

## Quantum Dot as Spin Filter and Spin Memory

Patrik Recher, Eugene V. Sukhorukov, and Daniel Loss

*Department of Physics and Astronomy, University of Basel, Klingelbergstrasse 82, CH-4056 Basel, Switzerland*  
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We consider a quantum dot in the Coulomb blockade regime weakly coupled to current leads and show that in the presence of a magnetic field it acts as an efficient spin filter (at the single-spin level), producing a spin-polarized current. Conversely, if the leads are fully spin polarized the up or the down state of the spin on the dot results in a large sequential or a small cotunneling current, and, thus, together with ESR techniques, the setup can be operated as a single-spin memory.

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An increasing number of spin-related experiments [1–6] show that the spin of the electron offers unique possibilities for finding novel mechanisms for information processing—most notably in quantum-confined semiconductors with unusually long spin-dephasing times approaching microseconds [2], and where spins can be transported coherently over distances of up to 100  $\mu\text{m}$  [2]. Besides the intrinsic interest in spin-related phenomena, spin-based devices hold promises for future applications in conventional [1] as well as in quantum computer hardware [7]. One of the challenging problems for such applications is to obtain sufficient control over the spin dynamics in nanostructures. In the following we address this issue and propose a quantum-dot setup which can be operated either as a spin filter (spin diode) to produce spin-polarized currents or as a device to detect (“readout”) and manipulate single-spin states (single-spin memory) [8]. Both effects occur at the single-spin level and thus represent the ultimate quantum limit of a spin filter and spin memory. In both cases, we will work in the Coulomb blockade regime [9] and consider sequential and cotunneling processes. A new feature of our proposal is that the spin degeneracy is lifted [10], with *different* Zeeman splittings in the dot and in the leads, which then results in Coulomb blockade peaks which are uniquely associated with a definite spin state on the dot.

**Formalism.**—Our system consists of a quantum dot (QD) connected to two Fermi-liquid leads which are in equilibrium with reservoirs kept at the chemical potentials  $\mu_l$ ,  $l = 1, 2$ , where outgoing currents can be measured; see Fig. 1. By using a standard tunneling Hamiltonian approach [11], we write for the full Hamiltonian  $H_0 + H_T$ , where  $H_0 = H_L + H_D$  describes the leads and the dot, with  $H_D$  including the charging and interaction energies of the dot electrons as well as the Zeeman energy  $g\mu_B B$  of their spins in the presence of an external magnetic field  $\mathbf{B} = (0, 0, B)$ , where  $g$  is the effective  $g$  factor. We concentrate first on unpolarized lead currents and assume that the Zeeman splitting in the leads is negligibly small compared to the one in the QD. This can be achieved, e.g., by using InAs for the dot ( $g = 15$ ) attached to GaAs two-dimensional electron gas (2DEG) leads ( $g = -0.44$ ), or by implanting a magnetic impurity (say, Mn) inside a

GaAs dot (again attached to GaAs 2DEG leads) with a strongly enhanced electron  $g$  factor due to exchange splitting with the magnetic impurity [12]. (Below we will also consider the opposite situation with a fully spin-polarized lead current and a much smaller Zeeman splitting on the dot.) The tunneling between leads and the QD is described by the perturbation  $H_T = \sum_{l,k,p,\sigma} t_{lp} c_{lk\sigma}^\dagger d_{p\sigma} + \text{H.c.}$ , where  $d_{p\sigma}$  and  $c_{lk\sigma}$  annihilate electrons with spin  $\sigma$  in the dot and in the  $l$ th lead, respectively. While the orbital  $k$  dependence of the tunneling amplitude  $t_{lp}$  can be safely neglected, this is not the case in general for the QD orbital states  $p$ . From now on, we concentrate on the Coulomb blockade (CB) regime [9], where the charge in the QD,  $\hat{N} = \sum_{p,\sigma} d_{p\sigma}^\dagger d_{p\sigma}$ , is quantized, i.e.,  $\langle \hat{N} \rangle = N$ . Next, turning to the dynamics induced by  $H_T$ , we introduce the reduced density matrix for the dot,  $\rho_D = \text{Tr}_L \rho$ , where  $\rho$  is the full stationary density matrix, and  $\text{Tr}_L$  is the trace over the leads. To describe the stationary limit, we use a standard master equation approach [9] formulated

