) , \quad (5)$$

where $\kappa(k_\perp) \equiv w(0,0,k_\perp)$. Since $\kappa(k_\perp)$ is a periodic function of $k_\perp$ with period 2π, $\kappa(k_\perp)$ has a convergent Fourier expansion of the form $\kappa(k_\perp) = \sum_n \kappa_n \cos n k_\perp$. We will parametrize the fixed point theory by the coefficients κ_n, which are smooth nonuniversal functions. In what follows we shall discuss the behavior of the simplified model with $\kappa(k_\perp) = \kappa_0 + \kappa_1 \cos k_\perp$. Here, κ_0 can be thought of as the intrastripe inverse Luttinger parameter, and κ_1 is a measure of the nearest neighbor interstripe coupling. For stability we require $\kappa_0 > \kappa_1$. Since it is unphysical to consider longer range interactions in H_{int} than are present in the fixed point Hamiltonian, we treat only perturbations with $n = 1$, whose dimensions are $\Delta_{\text{SC},1} \equiv \Delta_{\text{SC}} = \kappa_0 - \frac{\kappa_1}{2}$, and $\Delta_{\text{CDW},1} \equiv \Delta_{\text{CDW}} = 2/(\kappa_0 - \kappa_1 + \sqrt{\kappa_0^2 - \kappa_1^2})$. For a more general function $\kappa(k_\perp)$, operators with larger n must also be considered, but the results are qualitatively unchanged [12,15].

In Fig. 1 we present the phase diagram of this model. The dark AB curve is the set of points where

$\Delta_{\mathrm{CDW}} = \Delta_{\mathrm{SC}}$, and it is a line of first order transitions. To the right of this line the interstripe CDW coupling is the most relevant perturbation, indicating an instability of the system to the formation of a 2D stripe crystal [4]. To the left, Josephson tunneling (which still preserves the smectic symmetry) is the most relevant, so this phase is a 2D smectic superconductor. (Here we have neglected the possibility of coexistence since a first order transition seems more likely.) Note that there is a region of $\kappa_0 \geq 1$, and large enough κ_1, where the global order is superconducting although, in the absence of interstripe interactions (which roughly corresponds to $\kappa_1 = 0$), the superconducting fluctuations are subdominant. There is also a (strong coupling) regime above the curve CB where both Josephson tunneling and the CDW coupling are irrelevant at low energies. Thus, in this regime the smectic metal state is stable. This phase is a 2D smectic non-Fermi liquid in which there is coherent transport only along the stripes.

The phase transitions from the smectic metal to the 2D smectic superconductor and the stripe crystal are continuous. The three phase boundaries meet at the bicritical point B, where $\kappa_0 \approx 4$ and $\kappa_1 \approx 0.97\kappa_0$. While the details of the phase diagram are nonuniversal, the basic properties of this model are quite general: the interstripe long wavelength density-density coupling rapidly increases the scaling dimension of the interstripe CDW coupling while the scaling dimension of the interstripe Josphson coupling is less strongly affected. Although for this model the smectic metal has a small region of stability, we expect it to grow for longer range interactions.

The transport properties of isolated Luttinger liquids have been studied extensively [16], and many of these results can be applied in this context. At temperatures well above any ordering transition, we can use perturbation theory about the smectic fixed point in powers of the scaling variables $X \equiv (\mathcal{J}/v)(\Lambda v/T)^{2-\Delta_{\mathrm{SC}}}$ and $Y \equiv (\mathcal{V}/v)(\Lambda v/T)^{2-\Delta_{\mathrm{CDW}}}$, and for weak disorder we can similarly employ perturbation theory in powers of the backscattering interaction, V_{back}. (Electron-phonon coupling produces results similar to those of disorder, although

