Anharmonic Properties of the Double Giant Dipole Resonance

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(Received 15 May 2000)

A systematic microscopic study of the anharmonic properties of the double giant dipole resonance (DGDR) has been carried out, for the first time, for nuclei with mass number A spanning the whole mass table. It is concluded that the corrections of the energy centroid of the $J^\pi = 0^+$ and 2^+ components of the DGDR from its harmonic limit are negative, have a value of the order of a few hundred keV, and follow an A^{-1} dependence.

PACS numbers: 21.60.Jz, 21.10.Re, 24.30.Cz

Many-fermion systems, from metals in bulk to atomic nuclei, display collective degrees of freedom known as plasmons in the language of solid state physics [1] and as giant resonances within the framework of nuclear physics [2]. A very successful description of these modes is provided by the time-dependent mean-field theory [3], in the variety of versions known as quasiboson approximation, random phase approximation, linear response theory [4–6], time-dependent Hartree-Fock theory [7], time-dependent local density approximation [8], etc. At the basis of all these, to a large extent, equivalent theoretical descriptions of the sloshing back and forth of electrons against ions, of protons against neutrons, etc., one finds the small amplitude approximation which identifies these modes with the one-phonon state of an harmonic oscillator. While it is true that a couple of fermions (particle-hole excitation) has integer spin, its behavior cannot be identified for all relative energies and momenta with that of a real boson, the range of validity of this identification depending on the correlation energy of the pair. On the other hand, close to the ground state, fermion particle-hole excitations do behave as (quasi)bosons. In fact, the terms which, in the equations of motion, are related to the non-bosonic contributions of the commutation relations of pairs of fermions have random phases leading to cancellations which reduce conspicuously the contribution of the corresponding terms, eventually justifying the harmonic approximation [4–6,9]. In any case, all degrees of freedom of a many-fermion system are exhausted by the degrees of freedom of the particles. Consequently, although collective vibrations display small overlaps with each of the (particle-hole) components of the wave function describing the mode, a certain amount of overcounting is unavoidable.

With the advent of high fluency lasers and of high luminosity heavy ion beams at relativistic energies, it is now possible to study multiplasmon states in bulk matter and in clusters [10], as well as states of multiple excited giant dipole resonances in atomic nuclei [11], and thus test the limits of validity of the harmonic paradigm in many-fermion systems [12]. In particular, the discovery of the double giant dipole resonance (DGDR) in nuclei [11,18–21] and the observation of small deviations from the harmonic picture concerning the excitation energy and the spreading width, combined with the large (up to a factor 2–3 enhancement) deviations of the associated Coulomb excitation cross sections measured in relativistic heavy ion collisions [11], call for a better understanding of the role anharmonicities play in the spectrum of the DGDR. In fact, anharmonicities influence electromagnetic DGDR cross sections in several ways: (a) the energy shifts of the DGDR states from the harmonic values can affect in an important way the electromagnetic cross section, in keeping with the exponential dependence of these quantities with the Q value of the process [22], (b) anharmonicities lead to changes in the $E1$ transition matrix elements to preserve the energy weighted sum rule (EWSR) [23] which eventually reinforce these effects, (c) anharmonicities which are a consequence of the mixing of states with different number of phonons give rise to many paths, other than the (harmonic) two-step one, to excite the DGDR in electromagnetic processes. While all these questions inspired much theoretical work [24–34], no clear picture has emerged of the DGDR anharmonicity question, let alone an explanation of the "Coulomb excitation anomaly." In particular, no consensus exists concerning the mass-number dependence of the energy shifts from the harmonic values.

In this Letter we present the results of the first, systematic calculation of the spectrum of the DGDR, carried out in a complete one- and two-phonon basis (the effect of the three phonon states on the anharmonicity being arguably small [28]) for nuclei with mass number A spanning the whole mass table. It will be concluded that the energy shift (lowering) of the energy centroid of the $J^\pi = 0^+$ and 2^+ components of the DGDR from the harmonic limit is rather modest (few hundreds of keV) and display a clear A^{-1} dependence. The solution of the Coulomb excitation anomaly is thus likely to be found elsewhere [35].

The Hamiltonian used in describing the system contains, aside from a mean-field term which determines the single-particle motion of protons and neutrons, a monopole pairing interaction and a separable multipole-multipole force with strengths adjusted so as to reproduce the odd-even

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mass differences and the spectrum of low-lying vibrations and of giant resonances, respectively [2,7,15,17]. In particular, the strength of the isovector dipole-dipole term was fixed by fitting the observed energy centroid of the GDR in each nucleus or, lacking this information, the value emerging from the energy systematics ($80A^{-1/3}$ MeV).

