Extracting Weak Phase Information from $B \to V_1V_2$ Decays

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We describe a new method for extracting weak, CP-violating phase information, with no hadronic uncertainties, from an angular analysis of $B \to V_1V_2$ decays, where V_1 and V_2 are vector mesons. The quantity $\sin^2(2\beta + \gamma)$ can be cleanly obtained from the study of decays such as $B_d^0(t) \to D^{*\pm}\rho^{\mp}$, $D^{*\pm}a_1^{\mp}$, $\overset{(-)}{D}{}^{*0}\overset{(-)}{K}{}^{*0}$, etc. Similarly, one can use $B_s^0(t) \to D_s^{*\pm}K^{*\mp}$ to extract $\sin^2\gamma$. There are no penguin contributions to these decays. It is possible that $\sin^2(2\beta + \gamma)$ will be the second function of CP phases, after $\sin 2\beta$, to be measured at B factories.

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One of the most important open questions in particle physics is the origin of CP violation. According to the standard model (SM), CP violation is due to the presence of a nonzero complex phase in the Cabibbo-Kobayashi-Maskawa quark mixing matrix. This explanation can be tested in the B system. By measuring CP-violating rate asymmetries in B decays, one can extract α, β, and γ, the three interior angles of the unitarity triangle [1]. The measured values of these angles may be consistent with SM predictions, or they may indicate the presence of physics beyond the SM. Hopefully, it is this latter scenario which will be realized.

The reason that B decays are such a useful tool is that the CP angles can be obtained without hadronic uncertainties. The usual technique is to consider a final state f to which both B^0 and $\overline{B^0}$ can decay. Because of B^0-$\overline{B^0}$ mixing, CP violation then comes about due to an interference between the amplitudes $B^0 \to f$ and $B^0 \to \overline{B^0} \to f$. In the early days of the field, it was thought that the CP angles could be easily measured in $B_d^0(t) \to \pi^+\pi^-$ (α), $B_d^0(t) \to \Psi K_S$ (β), and $B_s^0(t) \to \rho K_S$ (γ). However, it soon became clear that things would not be so easy: the presence of penguin amplitudes [2] makes the extraction of α from $B_d^0(t) \to \pi^+\pi^-$ quite difficult, and completely spoils the measurement of γ in $B_s^0(t) \to \rho K_S$. Even in the gold-plated mode $B_d^0(t) \to \Psi K_S$, penguin contributions limit the precision with which β can be measured to about 2%. In part because of this, a great deal of work was then done developing new methods to cleanly obtain the CP angles from a wide variety of final states.

One class of final states that was considered consists of two vector mesons, V_1V_2. Because the final state does not have a well-defined orbital angular momentum, V_1V_2 cannot be a CP eigenstate. This then implies that, even if both B^0 and $\overline{B^0}$ can decay to the final state V_1V_2, one cannot extract a CP phase cleanly. However, this situation can be remedied with the help of an angular analysis [3]. By examining the decay products of V_1 and V_2, one can measure the various helicity components of the final state. Since each helicity state corresponds to a state of well-defined CP, an angular analysis allows one to use $B \to V_1V_2$ decays to obtain one of the CP phases cleanly. Thus, for example, the angle β can be extracted from the decay $B_d^0(t) \to \Psi K^*$: each helicity state of ΨK^* can be treated in the same way as ΨK_S.

In this paper, we show that the angular analysis is more powerful than has previously been realized. Because of the interference between the different helicity states, there are enough independent measurements that one can obtain weak phase information from the decays of B^0 and $\overline{B^0}$ to any common final state f. Furthermore, contrary to other methods, it is not necessary to measure the branching ratios of both $B^0 \to f$ and $\overline{B^0} \to f$. This is important for final states such as $D^{*\pm}\rho^{\mp}$, in which one of the two decay amplitudes is considerably smaller than the other one.

