

Gravitational Waves from Mesoscopic Dynamics of the Extra Dimensions

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Recent models which describe our world as a brane embedded in a higher dimensional space introduce new geometrical degrees of freedom, associated with spatial variations in the position of the brane and the size of the extra dimensions, that can be coherently excited by symmetry breaking in the early universe even on “mesoscopic” scales as large as 1 mm. The characteristic frequency and intensity of resulting gravitational radiation backgrounds are estimated. Extra dimensions with scale between 10^{-14} and 1 mm can produce detectable backgrounds at frequencies $f \approx 10^3$ to 10^{-4} Hz.

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I. Mesoscopic cosmology of the extra dimensions.—Recently there has been considerable interest in models of spacetimes with relatively large extra dimensions, in which the familiar standard model fields are confined to a “brane,” a three-dimensional defect in the larger space, while gravity propagates in all the dimensions (the “bulk”). In some models [1–3], the extra dimensions can be as large as the current direct experimental gravitational probes of the order of a millimeter [4]. The apparent (usual) Planck mass in $(3 + 1)D$, M_{Planck} , is related to the true fundamental scale M_* by $M_{\text{Planck}}^2 \approx M_*^2(M_*^n b^n)$ where b^n is the volume of n extra large dimensions. In another class of models (“nonfactorizable geometries” [5–7]), the extra dimensions can be even larger, but have curvature k which traps gravitons in a bound state close to a brane. The curvature radius $b \approx k^{-1} \approx M_{\text{Planck}}^2/M_*^3$ is again on a mesoscopic scale which may be as large as ≈ 1 mm.

The cosmology of these models might well include radical departures from the orderly evolution of the standard model, including episodes of violent mesoscopic geometrical activity at relatively late times. New classical geometrical effects remain important long after inflation, until $T \leq M_*$ and/or the Hubble length $H^{-1} \approx M_{\text{Planck}}/T^2$ is larger than the size or curvature radius b of the largest extra dimensions [3,8,9]. Of particular interest are two new geometrical degrees of freedom common to many of these models: “radion” modes controlling the size or curvature of the extra dimensions [8,10], and new Nambu-Goldstone modes corresponding to inhomogeneous displacements of the brane in the extra dimensions [3,11]. Previous treatments have focused on microscopic thermal and quantum emission in these modes, but cosmological symmetry breaking can also create large-amplitude, coherent classical excitations on much larger scales (of the order of H^{-1}) as the configuration of the extra dimensions and the position of the brane settle into their present state. This highly dynamic geometric activity generically produces a conspicuous detectable relic: an intense, classically generated background of gravitational radiation. In this paper I explore the possibility of a direct detection of a gravitational wave background

generated by coherently excited radion and Nambu-Goldstone modes on the threshold of standard cosmology.

II. Hubble frequency and energy equipartition.—The main features of the background spectra can be estimated from general scaling considerations, without reference to particular models.

a. Frequency: The characteristic gravitational (“Hubble”) frequency [12,13] redshifted to the present day is $f_{H0}(T) \equiv H(T)a(T)/a_0$ or

$$\begin{aligned} f_{H0} &= (7.65 \times 10^{-5} \text{ Hz}) (T/\text{TeV}) g_*^{1/6} (g_*/g_{*S})^{1/3} T_{2.728} \\ &= (9.37 \times 10^{-5} \text{ Hz}) (H \times 1 \text{ mm})^{1/2} \\ &\quad \times g_*^{-1/12} (g_*/g_{*S})^{1/3} T_{2.728}. \end{aligned} \quad (1)$$

The estimate is valid back to the threshold of the extradimensional dynamics, $H^{-1} \approx b$. The mapping between observed frequency, temperature T and Hubble length is shown by axis labels in Fig. 1 for frequencies accessible to the proposed space interferometer LISA. Note the weak dependence on the particle-physics uncertainties encapsulated in the number of effective relativistic degrees of freedom, defined in the usual way $g_* \equiv \sum_{\text{bosons}} (T_i/T)^4 g_i + (7/8) \sum_{\text{fermions}} (T_i/T)^4 g_i$ and $g_{*S} \equiv \sum_{\text{bosons}} (T_i/T)^3 g_i + (7/8) \sum_{\text{fermions}} (T_i/T)^3 g_i$. Even above 100 GeV we expect standard relativistic cosmology derived from relativity and thermodynamics to hold (e.g., [14]), $H = (2.07 \times 10^5 \text{ Hz}) g_*^{1/2} (T/\text{GeV})^2$ and $a(T)/a_0 = 3.70 \times 10^{-13} T_{2.728} g_{*S}^{-1/3} (\text{GeV}/T)$. Extra dimensions between $b \approx 1$ and 10^{-6} mm produce backgrounds peaked in the LISA band (10^{-1} to 10^{-4} Hz); ground-based observations with LIGO or VIRGO (up to ≈ 1000 Hz) can detect activity from dimensions as small as $b \approx 10^{-14}$ mm.