The basis of one-phonon states was obtained by diagonalizing the Hamiltonian in the quasiparticle random phase approximation. The basis of two-phonon states was constructed by coupling two one-phonon states to total angular momentum and parity $J^\pi = 0^+$ and 2^+, in keeping with the quantum numbers of the DGDR states. The two-phonon basis thus includes, aside from the states $[1_i^- \times 1_{i'}^-]_{0^+(2^+)}$, where the subindex i is used to distinguish between the different one-phonon dipole states arising from the shell structure of the system, also two-phonon states made up of 0^+, 1^-, 2^+, 3^-, and 4^+ phonons. All one-phonon states, up to 40–50 MeV of excitation energy and contributing with more than 1.0% to the EWSR (0.2% in the case of dipole modes) have been included in the calculations. This choice leads, for the $J^\pi = 2^+$ component of the DGDR in heavy nuclei, to a two-phonon basis containing of the order of 10^3 states.

The Hamiltonian written in terms of quasiparticles and phonons [17] is diagonal in the space of one- and two-phonon states separately, but contains terms coupling one- to two-phonon states. Diagonalizing this Hamiltonian, we obtain the total wave functions Ψ_J^ν and the corresponding eigenvalues from which the results displayed in Table I have been obtained. In the second column of this table, the percentage of the Thomas-Reiche-Kuhn EWSR exhausted by the selected one-phonon dipole states is displayed, while the third column contains the percentage of the EWSR for the DGDR (calculated in Ref. [23]) exhausted relatively to the sum of the 0^+ and 2^+ components. The small differences observed between the percentage of the EWSR exhausted by the DGDR and the GDR is mainly due to the fact that the ground state is considered in the calculations as the one-phonon vacuum, the ground state correlations arising from the interaction between multiphonon configurations not being taken into account. In columns four and five the energy shifts $\Delta E_c(J^\pi)$ of the centroids of the $J^\pi = 0^+$ and 2^+ members of the DGDR with respect to the harmonic predictions are reported. In Fig. 1 we show the quantity

$$B_\nu([E1 \times E1]_J) = \left| \sum_i \langle \Psi_J^\nu | E1 | \Psi_{1^-}^i \rangle \langle \Psi_{1^-}^i | E1 | \Psi_{\mathrm{g.s.}} \rangle \right|^2$$

for the different (two-phonon) states ν, eigenstates of the total Hamiltonian with angular momentum and parity 0^+ and 2^+ of the nucleus ^{136}Xe. The same calculations have been repeated in a basis containing a single two-phonon state $[1_{i_0}^- \times 1_{i_0}^-]_{0^+(2^+)}$, where $1_{i_0}^-$ is the GDR mode carrying the largest fraction of the EWSR, and all one-phonon states so as to reproduce as far as possible the harmonic scenario within the framework of the present microscopic calculation. We shall discuss these results before discussing those of the full calculation.

The diagonalization in the reduced space leads to a breaking of the $[1_{i_0}^- \times 1_{i_0}^-]_{0^+(2^+)}$, and thus to a set of states with $J^\pi = 0^+$ and 2^+, one of which carries about 95% of the two-phonon configuration oscillator strength. The energy shift of this state from the energy of the (noninteracting) two-phonon configuration is shown in Table II in the columns labeled "Sum." There are two mechanisms contributing to this shift: the first one is associated with the Pauli principle corrections. Excluding four-quasiparticle configurations which violate Pauli principle reduces somehow the collectivity of two-phonon configurations. One thus expects a downward shift for isovector phonons such as, e.g., the DGDR (cf. column I of Table II displaying

TABLE I. Percentage of the EWSR exhausted by the GDR and DGDR of the atomic nuclei indicated in the first column. In columns 4 and 5 is displayed the anharmonicity shift $\Delta E_c(J^\pi)$ of the energy centroid of the $J^\pi = 0^+$ and 2^+ components of the DGDR from its harmonic limit.

