Casimir Scaling as a Test of QCD Vacuum Models

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Recent accurate lattice measurements of static potentials between sources in various representations of the gauge group SU(3) performed by Bali, provide a crucial test of different QCD vacuum models. The Casimir scaling of the potential observed for all measured distances can be explained as being due to strong suppression of higher cumulants contribution.

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The structure of QCD in its nonperturbative domain has commanded attention of theorists for many years. Confinement and chiral symmetry breaking have been studied both using theoretical models and lattice simulations (for a review, see [1]).

However, most of the models are designed to describe confinement of color charge and anticharge in the fundamental representation of the gauge group SU(3), i.e., the area law for the simplest Wilson loop and hence linear potential between static quark and antiquark. Supplementary and very important information about QCD vacuum is provided by the investigation of interaction between static charges in higher SU(3) representations. In this way one can derive information about field correlators in the vacuum, which is not possible to obtain from fundamental charges alone.

The recent accurate measurements of the corresponding potential have been performed by Bali in [2] (see also review [3]). Preliminary physical analysis of the data from [2] was reported in [4]. We present in this paper more extended investigation and discuss new important information about the QCD vacuum and constraints on several QCD vacuum models.

The static potential between sources at the distance R in the given representation D is defined as

$$V_D(R) = -\lim_{T\to\infty} \frac{1}{T} \ln\langle W(C)\rangle, \qquad (1)$$

where the Wilson loop $W(C)$ for the rectangular contour $C = R \times T$ in the "34" plane can be formally expanded as

$$\langle W(C)\rangle = \left\langle \mathrm{Tr}_D \mathrm{P}\exp\left(ig \int_C A_\mu^a T^a dz_\mu \right) \right\rangle = \mathrm{Tr}_D \exp \sum_{n=2}^{\infty} \int_S (ig)^n \langle\langle F(u_1)\cdots F(u_n)\rangle\rangle \, d\sigma(u_1)\cdots d\sigma(u_n). \qquad (2)$$

This expansion is widely used in the stochastic vacuum picture [5], which takes it for granted that the non-Abelian Stokes theorem [6] and cluster expansion theorem used in deriving (2) are applicable in the case of interest. Here $F(u)d\sigma(u) = \Phi(x_0,u)E_3^a(u)T^a\Phi(u,x_0)d\sigma_{34}(u)$, where Φ is a parallel transporter, u and x_0 are the points on the surface S bound by the contour C. The double brackets $\langle\langle\ldots\rangle\rangle$ denote irreducible Green's functions proportional to the unit matrix in the color space. Since (2) is gauge invariant, one can use the generalized contour gauge [7], which is defined by the condition $\Phi(x_0, u^{(k)}) \equiv 1$, hence all Φ will be omitted in what follows.

The SU(3) representations labeled by $D = 3, 8, 6, 15a, 10, 27, 24, 15s$ are characterized by $3^2 - 1 = 8$ Hermitian generators T^a which satisfy the commutation relations $[T^a, T^b] = if^{abc}T^c$. One of the main characteristics of the representation is an eigenvalue of quadratic Casimir operator $C_D^{(2)}$, which is defined according to $C_D^{(2)} = T^aT^a = C_D \cdot \hat{1}$. Following the notations from [2], we introduce the Casimir ratio $d_D = C_D/C_F$, where the fundamental Casimir $C_F = (N_c^2 - 1)/2N_c$ equals to 4/3 for SU(3). The invariant trace is given by $\mathrm{Tr}_D \hat{1} = 1$.

The potential (1) has two different regimes at $R \leq R_c$ and $R \geq R_c$ (R_c is the critical distance, where the screening of charges takes place; see discussion below). In the region $R \leq R_c$ the potential admits the following decomposition

$$V_D(R) = d_D V^{(2)}(R) + d_D^2 V^{(4)}(R) + \ldots. \qquad (3)$$

It is worth mentioning, that the expansion (3) is not in one-to-one correspondence with the cluster expansion (2). So the quadratic cumulant $\langle\langle FF\rangle\rangle$ contributes only to $V^{(2)}(R)$, while higher ones contribute to $V^{(2)}(R)$ as well as to other terms. The part denoted by dots contains terms, proportional to the higher powers of the quadratic Casimir and to higher Casimirs [there are two independent Casimir operators for SU(3)].