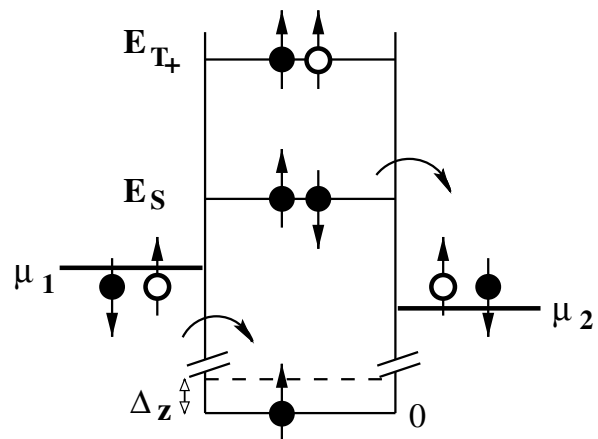


FIG. 1. The energy diagram of a QD attached to two leads is shown in the regime where the QD contains an odd number  $N$  of electrons with a topmost single electron in the ground state [ $\uparrow$  (filled circle), and  $E_T = 0$ ]. A cotunneling process is depicted (arrows) where two possible virtual states, singlet  $E_S$  and triplet  $E_{T+}$ , are shown. The parameter  $E_S - \mu_1$  can be tuned by the gate voltage to get into the sequential tunneling regime, defined by  $\mu_1 \geq E_S \geq \mu_2$ , where  $N$  on the QD fluctuates between odd and even. For  $N$  even, the ground state contains a topmost singlet state with  $E_S < \mu_1, \mu_2$ .

in terms of the dot eigenstates and eigenenergies,  $H_D|n\rangle = E_n|n\rangle$ , where  $n = (\mathbf{n}, N)$ . By denoting with  $\rho(n) = \langle n|\rho_D|n\rangle$  the stationary probability for the dot to be in the state  $|n\rangle$ , and with  $W(n', n)$  the transition rates between  $n$  and  $n'$ , the stationary master equation to be solved reads  $\sum_n [W(n', n)\rho(n) - W(n, n')\rho(n')] = 0$ .

The rates  $W$  can be calculated in a standard “golden rule” approach [13], where we go up to second order in  $H_T$ , i.e.,  $W = \sum_l W_l + \sum_{l', l} W_{l'l}$ , where  $W_l \propto t^2$  is the rate for a tunneling process of an electron from the  $l$ th lead to the dot and back, while  $W_{l'l} \propto t^4$  describes the simultaneous tunneling of two electrons from the lead  $l$  to the dot and from the dot to the lead  $l'$ . Thus, two regimes of transport through the QD can be distinguished: sequential tunneling (ST) and cotunneling (CT) [9,14]. The ST regime is at the degeneracy point, where  $\hat{N}$  fluctuates between  $N$  and  $N' = N \pm 1$ , and first order transitions are allowed by energy conservation with the explicit rates

$$W_l(n', n) = 2\pi\nu \{ f_l(\Delta_{n'n}) |A_{lnn'}^\sigma|^2 \delta_{N', N+1} + [1 - f_l(\Delta_{nn'})] |A_{lnn'}^\sigma|^2 \delta_{N', N-1} \}, \quad (1)$$

where  $\nu = \sum_k \delta(\varepsilon_F - \varepsilon_k)$  is the lead density of states per spin at the Fermi energy  $\varepsilon_F$ ,  $f_l(\varepsilon) = \{1 + \exp[(\varepsilon - \mu_l)/k_B T]\}^{-1}$  is the Fermi function at tempera-

