Barrier-Wave–Internal-Wave Interference and Airy Minima in $^{16}O + ^{16}O$ Elastic Scattering

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Taking $^{16}O + ^{16}O$ elastic scattering at 124 MeV as an example, we show that a barrier-wave–internal-wave decomposition of the elastic scattering amplitude provides valuable information on the light heavy-ion interaction and complements the more conventional nearside-farside decomposition. In particular, we show that the Airy minima present in the angular distributions are due to a barrier-wave–internal-wave interference mechanism, which sheds additional light on the exceptional transparency displayed by some light heavy-ion scattering systems. Extension of these ideas to other fields, like atomic and molecular collision physics, could prove rewarding.

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Decisive advances have been reported these last few years in the understanding of the interaction between light heavy ions [1]. In particular, the clear observation of rainbow scattering features in $^{16}O + ^{16}O$, $^{16}O + ^{12}C$, and $^{12}C + ^{12}C$ elastic scattering data has definitely established the fact that: (i) the real part of the light heavy-ion nucleus-nucleus optical potential is strongly attractive: the real part of the optical potential is *deep*. (ii) In some favorable cases (in particular, for the three aforementioned systems), the imaginary part of the potential is weak enough to allow some information to transpire from the nuclear interior in the elastic scattering differential cross section: the optical potential displays some *transparency*.

The combination of these two features—deep real potential and incomplete absorption—makes possible the observation in the elastic scattering data of distinctive refractive effects, like strong Airy minima, superimposed on more classic diffractive features. This refractive behavior is conspicuous, e.g., in the systematic analyses carried out very recently for the $^{16}O + ^{16}O$ system by Nicoli *et al.* [2] at incident energies between 75 and 124 MeV and by Khoa *et al.* [3] between 124 and 1120 MeV, and for the $^{16}O + ^{12}C$ system by Nicoli *et al.* [4] between 62 and 124 MeV and by Ogloblin *et al.* [5] at 132 MeV. Rainbow scattering and Airy minima have been observed for a long time in medium energy light-ion scattering [6–8], and are also familiar features in atomic and molecular collision processes [9].

Because the wavelengths associated with medium and high-energy heavy-ion scattering are small—and correspondingly because the number of partial waves involved is large—semiclassical approaches [10,11] have often proved invaluable for interpreting the optical model calculation results. The semiclassical analyses presented in this context have nearly invariably been performed [1,12,13] within the frame of the so-called "nearside-farside" decomposition of the elastic scattering amplitude proposed by Fuller [14]. In this approach, the scattering amplitude $f(\theta)$ is decomposed into its nearside $f_N(\theta)$ and farside $f_F(\theta)$ components, obtained simply from the partial wave expansion of $f(\theta)$:

$$f(\theta) = f_R(\theta) + \sum_{\ell} a_\ell P_\ell(\cos\theta), \qquad (1)$$

where f_R is the Rutherford scattering amplitude, by replacing the usual Legendre polynomials $P_\ell(\cos\theta)$ by suitable combinations $\tilde{Q}_\ell^{(-)}$ and $\tilde{Q}_\ell^{(+)}$, respectively, of Legendre polynomials and of Legendre functions of the second kind Q_ℓ:

$$P_\ell(\cos\theta) \rightarrow \tilde{Q}_\ell^{(\pm)} = \frac{1}{2}\left[P_\ell(\cos\theta) \mp i\,\frac{2}{\pi}\,Q_\ell(\cos\theta)\right], \qquad (2)$$

and by decomposing the Rutherford amplitude into its nearside-farside components $f_{R,N}(\theta)$ and $f_{R,F}(\theta)$ [14]. In a semiclassical interpretation, the nearside and farside contributions correspond to trajectories leading to positive and negative scattering angles, respectively.

In this decomposition, interference between f_N and f_F accounts for the familiar diffractive Fraunhofer oscillations at small angles. In contrast, the Airy minima observed for moderate absorption appear in the farside contribution to the amplitude, and are thus understood [1] to arise from the interference between contributions to f_F originating from different ranges of angular momenta $\ell_<$ and $\ell_>$, as schematically illustrated in Fig. 1(a). This interpretation is substantiated by the fact that an increase of the absorption, which affects preferentially the lower-ℓ range partial waves, tends to smooth out these minima. It is however difficult to separate directly f_F into subamplitudes corresponding to these two ranges; more generally, it is impossible to obtain the S-matrix contents of f_N and f_F, since the latter are expressed in terms of irregular Legendre functions.