We consider a final state f, consisting of two vector mesons, to which both B^0 and $\overline{B^0}$ can decay. We assume further that only one weak amplitude contributes to $B^0 \to f$ and $\overline{B^0} \to f$. We write the helicity amplitudes as follows:

$$A_\lambda \equiv \mathrm{am}(B^0 \to f)_\lambda = a_\lambda e^{i\delta_\lambda^a} e^{i\phi_a}, \tag{1}$$

$$A'_\lambda \equiv \mathrm{am}(\overline{B^0} \to f)_\lambda = b_\lambda e^{i\delta_\lambda^b} e^{i\phi_b}, \tag{2}$$

$$\overline{A}'_\lambda \equiv \mathrm{am}(B^0 \to \overline{f})_\lambda = b_\lambda e^{i\delta_\lambda^b} e^{-i\phi_b}, \tag{3}$$

$$\overline{A}_\lambda \equiv \mathrm{am}(\overline{B^0} \to \overline{f})_\lambda = a_\lambda e^{i\delta_\lambda^a} e^{-i\phi_a}, \tag{4}$$

where the helicity index λ takes the values $\{0, \|, \perp\}$. In the above, $\phi_{a,b}$ and $\delta_\lambda^{a,b}$ are the weak and strong phases, respectively.

Using CPT invariance, the total decay amplitudes can be written as

$$\mathcal{A} = \mathrm{am}(B^0 \to f) = A_0 g_0 + A_\| g_\| + i A_\perp g_\perp \,, \tag{5}$$

$$\overline{\mathcal{A}} = \mathrm{am}(\overline{B^0} \to \overline{f}) = \overline{A}_0 g_0 + \overline{A}_\| g_\| - i \overline{A}_\perp g_\perp \,, \tag{6}$$

$$\mathcal{A}' = \mathrm{am}(\overline{B^0} \to f) = A_0' g_0 + A_\parallel' g_\parallel - i A_\perp' g_\perp\,, \quad (7)$$

$$\overline{\mathcal{A}}' = \mathrm{am}(B^0 \to \overline{f}) = \overline{A}_0' g_0 + \overline{A}_\parallel' g_\parallel + i \overline{A}_\perp' g_\perp\,, \quad (8)$$

where the g_λ are the coefficients of the helicity amplitudes written in the linear polarization basis. The g_λ depend only on the angles describing the kinematics [4,5].

With the above equations, the time-dependent decay rate for $B^0(t) \to f$ can be written as

$$\Gamma[B^0(t) \to f] = e^{-\Gamma t} \sum_{\lambda \le \sigma} [\Lambda_{\lambda\sigma} + \Sigma_{\lambda\sigma} \cos(\Delta M t) - \rho_{\lambda\sigma} \sin(\Delta M t)] g_\lambda g_\sigma\,. \quad (9)$$

Thus, by performing a time-dependent study and angular analysis of the decay $B^0(t) \to f$, one can measure the observables $\Lambda_{\lambda\sigma}$, $\Sigma_{\lambda\sigma}$, and $\rho_{\lambda\sigma}$. In terms of the helicity amplitudes A_0, $A_\parallel$, $A_\perp$, these can be expressed as follows:

$$\begin{aligned}
\Lambda_{\lambda\lambda} &= \frac{|A_\lambda|^2 + |A_\lambda'|^2}{2}\,, & \Lambda_{\perp i} &= -\mathrm{Im}(A_\perp A_i^* - A_\perp' A_i'^*), & \Lambda_{\parallel 0} &= \mathrm{Re}(A_\parallel A_0^* + A_\parallel' A_0'^*)\,, \\
\Sigma_{\lambda\lambda} &= \frac{|A_\lambda|^2 - |A_\lambda'|^2}{2}\,, & \Sigma_{\perp i} &= -\mathrm{Im}(A_\perp A_i^* + A_\perp' A_i'^*), & \Sigma_{\parallel 0} &= \mathrm{Re}(A_\parallel A_0^* - A_\parallel' A_0'^*)\,, \\
\rho_{\lambda\lambda} &= \zeta_\lambda \mathrm{Im}(\tfrac{q}{p} A_i^* A_i')\,, & \rho_{\perp i} &= -\mathrm{Re}(\tfrac{q}{p}[A_\perp^* A_i' + A_i^* A_\perp']), & \rho_{\parallel 0} &= \mathrm{Im}(\tfrac{q}{p}[A_\parallel^* A_0' + A_0^* A_\parallel']),
\end{aligned} \quad (10)$$

where $i = \{0, \parallel\}$ and ζ_λ is 1 for $\lambda = \parallel, 0$ and -1 for $\lambda = \perp$. In the above, $q/p = \exp(-2i\phi_M)$, where $2\phi_M$ is the weak phase present in B^0-$\overline{B^0}$ mixing.