One usually takes the fundamental static potential to contain perturbative Coulomb part, confining linear and constant terms. The Coulomb part is now known up to two loops [8] and is proportional to C_D. The "Casimir scaling hypothesis" [9] declares that the confinement potential is also proportional to the first power of the quadratic Casimir C_D, i.e., all terms in the right-hand side of (3) are much smaller than the first one. In particular, for the string tensions one should get $\sigma_D/\sigma_F = d_D$.

This scaling law is in perfect agreement with the results found in [2]. Earlier lattice calculations of static potential between sources in adjoint representation [10] are in general agreement with [2], however, deviations from scaling at the level of 10% are found in [11]; in particular, the value of σ_8/σ_3 is closer to 2 than to 9/4 in [11], which might be a finite lattice spacing effect.

The first nontrivial Gaussian cumulant in (2) is expressed through C_D and representation-independent averages as

$$\mathrm{Tr}_D\langle F(1)F(2)\rangle = \frac{d_D}{2N_c}\langle F^a(1)F^a(2)\rangle, \qquad (4)$$

so Gaussian approximation satisfies "Casimir scaling law" exactly. It is worth mentioning, that this fact does not depend on the actual profile of the potential. It could happen that the linear potential observed in [2] is just some kind of intermediate distance characteristics and changes the profile at larger R (see below). The coordinate dependence of the potential is not directly related to the CS, which can be analyzed at the distances small enough not to be affected by the screening effects.

Using identities for f^{abc} and d^{abc}, one can find expansions in powers of Casimirs for the higher terms in (2). Mnemonically the C_D—proportional components arise from the diagrams where the noncompensated color flows inside while the C_D^2 and higher—components describe the interaction of two or more white objects. Therefore the Casimir scaling is not a property of the "ensemble of quasifree gluons," instead it roughly speaking means "ensemble of quasifree white dipoles."

The quantitative analysis of the data from [2] is presented in Table I (see also Figs. 2 and 3 from [2], where the quantity $[V_D(R) - d_D V_F(R)]$ versus distance R is depicted for $D = 8, 6$, respectively). We have already mentioned that the Coulomb potential between static sources is proportional to C_D up to the second loop, and hence we expect contributions proportional to $C_D^2 \sim d_D^2$ to the constant and linear terms, i.e., we rewrite (3) as $V^{(4)}(R) = v^{(4)} + \sigma^{(4)}R$, and all higher contributions are omitted. Here $v^{(4)}, \sigma^{(4)}$ measure the d_D^2 contribution of the cumulants higher than Gaussian to the constant term and string tension, respectively. Their possible dependence on D could come from terms, omitted in (3). Notice, that we do not need to specify the coordinate dependence of $V^{(2)}(R)$.

All numbers in Table I are dimensionless and given in lattice units. The author of [2] used anisotropic lattice with the spatial unit $a_s = 0.082$ Fm. The standard χ^2 fitting was performed for the data in the whole range of all measured R, since no fingerprint of screening is seen up to the largest distances explored in [2]. Errors shown in Table I include statistical and systematical ones. Since the proper extrapolation to the continuum limit was not performed in [2], it is difficult to estimate which kind of errors plays the major role.

TABLE I. The Casimir-scaling and Casimir-violating string tensions and shifts. Based on the lattice data from Bali, hep-lat/9908021. All quantities with the hats are scaled according to $\hat{u} = u \times 10^4$.