In this Letter, we show that a decomposition of the scattering amplitude into its barrier-wave and internal-wave components, first proposed by Brink and Takigawa [11,15], provides an illuminating understanding of the

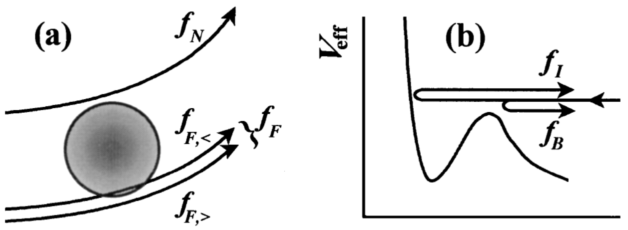

FIG. 1. Schematic representation of the semiclassical trajectories which contribute (a) to the nearside (f_N) and farside (f_F), and (b) to the barrier (f_B) and internal (f_I) components of the elastic scattering amplitude. In (a), f_F has been further split into "inner" ($f_{F,<}$) and "outer" ($f_{F,>}$) components.

salient features of light-heavy ion scattering when absorption is incomplete, in that it complements the information supplied by the nearside-farside decomposition.

The barrier-wave–internal-wave decomposition method has been applied to low-energy light-ion elastic scattering, where it has helped to understand the anomalous large angle scattering ("ALAS") observed for several α-nucleus systems (see, for example, Ref. [16]); curiously enough, this method has however been practically ignored in the context of light heavy-ion scattering. One of the advantages of the barrier-internal decomposition is to provide subamplitudes, with an intuitively very simple physical meaning, and associated with genuine S-matrices. This technique could be used with profit in other fields such as atomic and molecular collisions physics, where rainbow scattering and Airy minima have up to now also been discussed only in terms of the nearside-farside approach (see, e.g., Refs. [17,18]).

The barrier-wave–internal-wave decomposition makes sense only for potentials which are strong enough for the effective potentials active in the scattering to display a "potential pocket"; the scattering amplitude $f(\theta)$ can then be split into barrier $f_B(\theta)$ and internal $f_I(\theta)$ components, which correspond, respectively, to the part of the incident flux which is reflected at the barrier of the effective potential, and that which penetrates the nuclear interior and reemerges in the entrance channel after reflection from the most internal turning point; this decomposition is schematically explained in Fig. 1(b). Whereas the barrier contribution is rather insensitive to absorption, the internal-wave contribution survives only in the context of incomplete absorption; it has been shown to be responsible for the ALAS phenomenon observed in some light-ion systems, providing for the first time unquestionable evidence for transparency in the scattering of composite nuclear projectiles like the α particle [15].

Although the barrier-wave–internal-wave decomposition was initially introduced within a semiclassical context—it requires in principle the localization of complex turning points and the evaluation of action integrals in the complex plane—it was shown by Albiński and Michel [19] that it can be performed in many cases in an accurate way within a full quantum frame using any conventional optical model code. The method of Ref. [19] makes use of the response of the elastic scattering amplitude to small modifications of the optical potential inside of the barrier radius.

We used this method to decompose the $^{16}O + {}^{16}O$ elastic scattering amplitude at energies between 75 and 124 MeV, using the phenomenological optical potentials of Nicoli *et al.* [2]. We present here the results of our calculations at 124 MeV; this is one of the incident energies where a minimum is observed in the experimental angular distribution at 90° [2,20], which corresponds to a deep minimum in the excitation function at the same angle [21]. The existence of a minimum at 90° in the symmetrized cross section $\sigma_S(\theta) = |f(\theta) + f(\pi - \theta)|^2$ guarantees that such a minimum also exists in the unsymmetrized cross section $\sigma(\theta) = |f(\theta)|^2$ [22]. The differential cross sections $\sigma_B(\theta) = |f_B(\theta)|^2$ and $\sigma_I(\theta) = |f_I(\theta)|^2$, corresponding to the barrier- and internal-wave components, f_B and f_I, of the unsymmetrized scattering amplitude $f(\theta) = f_B(\theta) + f_I(\theta)$, are presented in Fig. 2(a). The decomposition was obtained using the parameter values $\rho = 3.25$ fm, $W_1 = 2.5$ MeV, and $W_2 = 20$ MeV in formulas (20) and (23) of Ref. [19]; we checked that the results do not depend sensitively on the precise values of these parameters. It is seen that the barrier contribution $\sigma_B(\theta)$ decreases steadily with angle; it dominates the scattering at small angles, where it accounts for the diffractive oscillations observed up to about 50°. On the other hand, the internal contribution $\sigma_I(\theta)$, which behaves smoothly up to about 120°, dominates the scattering beyond that angle. At intermediate angles (50° < θ < 120°), the two amplitudes interfere