Similarly, the decay rate for $B^0(t) \to \overline{f}$ can be obtained from Eq. (9) by replacing $\Lambda_{\lambda\sigma}$, $\Sigma_{\lambda\sigma}$, and $\rho_{\lambda\sigma}$ by $\overline{\Lambda}_{\lambda\sigma}$, $\overline{\Sigma}_{\lambda\sigma}$, and $\overline{\rho}_{\lambda\sigma}$, respectively, where the expressions for the observables $\overline{\Lambda}_{\lambda\sigma}$, $\overline{\Sigma}_{\lambda\sigma}$, and $\overline{\rho}_{\lambda\sigma}$ are similar to those given in Eq. (10), with the replacements $A_\lambda \to \overline{A}_\lambda'$ and $A_\lambda' \to \overline{A}_\lambda$.

With the above expressions for the various amplitudes, we now show how to extract weak phase information using the above measurements. First, we note that

$$\begin{aligned}
\Lambda_{\lambda\lambda} &= \overline{\Lambda}_{\lambda\lambda} = \frac{(a_\lambda^2 + b_\lambda^2)}{2}\,, \\
\Sigma_{\lambda\lambda} &= -\overline{\Sigma}_{\lambda\lambda} = \frac{(a_\lambda^2 - b_\lambda^2)}{2}\,.
\end{aligned} \quad (11)$$

Thus, one can determine the magnitudes of the amplitudes appearing in Eqs. (1)–(4), a_λ^2 and b_λ^2. However, we must stress that, in fact, knowledge of b_λ^2 will not be necessary within our method. This is important since some final states have $b_\lambda \ll a_\lambda$, and so the determination of b_λ^2 would be very difficult.

Next, we have

$$\begin{aligned}
\Lambda_{\perp i} = -\overline{\Lambda}_{\perp i} &= b_\perp b_i \sin(\delta_\perp - \delta_i + \Delta_i) - a_\perp a_i \sin(\Delta_i)\,, \\
\Sigma_{\perp i} = \overline{\Sigma}_{\perp i} &= -b_\perp b_i \sin(\delta_\perp - \delta_i + \Delta_i) - a_\perp a_i \sin(\Delta_i)\,,
\end{aligned} \quad (12)$$

where $\Delta_i \equiv \delta_\perp^a - \delta_i^a$ and $\delta_\lambda \equiv \delta_\lambda^b - \delta_\lambda^a$. Using Eq. (13) one can solve for $a_\perp a_i \sin\Delta_i$. We will see that this is the only combination needed to cleanly extract weak phase information.

The coefficients of the $\sin(\Delta m t)$ term, which can be obtained in a time-dependent study, can be written as

$$\overset{(-)}{\rho}_{\lambda\lambda} = \zeta_\lambda a_\lambda b_\lambda \sin(\phi \pm \delta_\lambda)\,, \quad (13)$$

where the "$\pm$" signs on the right correspond to ρ, $\overline{\rho}$, respectively. In the above, we have defined the CP phase $\phi \equiv -2\phi_M + \phi_b - \phi_a$. These quantities can be used to determine

$$\begin{aligned}
2b_\lambda \cos\delta_\lambda &= \pm \frac{\rho_{\lambda\lambda} + \overline{\rho}_{\lambda\lambda}}{a_\lambda \sin\phi}\,, \\
2b_\lambda \sin\delta_\lambda &= \pm \frac{\rho_{\lambda\lambda} - \overline{\rho}_{\lambda\lambda}}{a_\lambda \cos\phi}\,.
\end{aligned} \quad (14)$$

Similarly, the terms involving interference of different helicities are given as

$$\overset{(-)}{\rho}_{\perp i} = -a_\perp b_i \cos(\phi \pm \delta_i \mp \Delta_i) - a_i b_\perp \cos(\phi \pm \delta_\perp \pm \Delta_i)\,. \quad (15)$$