D	$\hat{\sigma}^{(4)}$	$\Delta\hat{\sigma}^{(4)}$	$\hat{v}^{(4)}$	$\Delta\hat{v}^{(4)}$	$\lvert\sigma^{(4)}/\sigma_D^{(2)}\rvert$	χ^2/dof
8	−3.5	1.2	−2.5	2.8	0.004	19/43
6	−6.4	1.2	1.0	2.6	0.007	26/42
15a	−5.2	0.6	−0.6	1.1	0.003	39/42
10	−4.9	0.5	0.2	1.0	0.003	22/41

Several comments are in order. First of all it is seen that the Casimir scaling behavior holds with very good accuracy, better than 1% in all cases in Table I with the reasonable χ^2/dof. The values of the constant term $v^{(4)}$ are found to be compatible with zero within the error bars for all considered D, while it is not the case for $\sigma^{(4)}$. The value of $\sigma^{(4)}$ for sextet, for example, is found to deviate from zero at the level of approximately five standard deviations. We have not found any strong systematic dependence of $\sigma^{(4)}$ on D, which presumably confirms the validity of the expansion (3) and shows that the omitted higher terms do not change the picture in a crucial way. At the same time, the CS violation seems to be statistically more significant for higher representations. Notice the negative sign of the $\sigma^{(4)}$ correction. In Euclidean metric it trivially follows from the fact, that the fourth order contribution is proportional to $(ig)^4 > 0$ while the Gaussian term is multiplied by $(ig)^2 < 0$.

As an example of the Casimir scaling violating model, based on the nonperturbative field configurations we mention here the model of the dilute instanton gas. One finds for the instanton-induced potential in the SU(3) case at small distances [12]: $V_D(R) \sim (N/V)\overline{\rho}R^2 \cdot \epsilon_D$ where instanton density $n_4 = N/V$ and instanton mean radius $\overline{\rho}$ have been introduced, and numerical coefficients ϵ_D for $D = 3, 8, 10$ are given by $\epsilon_3{:}\epsilon_8{:}\epsilon_{10} = 1{:}1.87{:}3.11$ instead of Casimir scaling ratios 1:2.25:4.5; a similar situation takes place for the large distance asymptotics of the instanton-induced potential (see details in [4]). Parametrically for $4d$ instantonlike configurations of the density n_4 and size $\overline{\rho}$ the correlator $\langle\langle F^{[n]}\rangle\rangle$ in the dilute gas approximation is proportional to $n_4[\overline{\rho}]^{2n-4}$ and the quartic contributions to the potential are $\sigma^{(4)} \sim n_4\overline{\rho}^2; v^{(4)} \sim n_4\overline{\rho}^3$. Hence the ratio $\sigma^{(4)}/\sigma_D^{(2)}$ is not small [4] which strongly contradicts the lattice data [2].

This sharp contradiction between the dilute instanton gas model calculation for the quark-antiquark potential and the Casimir scaling of this potential found on the lattice can be understood in one of two ways. Either instantons are strongly suppressed in the real(hot) QCD vacuum (as it was observed in [13]) while they are recovered by the cooling procedure. Or else instanton medium is dense and strongly differs from dilute instanton gas, in such a way that higher cumulant components of such collectivized instantons are suppressed. It is interesting to note that linear confinement missing in the dilute gas is recovered in this case.

There is another important consequence of the observed CS. It comes from the analysis of the confinement potential as being induced by the QCD string. In this case one has additional contribution to the confining potential besides the leading linear term, which comes from the transverse worldsheet vibrations. The simplest model in this respect is the Nambu-Goto (NG) string whose action is proportional to the area of the surface bounded by the static sources worldlines. It modifies the confining potential with respect to the classical case (nonvibrating string) as

$$\sigma R \rightarrow \sigma R - \frac{\pi}{12}\frac{1}{R} + \ldots, \tag{5}$$

where the term $-\pi/(12R)$ will be referred to as the string vibration (SV) term [14]. It is instructive to look whether or not the data [2] support the existence of such a term. It is a nontrivial task to separate the contributions of the discussed sort in the confining potential as it is because these corrections are essentially large distance effect, where they are subleading. But they have to become pronounced in expression (3) due to scaling violation. Namely, one has

$$\frac{V_D(R) - d_D V_F(R)}{d_D(d_D - 1)} = \frac{1}{d_D}\frac{\pi}{12}\frac{1}{R} + \ldots, \tag{6}$$

where the dots denote the terms, omitted in (5).