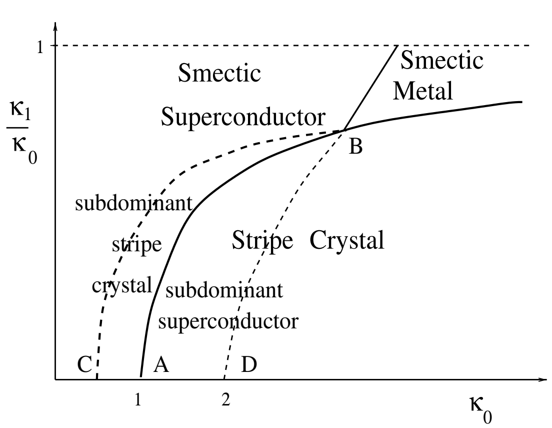

FIG. 1. Phase diagram for a system with a spin gap.

with a temperature dependent effective V_{back}.) However, because σ_{xx} and σ_{xy} are highly singular in the limit $V_{\mathrm{back}} \to 0$ (when the system is Galilean invariant along the stripes), we must resume the naive perturbation expansion of the Kubo formula to obtain perturbative expressions for the component of the resistivity tensor along a stripe ρ_{xx}, the Hall resistance ρ_{xy}, and the conductivity transverse to the stripe, σ_{yy}.

As is well known [16], $\rho_{xx} = 0$ for $V_{\mathrm{back}} = 0$, and develops a calculable power-law temperature dependence which, to leading order in V_{back}, is

$$\rho_{xx} = \frac{\hbar}{e^2 n_s v} \frac{|V_{\mathrm{back}}|^2}{T^2} \left(\frac{T}{v\Lambda}\right)^{\bar{\Delta}_{\mathrm{CDW}}} f_{xx}(X^2, Y^2) + \cdots, \tag{6}$$

where $f_{xx}(X,Y)$ is a scaling function and $f_{xx}(0,0) \sim 1$. Here, n_s is the density of stripes, and $\bar{\Delta}_{\mathrm{CDW}} \equiv \Delta_{\mathrm{CDW},\infty}$ is the dimension of the CDW order parameter.

Whether the interstripe Josephson coupling, $\mathcal{J}$, is irrelevant or relevant, so long as the temperature is not too low, the component of the conductivity tensor transverse to the stripe direction can be obtained from a perturbative evaluation of the Kubo formula to lowest order in powers of the leading coupling $\mathcal{J}$. Combining this result with a simple scaling analysis we find (to zeroth order in V_{back})

$$\sigma_{yy} = \frac{e^2}{h} n_s b^2 \Lambda \left(\frac{\mathcal{J}}{v}\right)^2 \left(\frac{T}{\Lambda v}\right)^{2\Delta_{\mathrm{SC}}-3} f_{yy}(X^2, Y^2), \tag{7}$$

where b is the spacing between stripes, f_{yy} is a scaling function, and $f_{yy}(0,0) \sim 1$. An interesting aspect of this expression is that, in the perturbative (high-temperature) regime, the temperature derivative of σ_{yy} changes from positive to negative at a critical value of $\Delta_{\mathrm{SC}} = 3/2$, whereas the actual superconductor to (CDW) insulator transition occurs somewhere in the range $1 < \Delta_{\mathrm{SC}} < 2$, depending on the value of κ_0/κ_1.

For a system with Galilean invariance along the stripes $\sigma_{xy} = n^{\mathrm{eff}} ec/B$, and, to leading order in V_{back},

$$\rho_{xy} = B/n^{\mathrm{eff}} ec + \cdots. \tag{8}$$

The physics governing n_{eff} is rather subtle—neglecting irrelevant couplings, the fixed point Hamiltonian is actually particle-hole symmetric, which implies $\rho_{xy} = 0$. Thus n^{eff} is determined by the leading irrelevant couplings which break particle-hole symmetry, terms of the form $(\partial_x \phi)^3$ and $(\partial_x \theta)^2 \partial_x \phi$. Generically, $1/n^{\mathrm{eff}}$ approaches a nonzero constant value at low temperatures. However, in special cases (e.g., the quarter-filled Hubbard chain in the infinite U limit) where there is an effective "particle-hole symmetry" at low energy, ρ_{xy} will vanish as a power of T [17].