Putting all the above information together, we are now in a position to extract the weak phase ϕ. Using Eq. (15), the expressions in Eq. (16) can be used to yield

$$\begin{aligned}
\rho_{\perp i} \pm \overline{\rho}_{\perp i} = {} & \mp(\tan\phi)^{\mp 1} a_i a_\perp \\
& \times \cos\Delta_i \left[\frac{\rho_{ii} \pm \overline{\rho}_{ii}}{a_i^2} - \frac{\rho_{\perp\perp} \pm \overline{\rho}_{\perp\perp}}{a_\perp^2}\right] \\
& - a_i a_\perp \sin\Delta_i \left[\frac{\rho_{ii} \mp \overline{\rho}_{ii}}{a_i^2} + \frac{\rho_{\perp\perp} \mp \overline{\rho}_{\perp\perp}}{a_\perp^2}\right].
\end{aligned} \quad (16)$$

Now, we already know most of the quantities in the above two equations: (i) $\rho_{\lambda\sigma}$ and $\overline{\rho}_{\lambda\sigma}$ are measured quantities, (ii) the a_λ^2 are determined from the relations in Eq. (12), and (iii) $a_i a_\perp \sin\Delta_i$ is obtained from Eq. (13). Thus, the two equations in Eq. (16) involve only two unknown quantities—$\tan\phi$ and $a_i a_\perp \cos\Delta_i$—and can easily be solved (up to a sign ambiguity in each of these quantities). In this way $\tan^2\phi$ (or, equivalently, $\sin^2\phi$) can be obtained from the angular analysis.

Note that this method relies on the measurement of the interference terms between different helicities. However, we do not actually require that all three helicity components of the amplitude be used. In fact, one can use observables involving any two of the largest helicity amplitudes. In the above description, one could have chosen "$\| 0$" instead of "$\perp \|$" or "$\perp 0$."

We now turn to specific applications of this method. Consider first the situation in which the final state is a CP eigenstate, $f = \pm\overline{f}$. In this case, the parameters of Eqs. (1)–(4) satisfy $a_\lambda = b_\lambda$, $\delta^a_\lambda = \delta^b_\lambda$ (which implies that $\delta_\lambda = 0$), and $\phi_a = -\phi_b$ (so that $\phi \equiv -2\phi_M + 2\phi_b$). As described above, a^2_λ can be obtained from Eq. (12). But now the measurement of $\rho_{\lambda\lambda}$ [Eq. (14)] directly yields $\sin\phi$. In fact, this is the conventional way of using the angular analysis to measure the weak phases: each helicity state separately gives clean CP-phase information. Thus, when f is a CP eigenstate, nothing is gained by including the interference terms.

Of course, in general, final states that are CP eigenstates will all receive penguin contributions at some level. Thus, these states violate our assumption that only one weak amplitude contributes to $B^0 \to f$ and $\overline{B^0} \to f$. The only quark-level decays which do not receive penguin contributions are $\overline{b} \to \overline{c}u\overline{d}$, $\overline{u}c\overline{d}$, as well as their Cabibbo-suppressed counterparts, $\overline{b} \to \overline{c}u\overline{s}$, $\overline{u}c\overline{s}$. These are, in fact, the types of decays for which our method is most useful, and we will give meson-level examples of each of these below.

Consider first the decays $B^0_d/\overline{B^0_d} \to D^{*-}\rho^+$, $D^{*+}\rho^-$ (which correspond to $\overline{b} \to \overline{c}u\overline{d}$, $\overline{u}c\overline{d}$ at the quark level). In this case we have $\phi_M = \beta$, $\phi_a = 0$, and $\phi_b = -\gamma$, so that $\phi = -2\beta - \gamma$. The method described above allows one to extract $\sin^2(2\beta + \gamma)$ from an angular analysis of the final state $D^{*\pm}\rho^{\mp}$.

In Ref. [6], Dunietz pointed out that $\sin^2(2\beta + \gamma)$ could, in principle, be obtained from measurements of $B^0_d(t) \to D^{\mp}\pi^{\pm}$. He used the method of Ref. [7], which requires the accurate measurement of the quantity $\Gamma(\overline{B^0_d} \to D^-\pi^+)/\Gamma(B^0_d \to D^-\pi^+)$. This ratio is essentially $|V_{ub}V^*_{cd}/V^*_{cb}V_{ud}|^2 \simeq 4 \times 10^{-4}$. Obviously, it will be very difficult to measure this tiny quantity with any precision, which creates a serious barrier to carrying out Dunietz's method in practice.