ture  $T$ ,  $\Delta_{n'n} = E_{n'} - E_n$  is the level distance, and we have introduced the matrix elements  $A_{lnn'}^\sigma = \sum_p t_{lp} \langle n'|d_{p\sigma}|n\rangle$  [note that  $n$  and  $n'$  in Eq. (1) fix the spin index  $\sigma$ ]. In the ST regime the current through the QD can be written as  $I_s = \pm e \sum_{n, n'} W_2(n', n)\rho(n)$ , where  $(\pm)$  stands for  $N' = N \mp 1$ . We emphasize that the rates  $W(n, n')$  and thus the current depend on the spin state of the dot electrons via  $n, n'$ . The ST current takes a particularly simple form if the voltage bias,  $\Delta\mu = \mu_1 - \mu_2 > 0$ , and the temperature are small compared to the level distance on the dot (the case of interest here),  $\Delta\mu, k_B T < |\Delta_{mn}|$ ,  $\forall m, n$ , and thus only the lowest energy levels participate in the transport [9]. The solution of the master equation gives, in this case, for the ST current

$$I_s = \frac{e\gamma_1\gamma_2}{\gamma_1 + \gamma_2} [f_1(\Delta_{n'n}) - f_2(\Delta_{n'n})], \quad N' = N + 1, \quad (2)$$

where  $\gamma_l = 2\pi\nu |A_{lnn'}^\sigma|^2$  is the tunneling rate through the  $l$ th barrier. For  $N' = N - 1$ , we again get Eq. (2) but with  $n \leftrightarrow n'$ . In the CT regime the only allowed processes are second order transitions with the initial and the final electron number on the QD being equal, i.e.,  $N = N'$ , and with the rate

$$W_{l'l}(n', n) = 2\pi\nu^2 \int d\varepsilon f_l(\varepsilon) [1 - f_{l'}(\varepsilon - \Delta_{n'n})] \sum_{\sigma, \sigma'} \left| \sum_{n_1} \frac{A_{ln'n_1}^{\sigma'} A_{lnn_1}^{\sigma*}}{\Delta_{nn_1} + \varepsilon} + \sum_{n_2} \frac{A_{l'n_2n}^{\sigma'} A_{ln_2n'}^{\sigma*}}{\Delta_{n'n_2} - \varepsilon} \right|^2, \quad (3)$$

where  $N_1 = N + 1$ , and  $N_2 = N - 1$ , and thus the two terms in Eq. (3) differ by the sequence of tunneling. Our regime of interest here is *elastic* CT, where  $E_{n'} = E_n$ , which holds for  $|\Delta_{mn}| > \Delta\mu, k_B T$ ,  $\forall m \neq n$ . This means that the system is always in the ground state with  $\rho(n) = 1$ , and thus the CT current is given by  $I_c = eW_{21}(n, n) - eW_{12}(n, n)$ . In particular, close to a ST resonance (but still in the CT regime) Eq. (3) considerably simplifies—only one term contributes—and, for  $\Delta\mu, k_B T < |\mu \pm \Delta_{nn_i}|$ , we obtain

$$I_c = \frac{e}{2\pi} \frac{\gamma_1\gamma_2\Delta\mu}{(\mu \pm \Delta_{nn_i})^2}, \quad (4)$$

where  $(+)$  stands for  $i = 1$ ,  $(-)$  represents  $i = 2$ , and  $\mu = (\mu_1 + \mu_2)/2$ . From Eqs. (2) and (4) it follows that  $I_s \sim \gamma_i$ , while  $I_c \sim \gamma_i^2$ , and therefore  $I_c \ll I_s$ . Thus, in the CB regime the current as a function of  $\mu$  (or gate voltage) consists of resonant ST peaks, where  $\hat{N}$  on the QD fluctuates between  $N$  and  $N \pm 1$ . The peaks are separated by plateaus, where  $N$  is fixed and the (residual) current is due to CT.

We note that the tunneling rates  $\gamma_l$  depend on the tunneling path through the matrix elements  $A_{lnn'}^\sigma$ . In general, this can lead to a spin dependence of the current, which, however, is difficult to measure [15]. In contrast to this, we now show that a much stronger spin dependence can come from the resonance character of the currents  $I_s$  and  $I_c$ , when the position of a resonance (as a function of gate voltage) depends on the spin orientation of the tunneling

electron. To proceed we first specify the energy spectrum of the QD more precisely. In general, the determination of the spectrum of a QD is a complicated many-electron problem [16]. However, it is known from experiment [17] that, especially away from orbital degeneracy points (which can be easily achieved by applying magnetic fields [17]), the spectrum is formed mainly by single-particle levels, possibly slightly renormalized by exchange interaction [18].