b. Amplitude: The mechanisms considered here excite geometrical degrees of freedom in approximate equipartition of energy density with the thermal relativistic matter. A stochastic background of gravitational radiation with rms metric perturbations $h_{\text{rms}}(f)$ over bandwidth Δf contributes a fraction of the critical density $\Omega_{\text{GW}}(f, \Delta f = f) \approx (f/H)^2 h_{\text{rms}}^2(f, \Delta f = f)$. In an

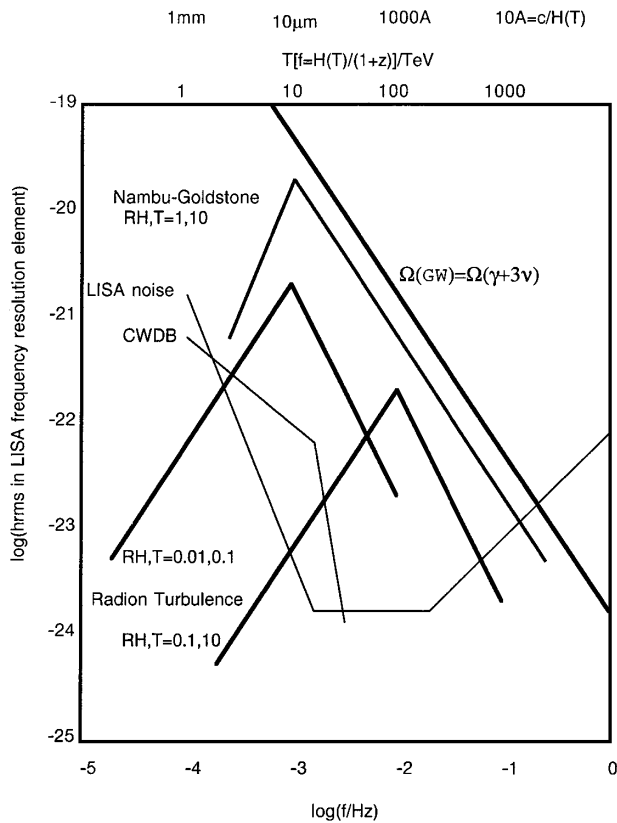


FIG. 1. Spectra of predicted gravitational wave backgrounds in the LISA band. The amplitude of metric strain in a frequency resolution element after one year of observations ($\Delta f = 3 \times 10^{-8}$ Hz) is plotted against frequency f . Sensitivity limits are shown from LISA shot and acceleration noise (1σ per resolution element after one year) and the confusion-limited astrophysical background from compact white dwarf binaries (CWDB). The uppermost curve shows the amplitude of a background which has a broad-band energy density equal to the known cosmological standard model relativistic degrees of freedom (photons and neutrinos). The other curves sketch predicted backgrounds from mesoscopic activity in new extra-dimensional modes. The spectrum from relativistic flows is shown for “maximal” turbulence sustained for a Hubble time, on scale $RH \approx 0.1$ at $T_* = 10$ TeV and for $RH \approx 0.01$ at $T_* = 0.1$ TeV. There is near degeneracy in the determination of parameters, in particular, a smaller nucleation scale RH at a lower redshift comes close to mimicking a larger one at a higher redshift. The spectrum from maximally excited Nambu-Goldstone modes of brane displacement is shown for the scale-free limiting case up to the damping epoch, taken here to be 10 TeV (or $b_0 = 10\mu$).

experiment such as LISA a stochastic background can only be distinguished from other sources of noise and astrophysical wave sources by resolving the background in frequency (and to some extent, direction and polarization). The rms strain produced in a LISA frequency resolution element is [15]

$$h_{\text{rms}}(f, \Delta f) = 4.74 \times 10^{-20} f_{\text{mHz}}^{-3/2} T_{2.728}^2 h_{70}^0 \times (\Delta f / 3 \times 10^{-8} \text{ Hz})^{1/2} \times [\Omega_{\text{GW}}(\Delta f = f) / \Omega_{\text{rel}}]^{1/2}. \quad (2)$$

The reference density is set [12,13] by the mean energy density in all relativistic species (photons and three massless neutrinos), $\Omega_{\text{rel}} = 8.51 \times 10^{-5} h_{70}^{-2} T_{2.728}^4$, where h_{70} refers to the Hubble constant. Since the energy density of gravitational waves redshifts like relativistic matter, this gives a maximal bound for primordial broad-band backgrounds, shown in Fig. 1 along with projected LISA sensitivity [16].