Let us now discuss what happens if both charge and spin excitations are gapless on the stripes. We now have two Luttinger fluids on each stripe for charge and spin respectively, represented by the fields ϕ_c and ϕ_s. SU(2) spin invariance requires $K_s = 1$ whereas $K_c = K$ as in the spin-gap case. Here we will discuss a system in which there is only a coupling of the charge densities between

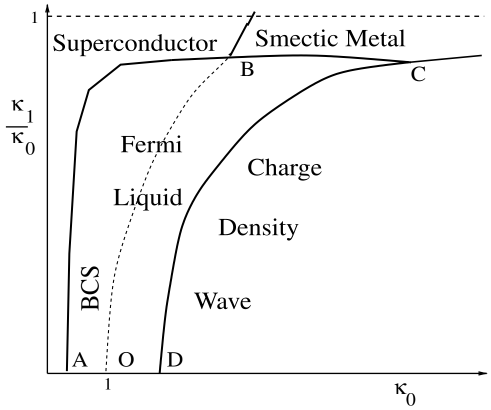

FIG. 2. Phase diagram for a system without a spin gap.

neighboring stripes and no exchange coupling. Since both spin and charge are gapless, electron tunneling has to be considered in addition to CDW coupling and Josephson tunneling. The dimensions of the most relevant CDW and Josephson interactions in the gapless spin case are $\Delta_{\rm CDW} = 1 + \Delta_{\rm CDW}^{(\rm Gap)}$, and $\Delta_{\rm SC} = 1 + \Delta_{\rm SC}^{(\rm Gap)}$, where $\Delta_{\rm CDW}^{(\rm Gap)}$ and $\Delta_{\rm SC}^{(\rm Gap)}$ are their dimensions in the spin-gap case, Eq. (5). The dimension of the nearest neighbor single electron tunneling operator is $\Delta_e = \frac{1}{4}(\Delta_{\rm SC}^{(\rm Gap)} + \Delta_{\rm CDW}^{(\rm Gap)} + 2)$. It is also easy to check that the dimensions of the $2k_F$ CDW and spin density wave (SDW) operators satisfy $\Delta_{\rm CDW} = \Delta_{\rm SDW}$. Similarly, the triplet and singlet superconductor couplings have the same dimension. The phase diagram is shown in Fig. 2. There is a large region of the phase diagram in which the electron tunneling operator is relevant, shown in Fig. 2 as the region below the curve *ABC* (defined by the marginality condition $\Delta_{e,1} = 2$). In this regime the system initially flows towards a 2D Fermi-liquid fixed point, which will itself exhibit a BCS instability in the presence of residual attractive interactions ($\kappa_0 < 1$). For stronger interstripe couplings the system crystallizes, and there are also strong coupling smectic metal (non-Fermi-liquid) and superconducting phases.

The non-Fermi-liquid smectic metal phase is a remarkable state of matter. Because interstripe tunneling of any type is irrelevant, the transport across the stripes is incoherent, whereas transport is coherent (and large) inside each stripe. Recently, evidence of the existence of a "metallic" stripe ordered state, which we tentatively identify as such a smectic, has been observed [18] in $\rm La_{1.4-x}Nd_{0.6}Sr_xCuO_4$: Glassy stripe order has been confirmed by neutron and x-ray scattering studies; the in-plane transport remains metallic (with at most a logarithmic increase) down to low temperatures while the interplane resistivity (which is perpendicular to the stripes) appears to diverge as $T \to 0$. On the same system photoemission experiments [19] have found strong evidence for one-dimensional electronic structure. Noda *et al.* [20] have found that, for $x \leq 1/8$, ρ_{xy} vanishes (roughly linearly) as $T \to 0$, while, for $x > 1/8$, although ρ_{xy} still decreases strongly at low temperatures, it appears to approach a finite value. They took this behavior to indicate a crossover from one- to two-dimensional metallic conduction at $x = 1/8$. We propose, instead, that the system is a smectic for a range of x, and that the crossover indicates that the stripes are nearly quarter filled, and have an approximate particle-hole symmetry for $x < 1/8$, while particle-hole symmetry is broken for $x > 1/8$. Finally, the present results suggest the existence of a smectic metal state of the 2DEG in large magnetic fields [4,5,21]; see however [22].