For a QD with  $N$  odd there is one unpaired electron in one of the two lowest energy states,  $|\uparrow\rangle$  and  $|\downarrow\rangle$ , with energies  $E_\uparrow$  and  $E_\downarrow$ , which become Zeeman split due to a magnetic field  $B$ ,  $\Delta_z = |E_\uparrow - E_\downarrow| = \mu_B |gB|$ . Let us assume that  $|\uparrow\rangle$  is the ground state, and set  $E_\uparrow = 0$  for convenience. For  $N$  even, the two topmost electrons (with the same orbital wave function) form a spin singlet,  $(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$ , with energy  $E_S$ . This is the ground state. The other states, such as three triplets  $|T_+\rangle = |\uparrow\uparrow\rangle$ ,  $|T_-\rangle = |\downarrow\downarrow\rangle$ , and  $|T_0\rangle = (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2}$  with energies  $E_{T_\pm}$  and  $E_{T_0}$ , are excited states, because of higher (mostly) single-particle orbital energy.

*Sequential tunneling.*—First we consider the ST peak, which separates two plateaus with  $N$  and  $N + 1$  electrons on the dot, where  $N$  is odd (odd-to-even transition). In the regime  $E_{T_+} - E_S, \Delta_z > \Delta\mu, k_B T$ , only ground-state transitions are allowed by energy conservation. The tunneling of spin-up electrons is blocked by energy conservation, i.e.,  $I_s(\uparrow) = 0$ , because it involves excited states  $|T_+\rangle$  and  $|\downarrow\rangle$ . The only possible process is the tunneling of

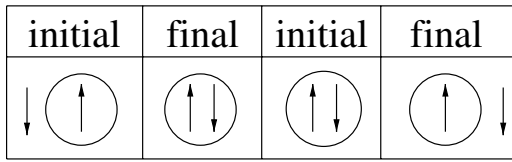


FIG. 2. The only allowed processes for charge transport through the dot in the ST regime at the odd-to-even transition. A spin-down electron tunnels from lead 1 to the dot, forming a singlet, and tunnels out again into lead 2. Tunneling of spin-up electrons into the dot is forbidden by energy conservation since this process involves excited states. The resulting current,  $I_s(\downarrow)$ , is spin polarized.

spin-down electrons, as shown in Fig. 2, which leads to a *spin-polarized* current,  $I_s(\downarrow)$ , given by Eq. (2), where  $\Delta_{n'n} = E_S > 0$  (since  $E_{\uparrow} = 0$ ). Thus, we have

$$I_s(\downarrow)/I_0 = \theta(\mu_1 - E_S) - \theta(\mu_2 - E_S),$$

$$k_B T < \Delta\mu, \quad (5)$$

$$I_s(\downarrow)/I_0 = \frac{\Delta\mu}{4k_B T} \cosh^{-2} \left[ \frac{E_S - \mu}{2k_B T} \right], \quad k_B T > \Delta\mu, \quad (6)$$

where  $I_0 = e\gamma_1\gamma_2/(\gamma_1 + \gamma_2)$ . Hence, in the specified regime the dot acts as a spin filter through which only spin-down electrons can pass [19]. Second, we consider the ST peak at the transition from even to odd, i.e., when  $N$  is even. The current is then given by Eq. (2) with  $\Delta_{n'n} = -E_S > 0$ . The spin-down current is now blocked,  $I_s(\downarrow) = 0$ , while the spin-up electrons can pass through the QD, with the current  $I_s(\uparrow)$  given by (5) and (6), where  $E_S$  has to be replaced by  $-E_S$ . Because this case is very similar to the previous one with  $\downarrow$  replaced by  $\uparrow$ , we will concentrate on the odd-to-even transition only. Next, we will demonstrate that, although CT processes can in general lead to a leakage of unwanted current, this turns out to be a minor effect, and spin filtering works also in the CT regime.