On the other hand, our method does not suffer from this problem. In our notation [Eqs. (1)–(4)], the rate $\Gamma(\overline{B^0_d} \to D^{*-}\rho^+)$ is proportional to b^2_λ. However, as we have already emphasized in the discussion following Eq. (12), a determination of this quantity is not needed to extract $\sin^2(2\beta + \gamma)$ using the angular analysis: *none of the observables or combinations required for the analysis are proportional to* b^2_λ. Thus, we avoid the practical problems present in Dunietz's method.

One disadvantage of the final states $D^{*\pm}\rho^{\mp}$ is that the two decay amplitudes are very different in size (hence the small value of b_λ). This results in a very small CP-violating asymmetry whose size is approximately $|V_{ub}V^*_{cd}/V^*_{cb}V_{ud}| \approx 2\%$. Since the total number of B's required to make the measurement is inversely proportional to the square of the asymmetry A_f, $N_B \propto 1/[BR(B^0_d \to f)A^2_f]$, this is a potential problem, even though the branching ratio for the decay $B^0_d \to D^{*-}\rho^+$ is quite large, roughly 1%.

One can avoid the problem of a small asymmetry by instead using the Cabibbo-suppressed decays $B^0_d \to \overline{D}^{*0}K^{*0}$, $D^{*0}K^{*0}$ and $\overline{B^0_d} \to D^{*0}\overline{K}^{*0}$, $\overline{D}^{*0}\overline{K}^{*0}$ (corresponding to the quark-level decays $\overline{b} \to \overline{c}u\overline{s}$, $\overline{u}c\overline{s}$) [8]. (Here it is assumed that both K^{*0} and $\overline{K}^{*0}$ decay to the same state $K_S\pi^0$.) In this case the two amplitudes are much more equal in size, leading to a large asymmetry of about $|V_{ub}V^*_{cs}/V^*_{cb}V_{us}| \approx 40\%$. The disadvantage, of course, is that the branching ratios for such Cabibbo-suppressed decays are much smaller than those for $B^0_d/\overline{B^0_d} \to D^{*\pm}\rho^{\mp}$. We estimate that $B(B^0_d \to \overline{D}^{*0}K^{*0}) \approx \lambda^2 B(B^0_d \to \Psi K^{*0}) = 7 \times 10^{-5}$, which, when combined with $B(K^{*0} \to K_S\pi^0) = \frac{1}{3}$, yields a net branching ratio of about 2×10^{-5}. Even though this branching ratio is quite a bit smaller than that for $B^0_d \to D^{*-}\rho^+$, the much larger asymmetry makes up for it. We see that the measurement of $\sin^2(2\beta + \gamma)$ using $B^0_d(t) \to \overset{(-)}{D}{}^{*0}\overset{(-)}{K}{}^{*0}$ requires roughly the same number of B's as if $B^0_d(t) \to D^{*\pm}\rho^{\mp}$ were used.

Of course, this leads to an important question: just how many B's are needed for such a measurement? The CLEO Collaboration has already performed an angular analysis of $B^0_d \to D^{*-}\rho^+$ with a sample of 197 ± 15 events [9], and has been able to measure some of the interference terms. [Of course, since they do not have an asymmetric collider, it is not possible for them to measure the $\sin(\Delta Mt)$ terms.] In addition, in our method it is necessary to tag the decaying $B^0_d/\overline{B^0_d}$. Taking the tagging efficiency to be about 30% [1], and using the above values for the branching ratios and asymmetries, we estimate the total number of B's required to measure $\sin^2(2\beta + \gamma)$ using our method to be roughly 10^8. This number may be reduced if it is possible to combine the various final states ($D^{*\pm}\rho^{\mp}, D^{*\pm}a_1^{\mp}, \overset{(-)}{D}{}^{*0}\overset{(-)}{K}{}^{*0}$, etc.). We therefore conclude that this measurement will probably be possible at a first-generation B factory, though it may take several years of data accumulation.