A chaotic cosmology is almost unconstrained by other data if it happens early enough. Scalar inhomogeneities (including entropy perturbations) from subhorizon events earlier than about 100 GeV are erased by neutrino and nucleon diffusion before nucleosynthesis begins [17,18], so gravitational waves may be the only direct surviving relic. Overproduction of black holes or other stable relics must be avoided but these are very model dependent [19]. Existing data constrain gravitational waves only at much lower frequencies; above $f_{\text{msp}} = 4.4 \times 10^{-9}$ Hz [corresponding to $\approx f_{H0}(T \approx 60 \text{ MeV}, H^{-1} \approx 453 \text{ km})$], millisecond pulsar timing gives a limit at 90% confidence [20], $\Omega_{\text{GW}} < 1.14 \times 10^{-4} (f/f_{\text{msp}})^2 T_{2.728}^{-4} h_{70}^0 \Omega_{\text{rel}}$.

III. Turbulent flow from a first-order radion transition.—The radion can be a significant source of mesoscopic activity if its potential has a first-order phase transition. The radion is stuck initially in a metastable state with some initial value b_i and thermal or quantum nucleation leads to randomly nucleated regions corresponding to the final value b_0 , accompanied by a release of internal energy. The gravitational waves from this mode are easiest to describe if for the largest extra dimensions, $b_0 \leq H^{-1}(T = M_*)$: the final dimensional stabilization (at M_*) then happens within the context of an approximately $(3 + 1)$ -dimensional cosmology.

On our brane this process resembles nucleation of bubbles or vacuum domains of Higgs scalars [21], with the extra complication of modifications in gravity. In an expanding universe [22–24] the bubbles collide and overlap, creating flows of energy with velocities of the order of unity. The coherence scale R of the flows is determined by nucleation dynamics which generally yields [25] $R \leq \log(HT)/H \approx 10^{-2} H^{-1}$. A number of models have been used to estimate the gravitational radiation in similar situations, from colliding bubbles and the resulting energy flows, in the context of QCD and electroweak phase transitions [26–29]. In the absence of a definite model of radion bubbles, we estimate the maximal gravitational wave spectrum, based on establishing a sustained relativistic turbulent cascade up to the scale R for a time H^{-1} .

The power output of a system in gravitational waves [30,31], $L_{\text{GW}} \approx 0.1 L_{\text{internal}}^2 / L_0$, is determined by the (changing quadrupolar) flow of mass energy L_{internal} , where $L_0 = c^5/G = M_{\text{Planck}}^2$ and the numerical factor ≈ 0.1 is typical of simple asymmetric geometries. Flows of mass energy on the Hubble scale produce a gravitational wave power per volume close to $10^{-1} H^3 L_0$, and integrated

over time H^{-1} produce broad-band $\Omega_{\text{GW}} \approx 10^{-1} \Omega_{\text{rel}}$ at characteristic frequency H (now shifted to f_{H0}). Flows on smaller scale R create a spectrum peaked at higher frequency $f_{\text{peak}} \approx f_{H0}(RH)^{-1}$ and with a smaller amplitude. Relativistic motions in a volume R^3 with density perturbations of order unity, $M \propto R^3$, create a mass-energy flow $L_{\text{internal}} \propto (M/R) \propto R^2$ which if sustained for time H^{-1} gives $\Omega(\Delta f = f) \propto R \propto f_{\text{peak}}^{-1}$. In a narrow band this translates to amplitude $h_{\text{rms}}(f, \Delta f = 3 \times 10^{-8} \text{ Hz}) \approx \Omega(\Delta f = f)^{1/2} (\Delta f / f)^{1/2} (H / f) \propto f_{\text{peak}}^{-2}$. To estimate the low-frequency spectrum consider motions of smaller velocity on the same length scale, $v \approx Rf < 1$. The mass-energy flow is $L_{\text{internal}} \approx Mv^2(v/R) \propto v^3$, hence the gravitational wave power $\propto f^6$. The maximal tail of low-frequency waves then has $\Omega(\Delta f = f) \propto f^6$, hence $h_{\text{rms}}(f, \Delta f = 3 \times 10^{-8} \text{ Hz}) \propto f^{3/2}$. These estimates lead to the maximal spectrum shown in Fig. 1 for $R, T = 0.1H^{-1}$, 10 TeV, and $R, T = 0.01H^{-1}$, 100 GeV. [In the quieter case where turbulence is not sustained, the amplitude is less than this estimate by $(RH)^{-1/2}$.]