*Cotunneling.*—Above or below a ST resonance, i.e., when  $E_S > \mu_{1,2}$  or  $E_S < \mu_{1,2}$ , the system is in the CT regime. Close to this peak the main contribution to the transport is due to two processes (a) and (c); see Fig. 3, where the energy deficit of the virtual states,  $|\mu - E_S|$ , is minimal. According to Eq. (4), we have

$$I_c(\downarrow) = \frac{e}{2\pi} \frac{\gamma_1\gamma_2\Delta\mu}{(\mu - E_S)^2}. \quad (7)$$

Thus, we expect the spin filtering of down electrons to work even in the CT regime close to the resonance. However, there are additional CT processes, 3(b) and 3(d), which involve tunneling of spin-up electrons and lead to a leakage of up-spin. If  $N$  is odd (below the resonance), the dot is initially in its ground state ( $\uparrow$ ), and an incoming spin-up electron can form only a virtual triplet state  $|T_+\rangle$  [process (b) in Fig. 3]. This process contributes to the rate (2) with an energy deficit  $E_{T_+} - \mu$ , so that for the efficiency of spin filtering [defined as  $I(\downarrow)/I(\uparrow)$ ] we obtain, in this regime,

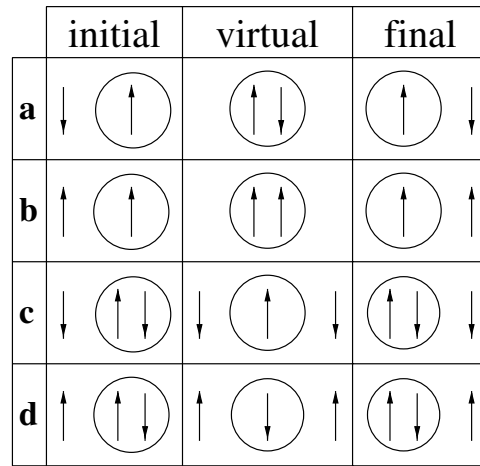


FIG. 3. (a) and (b) are the main processes in the cotunneling regime with  $N$  odd when inelastic processes and processes where the dot is not in the ground state are suppressed by the Zeeman energy  $\Delta_z$ . Only the leading virtual transitions are shown. (c) and (d) visualize the leading cotunneling processes for  $N$  even. Here, other processes are suppressed by the energy difference between singlet and triplet,  $E_{T_+} - E_S$ .

$$I_c(\downarrow)/I_c(\uparrow) \sim \left( 1 + \frac{E_{T_+} - E_S}{E_S - \mu} \right)^2, \quad N \text{ odd}. \quad (8)$$

Above the resonance, i.e., when  $N$  is even and the ground state is the spin singlet  $|S\rangle$ , the tunneling of spin-up electrons occurs via the virtual spin-down state [process (d) in Fig. 3] with an energy deficit  $\Delta_z + \mu - E_S$ , which has to be compared to the energy deficit  $\mu - E_S$  of the main process [Fig. 3(c)]. Thus, we obtain, for the efficiency of the spin filtering in the CT regime,

$$I_c(\downarrow)/I_c(\uparrow) \sim \left( 1 + \frac{\Delta_z}{\mu - E_S} \right)^2, \quad N \text{ even}. \quad (9)$$

We see that in both cases, above and below the resonance, the efficiency can be made large by tuning the gate voltage to the resonance, i.e.,  $|\mu - E_S| \rightarrow 0$ . Eventually, the system goes to the ST regime,  $|\mu - E_S| \lesssim k_B T, \Delta\mu$ . Now, by using Eqs. (4)–(6) we can estimate the efficiency of spin filtering in the ST regime,

$$I_s(\downarrow)/I_c(\uparrow) \sim \frac{\Delta_z^2}{(\gamma_1 + \gamma_2) \max\{k_B T, \Delta\mu\}}, \quad (10)$$