In fact, the extraction of $\sin^2(2\beta + \gamma)$ may well turn out to be the second clean measurement to be made at B factories [$\sin 2\beta$ will clearly be measured first via $B^0_d(t) \to \Psi K_S$]. As discussed above, the angle α cannot be obtained cleanly from $B^0_d(t) \to \pi^+\pi^-$ due to the presence of penguin contributions. This difficulty can be resolved with the aid of an isospin analysis [10], but this technique requires measuring the branching ratio for $B^0_d \to \pi^0\pi^0$, which may be quite small. It is also possible to extract α with no hadronic uncertainties using a Dalitz-plot analysis of $B^0_d(t) \to \pi^+\pi^-\pi^0$ decays [11]. Here the idea is to isolate the resonant contributions from intermediate $\rho\pi$ states, to which certain isospin relations apply. However, one has to be sure that the nonresonant contributions are well understood, which requires some theoretical input. In any case, it is estimated that this measurement will take roughly six years to complete. As for the angle γ, the original suggestion for measuring it cleanly involved the decays $B^{\pm} \to D^0 K^{\pm}$, $\overline{D^0} K^{\pm}$, $D^0_{CP} K^{\pm}$ [12]. However, it was subsequently shown that this type of analysis runs into problems because it is virtually impossible to tag the flavor of the final-state D-meson [13], and so one cannot distinguish $B^{\pm} \to D^0 K^{\pm}$ from $B^{\pm} \to \overline{D^0} K^{\pm}$ decays. One can still obtain γ cleanly by studying decays such as $B^+ \to (K^+ \pi^-)_D K^+$ and $B^+ \to (K^+ \rho^-)_D K^+$, along with their CP conjugates, but this requires many more B's, so that it is unlikely such measurements can be carried out in the first round of B-factory experiments. Finally, there has been much work recently looking at the possibilities for extracting γ from $B \to \pi K$ decays [14]. However, all of these methods use flavor SU(3) symmetry, and so rely heavily on theoretical input. In view of all of this, it is thus quite conceivable that the second clean extraction of CP phases at B factories will be the measurement of $\sin^2(2\beta + \gamma)$ using the method described in this paper.

Note that the measurement of $\sin^2(2\beta + \gamma)$ may turn out to be very useful in looking for physics beyond the SM. If new physics is present, it will affect the CP asymmetries principally through its contributions to B^0-$\overline{B^0}$ mixing [15]. The most straightforward way of searching for this new physics is to consider two distinct decay modes which, in the SM, probe the same CP angle. A discrepancy between the two values would be clear evidence of physics beyond the SM. For example, the angle γ can be measured using rate asymmetries in $B^{\pm}$ decays as described above ($B^{\pm} \to DK^{\pm}$ [12,13]), or in $B^0_s/\overline{B^0_s}$ decays [$B^0_s(t) \to D_s^{\pm} K^{\mp}$ [16] or $B^0_s(t) \to D_s^{*\pm} K^{*\mp}$ (see below)]. If there is new physics in B^0_s-$\overline{B^0_s}$ mixing, with new phases, one will obtain different values of γ from these two systems. Unfortunately, as argued above, it will be difficult to use $B^{\pm}$ decays to obtain γ, at least in the short term, so that we will not have two independent values of γ to compare. However, this is where the measurement of $\sin^2(2\beta + \gamma)$ will be useful: using the value of 2β as measured in $B^0_d(t) \to \Psi K_S$, one can obtain γ, up to discrete ambiguities. If none of these values of γ coincide with those given by the measurement of $\sin^2\gamma$ in the B_s system, this will be a clear signal of new physics.

Finally, one can also consider B^0_s and $\overline{B^0_s}$ decays corresponding to the quark-level decays $\overline{b} \to \overline{c} u \overline{d}$, $\overline{u} c \overline{d}$, or $\overline{b} \to \overline{c} u \overline{s}$, $\overline{u} c \overline{s}$. The most promising processes are the Cabibbo-suppressed decay modes $B^0_s/\overline{B^0_s} \to D_s^{*\pm} K^{*\mp}$. Here $\phi_M = 0$, so that the quantity $\sin^2\gamma$ can be extracted from the angular analysis of $B^0_s(t) \to D_s^{*\pm} K^{*\mp}$. This is therefore a new method of obtaining the CP phase γ [17]. Note that $\sin^2\gamma$ can also be obtained from a measurement of $B^0_s(t) \to D_s^{\pm} K^{\mp}$ using a different method [16]. The advantage of our method is that the branching ratios are likely to be larger. On the other hand, one must also perform an angular analysis, which is likely to require more B's. We therefore conclude that the two methods will probably be of equal difficulty experimentally. Thus, this gives two independent ways of extracting $\sin^2\gamma$ from similar final states.