A	EWSR, %		$\Delta E_c(J^\pi)$, keV	
Nucl.	GDR	DGDR	$J^\pi = 0^+$	$J^\pi = 2^+$
^{40}Ca	104	103	−643	−740
^{58}Ni	104	103	−476	−495
^{86}Kr	106	105	−309	−271
^{120}Sn	106	105	−199	−194
^{136}Xe	103	102	−203	−179
^{208}Pb	94	94	−108	−158

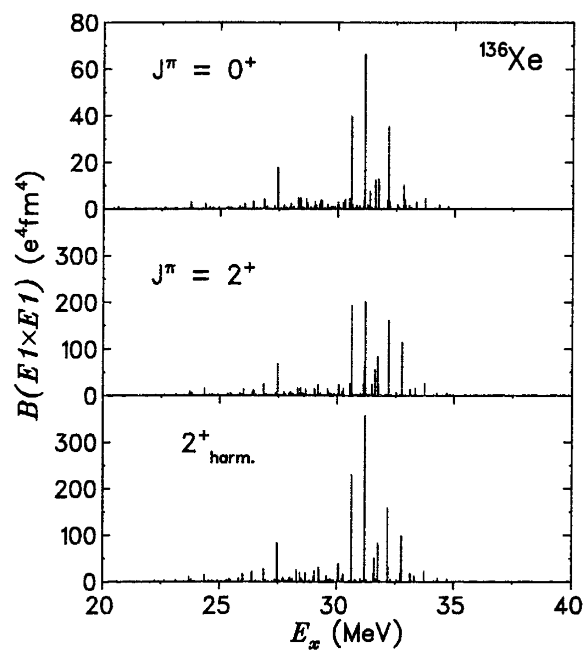

FIG. 1. Energy distributions of the $B(E1 \times E1)$ values associated with the excitation of the 0^+ and 2^+ components of the DGDR in ^{136}Xe, in comparison with the same quantity for the 2^+ component in the harmonic limit. Scales are chosen proportionally to $(2J + 1)$.

TABLE II. Energy shift of the two states $[1^-_{i_0} \times 1^-_{i_0}]_{J^\pi}$ ($J^\pi = 0^+$ and 2^+) with respect to the harmonic value $2\hbar\omega(1^-_{i_0})$. The label i_0 indicates the component of the GDR carrying the largest fraction of the EWSR. The calculations have been carried out in a basis which includes only the two-phonon configuration $[1^-_{i_0} \times 1^-_{i_0}]_{J^\pi}$ and a complete set of 0^+ (2^+) one-phonon states. The contributions to the energy shift arising from Pauli principle corrections and due to the interaction of the two-phonon configuration with one-phonon configurations are shown separately in I and II, respectively.

A		0^+			2^+	
Nucl.	I	II	Sum	I	II	Sum
^{40}Ca	−577	+274	−302	−740	+534	−206
^{58}Ni	−387	+667	+280	−486	+507	+21
^{86}Kr	−240	+103	−137	−291	+227	−64
^{120}Sn	−163	+181	+18	−223	+204	−19
^{136}Xe	−142	+87	−55	−186	+171	−15
^{208}Pb	−104	+129	+24	−137	+93	−44

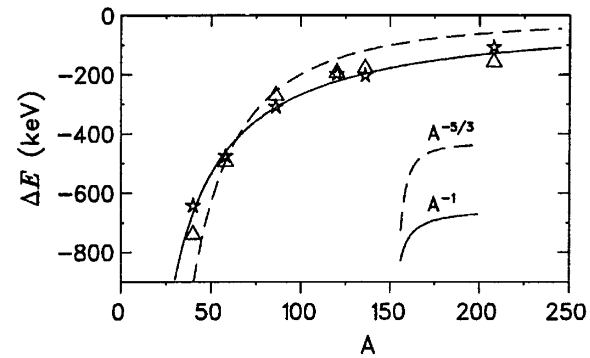

FIG. 2. Shift of the DGDR energy centroid (0^+ stars and 2^+ triangles) from the harmonic limit. The continuous and dashed curves represent fits, assuming an A^{-1} and an $A^{-5/3}$ dependence, respectively, of the results of the microscopic calculations.

the results obtained including only Pauli principle like processes). This shift is found to scale with A^{-1} as expected from general arguments [36] and simple models [15,33,34,39]. The second mechanism arises from the interaction of the $[1^-_{i_0} \times 1^-_{i_0}]_{J^\pi}$ configuration with all one-phonon states. The energy shifts arising from this interaction are given in columns II of Table II. A strong cancellation with the first contribution is found, although not as complete as that reported in Ref. [33], where estimates of the two contributions to the total energy shift under discussion have been carried out within a schematic model. This (second) contribution arising from the interaction of two-phonon configurations with one-phonon states is found, in the present simplified calculations, not to have any simple dependence with A, as it arises from the coupling of the single two-phonon configuration chosen, to relatively few one-phonon configurations lying close in energy and displaying a moderate value of the coupling matrix elements.