On general grounds one expects that the string picture should work at distances $R \geq 1/\sqrt{\sigma_D}$ and also $R \geq T_g$, where T_g is the thickness of the string. It is important to stress that T_g is D independent in Gaussian approximation and is defined instead by the correlation length of the vacuum correlator (2). We expect, therefore, that the data [2] allow one to extract the possible contribution from the CS-violating SV term at the distances $\sim$1 Fm. Taking as an example $R = 12a_s$ and $D = 8$ one can easily conclude from Table I, that the left-hand side of (6) is equal to $(-4.6 \pm 1.5) \times 10^{-3}$, while the right-hand side of (6) is equal to 9.7×10^{-3} (in the units of a_s^{-1}). Notice, that even the sign of the NG SV correction is opposite to what has actually been observed for scaling violation in [2].

One way to escape from this strong bound was proposed already in [9]—just to multiply all the potential (5) with d_D. It should be stressed, however, that it implies different physical mechanisms responsible for the creation of the string and its quantum fluctuations and presents actually a model, different from NG. The situation with direct lattice measurements of SV corrections is not yet clear. While the authors of [15] claim the disagreement between hybrid spectrum and the NG string picture, there is, however, some evidence in favor of the SV term [16]. The question certainly deserves further study.

From a theoretical point of view, nobody has proved up to now that the simplest bosonic NG string model should properly describe the dynamics of the QCD string, and the theoretical background of (5) is not clear. Just the opposite is true—there are many reasons why it is not the case (see the discussion in [17]). The theory of the QCD string—whatever it will be—must explain the observed CS of the potential at intermediate distances.

One important effect, which has not yet been discussed, is the string breaking in the triality zero representations. To take into account effects of screening, one should specify the meaning of the averaging process, denoted in (1), (2) by angular brackets $\langle \cdots \rangle$. The screening can be explained as being due to the appearance of quark loops (in an unquenched fundamental case) and due to dynamical gluon loops (in the case of adjoint charges). For higher D the screening may be partial and may need more loops. Here we concentrate on the adjoint case only for the lack of space. To this end we use modified background perturbation theory (see all details in [18]) and split the gluon field A_μ as $A_\mu = B_\mu + g a_\mu$, where B_μ represents the confining background and a_μ—the valence gluon field. One gets the valence gluon Green's function in the background Feynman gauge in the form $G_{\mu\nu}(B) = [D^2(B)\delta_{\mu\nu} + 2iF_{\mu\nu})]^{-1}$ and the result of integration over valence gluons at the lowest order yields in the partition function the factor $[\mathrm{Det}G(B)]^{-(1/2)}$. The averaging in (1), (2) turns out to be

$$\langle W \rangle_{B,a} = \frac{\langle [\mathrm{Det}G(B)]^{-(1/2)} W(B) \rangle_B}{\langle [\mathrm{Det}G(B)]^{-(1/2)} \rangle_B} + \ldots . \tag{7}$$

In a similar way quark loops are accounted for by the factor $\mathrm{Det}(\hat{D} + im)$ instead of $[\mathrm{Det}G(B)]^{-(1/2)}$ in (7). Higher terms in $g a_\mu$ expansion can be calculated systematically.