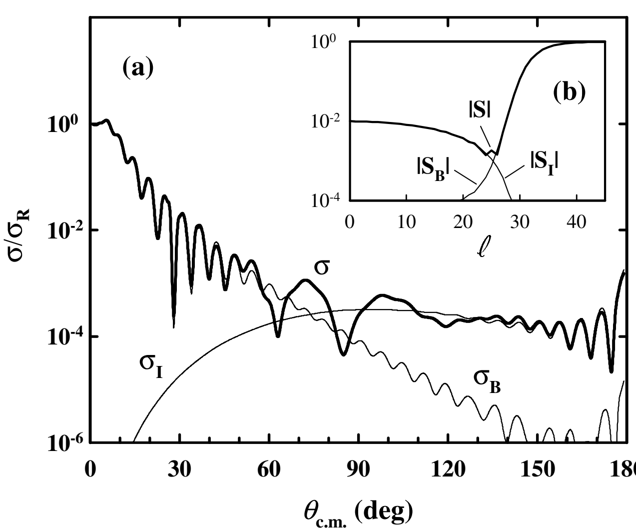

FIG. 2. (a) Ratio to Rutherford of the barrier-wave (σ_B) and internal-wave (σ_I) contributions to the unsymmetrized $^{16}O + {}^{16}O$ elastic scattering cross section σ at 124 MeV; (b) moduli of the barrier (S_B) and internal (S_I) components of the elastic S matrix.

strongly; this interference is responsible for the deep minima seen around 60° and 90° and the shallower one around 120° in the unsymmetrized cross section $\sigma(\theta)$. Similar results were obtained at the other energies; the most significant trend is a slow and regular decrease of the internal contribution at large angles which, because of the increase of absorption, drops by 2 orders of magnitude between 75 and 124 MeV.

Figure 2(a) shows that the presence of an internal-wave contribution is essential to the building up of the cross section pattern from 50° onwards, and, in particular, in the reproduction of the minima seen around 60°, 90°, and 120°. Since this contribution arises from the part of the incident flux which probes the nuclear interior, our result demonstrates clearly the exceptional transparency of the $^{16}O + ^{16}O$ interaction at the considered energy, which turns out to be comparable to that displayed by some very transparent light-ion systems like $\alpha + ^{16}O$ at a similar incident energy per nucleon [23,24]. We note however that the modulus of the internal component, $S_{I,\ell}$, of the elastic S matrix S_ℓ, which is plotted together with that of the barrier component $S_{B,\ell}$ as a function of the angular momentum ℓ in Fig. 2(b), is about 10 times less than in the $\alpha + ^{16}O$ case [23].

Minima like those observed around 60° and 90° in the full (unsymmetrized) cross section, which appear here as arising from an interference between the barrier and internal contributions to the scattering amplitude, have repeatedly been interpreted as Airy minima [1,7,8], due to the interference in the farside amplitude of two ranges of angular momenta, $\ell_<$ and $\ell_>$, contributing to the same (negative) deflection angle [Fig. 1(a)]. The contributions of these two ranges, $\sigma_{F,<}$ and $\sigma_{F,>}$, to the farside cross section σ_F have sometimes been estimated in an approximate way by using the so-called interpolated-envelope technique [7,25].

We found that these two contributions can be obtained in a much more natural—and at the same time more rigorous—way by subjecting the barrier- and internal-wave amplitudes themselves to a nearside-farside decomposition; this decomposition is performed simply by modifying the partial wave series for f_B and f_I according to Eq. (2). The nearside and farside components, $\sigma_N = |f_N|^2$ and $\sigma_F = |f_F|^2$, of the unsymmetrized $^{16}O + ^{16}O$ cross section σ at 124 MeV, are plotted in Fig. 3(a); these components are very similar to those obtained by Nicoli *et al.* [2] from their folding model calculations. The farside contributions, $\sigma_{B,F} = |f_{B,F}|^2$ and $\sigma_{I,F} = |f_{I,F}|^2$, to the barrier-wave and internal-wave cross sections σ_B and σ_I, are presented in Fig. 3(b), together with the total farside cross section σ_F.