Carrying the diagonalization in the full two- and one-phonon space, the contribution to the DGDR energy centroid associated with the second mechanism vanishes because the trace of the matrices is conserved independently on the values of nondiagonal matrix elements. Consequently, the centroid of each of the DGDR configurations (diagonal elements) remains at the noninteractive value, eventually modified by the Pauli principle for two-phonon corrections. The corresponding energy shifts $\Delta E_c(J^\pi)$ for the double magic nuclei ^{40}Ca and ^{208}Pb obtained in the present calculation (cf. Table I columns four and five) are of the same order of magnitude as those obtained in microscopic calculations in Refs. [24,26,32]. On the other hand, the mixing between one- and two-phonon states obtained for the other nuclei (cf. Table II) are larger than for ^{40}Ca and ^{208}Pb. This is in keeping with the fact that doubly magic nuclei are much more rigid than the semimagic ones. The most collective $[1^-_i \times 1^-_i]_{0^+(2^+)}$ configuration in general prefers to mix with either [LEOR × HEOR] or other $[1^-_i \times 1^-_{i'}]_{0^+(2^+)}$ configurations where L(H)EOR is the low (high) energy octupole resonance.

From the systematic calculations one can extract the A dependence of the energy shifts of the centroids from the harmonic limits. For this purpose, the values of the energy shifts of the DGDR are shown in Fig. 2 as functions of A. The continuous curve represents an A^{-1} fitting to the data, while the dashed line shows the $A^{-5/3}$ dependence obtained in the variational time-dependent approach of Ref. [31]. The results of our calculations follow quite accurately the A^{-1} behavior, even if both doubly and semimagic nuclei have been included in the systematics. Weighting equally the 0^+ and 2^+ components of the DGDR we obtain from a χ^2 analysis of the results displayed in Fig. 2, $\Delta E = b \cdot A^{-\alpha}$ with $\alpha = 1.08 \pm 0.06$ and $b = -37 \pm 8$ MeV.

Although our conclusion on the A^{-1} dependence is based on calculations within a specific model, general arguments [36] and estimates [15,33,34,39] support it. Different A dependence of the energy shift is discussed in [34], in terms of the number of active nucleons. The present results indicate that in the case of the GDR this number is indeed of the order of $A^{2/3}$, as for the Ω factor in [36].

We conclude that the deviation of the energy centroid of the double giant dipole resonance from the harmonic limit displays a behavior with mass number A typical of that associated with the global properties characterizing the system, such as, e.g., the energy centroid of the giant dipole resonance.

Discussions with G. F. Bertsch, K. Hagino, I. Hamamoto, and B. Mottelson are gratefully acknowledged. We thank J. Bryssinck for help. V. Yu. P. acknowledges support from INFN and NATO.

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[35] As a matter of fact and strictly speaking, such a conclusion must wait the explicit calculation of the ratio of the Coulomb excitation cross section associated with the (mainly) two-phonon and (mainly) one-phonon states. In fact, to avoid the complications of combining nuclear structure with nuclear reaction theories to eventually carry out a comparison with the experimental data, and be able to extract, in the simplest way, the A dependence of the two-phonon anharmonicities, we have avoided explicit reference to the one-phonon states. In keeping with the fact that the coupling to one-phonon states leads only to a breaking of the strength (damping width), but not to a change in the energy centroid, we are able to concentrate on the central issue which is at the basis of the DGDR energy centroid shift from the harmonic value, namely, the modification of the phonon structure due to Pauli principle corrections.

[36] The lowest order, Pauli principle correction corresponds to fourth order ("butterfly") diagrams (cf., e.g., Fig. 9 of Ref. [37]) leading to the energy shift which can be quantitatively written as $\Delta E^{(4)}_{\mathrm{Pauli}} \sim \Lambda^4 \Omega / Den^3$. In this expression $\Lambda \sim (\hbar\omega/2C)^{1/2}$ is the particle-vibration coupling strength, $\hbar\omega \sim A^{-1/3}$ being the energy of the mode (resonance) to which the particle couples, $C \sim A$ the associated restoring force. Consequently $\Lambda \sim A^{-2/3}$. The energy denominators arise from energy differences between giant resonances and quasiparticle pairs and display the typical A dependence of the GDR energy, thus $Den \sim A^{-1/3}$. The quantity $\Omega \sim A^{2/3}$ (cf., e.g., [38]) measures the degeneracy of the phase space where fermions are allowed to correlate to produce the collective GDR (closed loop summations in field theoretical terms). Consequently $\Delta E^{(4)}_{\mathrm{Pauli}} \sim A^{-1}$.

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