The next step is the standard loop expansion of the determinant augmented by the world-line (Feynman-Schwinger) formalism: $[\mathrm{Det}G(B)]^{-(1/2)} = \exp\{W_{\mathrm{adj}}(B)\}/2$, where the curly brackets stand for the path integral over contours, forming the loop and the corresponding proper time integration (see [18,19]). Expanding the exponent and keeping only the first two terms one has

$$\langle W[C] \rangle = \langle W(B) \rangle_B + \frac{1}{2}\langle\langle W(B)\{W_{\mathrm{adj}}(B)\}\rangle\rangle_B + \ldots, \tag{8}$$

where double brackets $\langle\langle \ldots \rangle\rangle$ denote the irreducible correlator. The first term in the right-hand side of (8) has the form $\exp(-\sigma_8 RT)$, while the second one represents Green's function of two gluelumps [19], and is given by $\eta \cdot \exp(-2M_{\mathrm{gl}}T)$ [9,20], where the external loop $C = R \times T$ and $\eta \sim \mathcal{O}(N_c^{-2})$. At the distances actually explored in [2], the first term in (8) dominates, while the second term yields, at least for lowest representations, the scaling violation within the error limits of the Table I. Notice that the correction to the potential (3) from the second term in (8) at the region $R < R_c$, $R_c \approx 2M_{\mathrm{gl}}/\sigma_8$ is exponentially suppressed in the limit of large T. Numerically, estimating M_{gl} as 1.4 GeV for the adjoint source [19], one gets for the critical distance $R_c \approx 2M_{\mathrm{gl}}/\sigma_8 \approx 1.5$ Fm, which is larger than the maximal distance explored in [2]. Additional measurements at larger distances could hopefully shed some light on the string breaking and the physics of gluelumps and establish the "CS region." The reader is referred to papers [3,21], where different issues of the QCD string breaking on the lattice are discussed.

Let us make a few concluding remarks concerning the pictures of the QCD vacuum, suggested in different models from the Casimir scaling point of view. The Abelian projection method supplied by the Abelian dominance hypothesis is in wide use nowadays as one of the most adequate for the dual Meissner scenario of confinement. Serious difficulties in the explanation of Casimir scaling encountered in this approach are discussed in [22] (for the case of adjoint sources). The observed adjoint string tension (at intermediate distances) arises from the interaction of diagonal Abelian projected gluons with the part of the adjoint source doubly charged with respect to the Cartan subgroup. If one naively omits the corresponding Faddeev-Popov determinant it gives $\sigma_{\text{adj}} = 4\sigma_{\text{fund}}$. It is expected that the loop expansion of the determinant produces terms, correcting the above behavior to the Casimir scaling ratio. From a physical point of view to reproduce Casimir scaling, which is a genuine non-Abelian feature, one needs to restore the original non-Abelian gauge invariance broken by hand in the Abelian projection method.

Another popular confining mechanism is the model of fat center vortices [23]. While the original center vortex picture yields vanishing potential for charges of zero N-ality, introducing finite thickness of the vortex makes it possible to obtain approximate Casimir ratios for the string tensions [23]. The crucial feature of this scenario is the smooth flattening of any zero N-ality potential with distance in contrast with the sharp (in the limit $T \rightarrow \infty$) transition dictated by (8). Thus, there is no strictly speaking constant string tension at intermediate distances in this model. Unfortunately, the exact value of the CS violation is strongly model dependent in this approach, which makes it difficult to put stringent bounds on the parameters of fat vortices model from the data [2]. For example, in the SU(2) case the deviation of the center-vortex-induced potential from CS behavior is about 30% for $j = 1$ (adjoint) and ~80% for $j = 3/2$ at large distances [23], instead of a percent level violation seen in Table I.

An old proposal $\sigma_D/\sigma_F = \sqrt{d_D}$ advocated in [24] in the bag model framework leads to the CS violating string tension which is approximately 30 times (for adjoint) larger than actually observed (see Table I).

In the gauge-invariant formalism [5], the Casimir scaling has two possible explanations. According to the first one, CS is just a consequence of Gaussian dominance, since Gaussian correlator provides the exact CS. Physically, it means the picture of the QCD vacuum, made of relatively small color dipoles with weak interactions between them. There is also another scenario, when each higher term in the expansion (2) is not small, but their sum demonstrates delicate cancellations of Casimir scaling violating terms (which might be relevant to perturbative QCD series). These pictures are in close correspondence to the stochastic versus coherent vacuum scenario [1]. Work is in progress to distinguish between them on the lattice.

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