where we assumed that  $\Delta_z < |E_{T_+} - E_S|$ , and, for simplicity,  $\gamma_1 \sim \gamma_2$ , which is typically the case in quantum dots [9,17]. In the ST regime considered here we have  $\gamma_i < k_B T, \Delta\mu$  [9]. Therefore, if the requirement  $k_B T, \Delta\mu < \Delta_z$  is satisfied, filtering of spin-down electrons in the ST regime is very efficient, i.e.,  $I_s(\downarrow)/I_c(\uparrow) \gg 1$ . In the quantum regime,  $\gamma_i > k_B T, \Delta\mu$ , tunneling occurs as Breit-Wigner resonance [9], and  $\max\{k_B T, \Delta\mu\}$  in Eq. (10) has to be replaced by  $\gamma_i$ . Finally, we note that the spin polarization of the transmitted current oscillates between up and down as we change the number of dot electrons  $N$  one-by-one. The functionality of the spin filter can be tested, e.g., with the use of a p-i-n diode [3,4] which is placed in the outgoing lead 2. Via

excitonic photoluminescence, the diode transforms the spin polarized electrons (entering lead 2) into correspondingly circularly polarized photons which can then be detected.

*Spin readout and memory.*—We now consider the opposite case where the current in the leads is fully spin polarized. Recent experiments have demonstrated that fully spin-polarized carriers can be tunnel injected from a spin-polarized magnetic semiconductor (III-V or II-VI materials) with a large effective  $g$  factor into an unpolarized GaAs system [3,4]. Another possibility would be to work in the quantum Hall regime, where spin-polarized edge states are coupled to a quantum dot [21]. To be specific, we consider the case where  $E_{T_+} - E_S + \Delta_z > \Delta\mu, k_B T$  with  $E_{T_+} > E_S$  [ $\Delta_z > k_B T$  is not necessary as long as the spin relaxation time is longer than the measurement time (see below)]. We assume that the spin polarization of both leads is, say, up and  $N$  is odd. There are now two cases for the current,  $I^\uparrow$  or  $I^\downarrow$ , corresponding to a spin-up or spin-down on the QD. First, we assume the QD to be in the ground state with its topmost electron spin pointing up. According to previous analysis [see paragraph before Eqs. (5) and (6)], the ST current vanishes, i.e.,  $I_s^\uparrow = 0$ , since the tunneling into the level  $E_{T_+}$  (and higher levels) is blocked by energy conservation, while the tunneling into  $E_S$  is blocked by spin conservation (the leads can provide and take up only electrons with spin-up). However, there is again a small CT current,  $I_c^\uparrow$ , which is given by Eq. (7). We now compare this to the second case, where the topmost dot spin is down with additional Zeeman energy  $\Delta_z > 0$ . Here, the ST current  $I_s^\downarrow$  is finite, and given by Eqs. (5) and (6) with  $E_S$  replaced by  $E_S - \Delta_z$ . In the limit  $E_S > \Delta_z$ , we get  $I_s^\downarrow = I_s(I)$ , and thus the ratio  $I_s^\downarrow/I_c^\uparrow$  is again given by Eq. (10). Hence, we see that the dot with its spin-up transmits a much smaller current than the dot with spin-down. This fact allows the readout of the spin state of the (topmost) dot electron by comparing the measured currents. Furthermore, the spin state of the QD can be changed (“read in”) by ESR techniques, i.e., by applying a pulse of an ac magnetic field (perpendicular to  $\mathbf{B}$ ) with resonance frequency  $\omega = \Delta_z$  [22]. Thus the proposed setup functions as a single-spin memory with readin and readout capabilities, the relaxation time of the memory given by the spin relaxation time  $\tau_S$  on the QD (which can be expected to exceed hundreds of nanoseconds [2]). We note that this  $\tau_S$  can be easily measured since it is the time during which  $I_s^\downarrow$  is finite before it strongly reduces to  $I_c^\uparrow$ . Finally, this scheme can be upscaled: In an array of such QDs where each dot is separately attached to ingoing and outgoing leads (for readout) we can switch the spin state of each dot individually by locally controlling the Zeeman splitting  $\Delta_z$ . This can be done [7], e.g., by applying a gate voltage on a particular dot that pushes the wave function of the dot electrons into a region of, say, higher effective  $g$  factor (the induced level shift in the QD can be compensated for by the chemical potentials).

In conclusion, we have shown that quantum dots in the Coulomb blockade regime and attached to leads can be operated as efficient spin filters at the single-spin level. Conversely, if the leads are spin polarized, the spin state of the quantum dot can be read out by a traversing current which is (nearly) blocked for one spin state while it is unblocked for the opposite spin state.

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