We thank S. Bacci, D. Barci, H. Esaki, M. P. A. Fisher, and Z. X. Shen for useful discussions. E. F. and S. A. K. are grateful to S. C. Zhang and the Department of Physics of Stanford University for their hospitality. This work was supported in part by the NSF, Grants No. DMR98-08685 (S. A. K.), No. DMR98-17941 (E. F.), and No. DMR97-30405 (T. C. L.), and at and by DMS, USDOE Contract No. DE-AC02-76CH00016 (V. J. E.).

Note added.—After this paper was sent for publication we became aware of Ref. [23].

[1] For a recent perspective on stripe phases, see V. J. Emery *et al.,* Proc. Natl. Acad. Sci. U.S.A. **96**, 8814 (1999).
[2] C. H. Chen and S.-W. Cheong, Phys. Rev. Lett. **76**, 4042 (1996).
[3] M. P. Lilly *et al.,* Phys. Rev. Lett. **82**, 394 (1999).
[4] S. A. Kivelson *et al.,* Nature (London) **393**, 550 (1998).
[5] E. Fradkin and S. A. Kivelson, Phys. Rev. B **59**, 8065 (1999).
[6] E. Fradkin *et al.,* Phys. Rev. Lett. **84**, 1982 (2000).
[7] See V. Emery, in *Highly Conducting One-Dimensional Solids,* edited by J. Devreese *et al.* (Plenum, New York, 1979).
[8] P. W. Anderson, Phys. Rev. Lett. **67**, 3844 (1991).
[9] R. A. Klemm and H. Gutfreund, Phys. Rev. B **14**, 1086 (1976); C. Bourbonnais and L. G. Caron, Int. J. Mod. Phys. B **5**, 1033 (1991); H. J. Schulz, J. Phys. C **16**, 6769 (1983).
[10] S. Chakravarty and P. W. Anderson, Phys. Rev. Lett. **72**, 3859 (1994).
[11] L. Golubović and M. Golubović, Phys. Rev. Lett. **80**, 4341 (1998); Erratum **81**, 5704 (1998); C. S. O'Hern and T. C. Lubensky, *ibid.* **80**, 4345 (1998).
[12] C. S. O'Hern *et al.,* Phys. Rev. Lett. **83**, 2745 (1999).
[13] V. J. Emery *et al.,* Phys. Rev. B **56**, 6120 (1997).
[14] A. Luther and V. J. Emery, Phys. Rev. Lett. **33**, 589 (1974).
[15] $\Delta_{\rm SC,2}$ is the most relevant operator. For a model with $\kappa(k_\perp) = [\kappa_0 + \kappa_1 \cos(k_\perp)]^2$, all perturbations are irrelevant for large κ_0 and small $|\kappa_0 - \kappa_1|$.
[16] A. Luther and I. Peschel, Phys. Rev. Lett. **32**, 922 (1974); W. Götze and P. Wölfle, Phys. Rev. B **6**, 1226 (1972).
[17] These results also follow from the Kubo formula.
[18] J. M. Tranquada, Nature (London) **375**, 561 (1995).
[19] X. J. Zhou *et al.,* Science **286**, 269 (1999).
[20] T. Noda *et al.,* Science **286**, 265 (1999).
[21] H. A. Fertig, Phys. Rev. Lett. **82**, 3693 (1999).
[22] A. H. MacDonald and M. P. A. Fisher, Phys. Rev. B **61**, 5724 (2000); H. Yi *et al.,* cond-mat/0003139.
[23] A. Vishwanath and D. Carpentier, cond-mat/0003036.