In summary, we have presented a new method of using the angular analysis of $B \to V_1 V_2$ decays to extract weak, CP-violating phases with no hadronic uncertainties. Its most useful application involves the quark-level decays $\overline{b} \to \overline{c} u \overline{d}$, $\overline{u} c \overline{d}$, and $\overline{b} \to \overline{c} u \overline{s}$, $\overline{u} c \overline{s}$. We have shown that the quantity $\sin^2(2\beta + \gamma)$ can be cleanly obtained from the study of the decays $B^0_d(t) \to D^{*\pm} \rho^{\mp}$, $D^{*\pm} a_1^{\mp}$, $\overset{(-)}{D}{}^{*0} \overset{(-)}{K}{}^{*0}$, etc. Similarly, $\sin^2\gamma$ can be extracted from $B^0_s(t) \to D_s^{*\pm} K^{*\mp}$. In all of these cases, there are no penguin contributions to the decays. Finally, we have argued that, due to difficulties with other methods of measuring CP phases, $\sin^2(2\beta + \gamma)$ may well be the second clean measurement, after $\sin 2\beta$, which will be made at B factories.

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[1] For a review, see *The BaBar Physics Book,* edited by P. F. Harrison and H. R. Quinn, SLAC Report No. 504 (Stanford Linear Acceleration Center, Stanford, CA, 1998).
[2] D. London and R. Peccei, Phys. Lett. B **223**, 257 (1989); M. Gronau, Phys. Rev. Lett. **63**, 1451 (1989); Phys. Lett. B **300**, 163 (1993); B. Grinstein, Phys. Lett. B **229**, 280 (1989).
[3] I. Dunietz, H. R. Quinn, A. Snyder, W. Toki, and H. J. Lipkin, Phys. Rev. D **43**, 2193 (1991).
[4] N. Sinha and R. Sinha, Phys. Rev. Lett. **80**, 3706 (1998).
[5] A. S. Dighe, I. Dunietz, and R. Fleischer, Eur. Phys. J. C **6**, 647 (1999).
[6] I. Dunietz, Phys. Lett. B **427**, 179 (1998).
[7] R. Aleksan, I. Dunietz, B. Kayser, and F. Le Diberder, Nucl. Phys. **B361**, 141 (1991).
[8] For a discussion of CP violation in $B^0_d/\overline{B^0_d} \to DK_S$ decays, see B. Kayser and D. London, Phys. Rev. D **61**, 116013 (2000).
[9] CLEO Collaboration Report No. CLEO CONF 98-23, http://w4.lns.cornell.edu/public/CONF/.
[10] M. Gronau and D. London, Phys. Rev. Lett. **65**, 3381 (1990).
[11] A. E. Snyder and H. R. Quinn, Phys. Rev. D **48**, 2139 (1993); H. R. Quinn and J. P. Silva, hep-ph/0001290.
[12] M. Gronau and D. Wyler, Phys. Lett. B **265**, 172 (1991).
[13] D. Atwood, I. Dunietz, and A. Soni, Phys. Rev. Lett. **78**, 3257 (1997). See also Ref. [4].
[14] For a review of methods for measuring γ, see M. Gronau, hep-ph/0001317, and references therein.
[15] C. O. Dib, D. London, and Y. Nir, Int. J. Mod. Phys. A **6**, 1253 (1991).
[16] R. Aleksan, I. Dunietz, and B. Kayser, Z. Phys. C **54**, 653 (1992).
[17] Another method for obtaining γ from $B^0_s(t) \to D_s^{*\pm} K^{*\mp}$ relies on a nonzero width difference $\Delta\Gamma_s$, see I. Dunietz and R. Fleischer, Phys. Lett. B **387**, 361 (1996).