IV. Gravitational waves from brane displacement.—In many scenarios there is another degree of freedom, the position of the brane. Spatial inhomogeneities in this displacement correspond to nearly massless Nambu-Goldstone modes [3,11]. In the cosmological formation of the brane/defect, the position is in general a random variable uncorrelated on large scales, and large-scale modes are excited by the Kibble mechanism. Since the modes also represent curvature of the brane, they are efficiently coupled into gravitational waves as viewed on the brane. This mode is generally excited if for the largest extra dimension, $b_0 \geq H^{-1}(T = M_*)$: the 3-brane condenses as a defect before cosmology enters the $(3 + 1)\text{D}$ era.

Suppose our brane forms at some early time as a defect in $3 + n + 1$ space, spontaneously breaking the Poincaré symmetry of the full theory. The position of the brane in each higher dimension j can be described by a field $y_j(x_i)$ which depends on the coordinate x_i on the brane. [The y_j 's are analogous [3,11] to group-transformation angles θ in the Goldstone description of the pion or axion, but have dimensions of length. They are related to the canonically normalized physical fields π by $y = \pi/f_b^2$ (rather than $\theta = \pi/f_b$) where f_b^4 is the brane tension. When a y is “eaten” it gets a mass $m_y \approx f_b^2/M_{\text{Planck}}$ or $\approx (1 \text{ mm})^{-1}$ for $f_b \approx \text{TeV}$.] These fields represent propagating Nambu-Goldstone bosons, new modes in addition to the standard model fields on the brane and gravitation propagating in the bulk.

In a cosmological setting, the initial values of the y_j 's are not generally correlated on large scales but are random for points with large separation, since the position of the brane, when it condenses as a defect, is determined by local accidents. A topologically stable 3-wall forms, for example, if vacua in a 4-bulk fall into two discrete degenerate minima; the spontaneous choice of one minimum or

the other is a random variable at large separation, so the wall is not formed initially in its (flat) ground state, but in an excited (wrinkled) state [32,33]. This is the same “Kibble mechanism” which leads in other contexts to formation of cosmic strings, axion miniclusters, or Goldstone excitations of scalar fields (which are also gravitational wave sources, e.g., [12,34–37]).

If the cosmology thereby excites these modes, the brane is wrinkled on all scales up to the dimensional size or curvature radius b_0 (which we take to be fixed here), curving through the extra dimensions. Ignoring the subtleties of the cosmological metric and imagining the brane in a flat spacetime, displacements directly induce an effective 4-metric on the brane $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, where the perturbed 4-metric is $h_{\mu\nu} = \sum_{j=1}^n \partial_\mu y_j \partial_\nu y_j$. Although these initial perturbations are scalars, no symmetry prevents transverse-traceless tensor components [30] from being excited dynamically for waves with $f \geq H$, creating classical gravitational waves. Kibble excitations corresponding to perturbations of magnitude δy on scale f^{-1} thereby lead to wave amplitudes of the order of $h_{\text{rms}}(f) \approx |h_{jk}^{TT}| \approx |\delta y f|^2$.

In the cosmological context, the Kibble excitation is regulated by the expansion. The brane location is initially uncorrelated on large scales, so on the Hubble scale variations in y are of the order of H^{-1} , producing waves with amplitude $h_{\text{rms}} \approx H^2 \delta y^2 \approx O(1)$, comparable in total energy with the other forms of energy on the brane ($\rho_{\text{rel}} \approx H^2 M_{\text{Planck}}^2$) which now appear in Ω_{rel} . These waves show up today as gravitational waves at the redshifted Hubble frequency $f_{H0}(H)$. The maximal spectrum corresponds to a background of the order of the energy density in brane fields (that is, $h = O(1)$ on each scale as it enters the horizon), leading to constant Ω_{GW} over a range of higher frequencies. The amplitude of excitation is reduced after the universe enters the classical $(3 + 1)\text{D}$ era, when $H^{-1} \geq b_0$ (or after $H \leq m_y$); variations in y are at most of order b_0 (or m_y), leading to $h_{\text{rms}} \approx (Hb_0)^2$ and to a spectrum with a corresponding $h \propto f^{5/2}$ roll-off at lower frequencies. The damping scale and low-frequency spectrum contain information on the scale of extra dimensions and/or the stabilization of the brane. The spectrum at higher frequencies is a probe of the cosmological model during the pre- $(3 + 1)\text{D}$ period, including the excitation mechanism.

The predicted backgrounds can be distinguished from other expected signals by their isotropy, and by the distinctive spectra with roll-offs at both low and high frequencies. Under some circumstances these backgrounds could be stronger than previously contemplated sources [15,31,37].

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