It is seen that the strongly oscillating farside cross section σ_F, whose behavior accounts for the Airy structure of the full cross section, has been decomposed into two components which, as they originate from the barrier and internal contributions to the cross section, necessarily

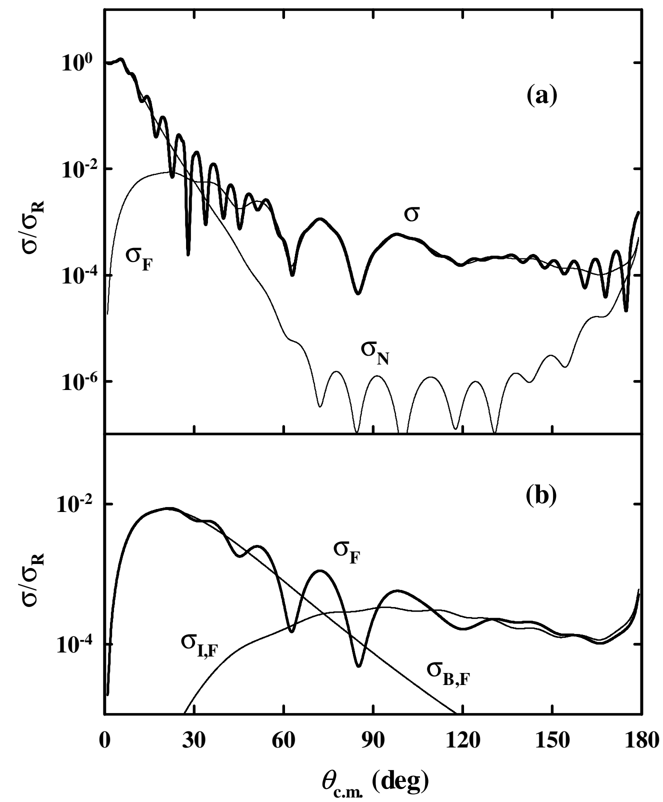

FIG. 3. (a) Ratio to Rutherford of the nearside (σ_N) and farside (σ_F) contributions to the unsymmetrized $^{16}O + ^{16}O$ elastic scattering cross section σ at 124 MeV; (b) farside contributions to the barrier ($\sigma_{B,F}$) and to the internal ($\sigma_{I,F}$) components of the full farside cross section σ_F.

correspond to different ranges of angular momentum—indeed the barrier and internal contributions are nearly decoupled in angular momentum space [Fig. 2(b)]. Moreover these components now vary smoothly with angle, like those extracted empirically by the interpolated-envelope technique [7,25]. These properties prove the identity of the "inner" $\sigma_{F,<}$ and "outer" $\sigma_{F,>}$ cross sections with our components $\sigma_{I,F}$ and $\sigma_{B,F}$, which are more firmly grounded within a quantum context and can be computed much more easily than within a semiclassical approach [26,27]. We like to note that a calculation of the nearside and farside components of the barrier and internal contributions to the $^{12}C + ^{12}C$ elastic scattering amplitude at 51 MeV center of mass energy was presented in the pioneering paper of Rowley, Doubre, and Marty [22]; at that time however, the beat structure observed in the farside amplitude—which was found here to be due to the interference between the farside contributions to the barrier and internal components—was not associated with an Airy mechanism, and the physical significance of the deep potentials needed to produce these features was unclear in the context of composite nuclear scattering; to the best of our knowledge, this type of decomposition was no more attempted in later studies.

To summarize, we have shown, taking $^{16}O + ^{16}O$ elastic scattering at 124 MeV as an example, that a barrier-wave–internal-wave decomposition of the elastic scattering amplitude provides valuable information on the light heavy-ion scattering mechanism, and that it complements nicely the nearside-farside decomposition, which has been widely used in this context. In particular, the Airy minima observed in the $^{16}O + ^{16}O$ angular distributions appear in our interpretation to be due to a barrier-wave–internal-wave interference mechanism, which sheds additional light on the exceptional transparency displayed by this system. Moreover, the two components of the farside amplitude, introduced in the nearside-farside approach to account for the Airy oscillations seen in the farside cross section, are neatly disentangled by the barrier-wave–internal-wave decomposition and acquire a clear physical interpretation.

Further investigation of light heavy-ion scattering within this approach might help to clarify the mechanism of the nucleus-nucleus interaction, and thus pave the way to a better understanding of the cluster structure of the unified nuclear system. Moreover we believe that extension of these ideas to other fields, like atomic and molecular collision physics, could prove rewarding.

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