Complete One-Loop Analysis of the Nucleon's Spin Polarizabilities

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We present a complete one-loop analysis of the four nucleon spin polarizabilities in the framework of heavy baryon chiral-perturbation theory. The first nonvanishing contributions to the isovector and first corrections to the isoscalar spin polarizabilities are calculated. No unknown parameters enter these predictions. We compare our results to various dispersive analyses. We also discuss the convergence of the chiral expansion and the role of the delta isobar.

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Low energy Compton scattering off the nucleon is an important probe to unravel the nonperturbative structure of QCD since the electromagnetic interactions in the initial and final state are well understood. In the long wavelength limit, only the charge of the target can be detected. At higher energies, $50 < \omega < 100$ MeV, the internal structure of the system slowly becomes visible. This nucleon structure-dependent effect in *unpolarized* Compton scattering was taken into account by introducing *two* free parameters into the cross-section formula, commonly denoted the *electric* $(\bar{\alpha})$ and *magnetic* $(\bar{\beta})$ polarizabilities of the nucleon in analogy to the structure-dependent response functions for light-matter interactions in classical electrodynamics. Over the past few decades several experiments on low energy Compton scattering off the proton have taken place, resulting in several extractions of the electromagnetic polarizabilities of the proton. At present, the commonly accepted numbers are $\bar{\alpha}^{(p)} = (12.1 \pm 0.8 \pm 0.5) \times 10^{-4}\ \mathrm{fm}^3$, $\bar{\beta}^{(p)} = (2.1 \mp 0.8 \mp 0.5) \times 10^{-4}\ \mathrm{fm}^3$ [1], indicating that the proton compared to its volume of $\sim 1\ \mathrm{fm}^3$ is a rather stiff object. At present, several quite different theoretical approaches find qualitative and quantitative explanations for these 2 polarizabilities, but they also constitute one of the striking successes of chiral perturbation theory [2,3].

Quite recently, with the advent of polarized targets and new sources with a high flux of polarized photons, the case of *polarized* Compton scattering off the proton $\vec{\gamma}\vec{p} \to \gamma p$ has come close to experimental feasibility. On the theoretical side it has been shown [4] that one can define four spin-dependent electromagnetic response functions γ_i, $i = 1, \ldots, 4$, which in analogy to $\bar{\alpha}, \bar{\beta}$ are commonly called the "spin polarizabilities" of the proton. First studies have been published [5,6], claiming that the parametrized information on the low energy spin structure of the proton can really be extracted from the upcoming double-polarization Compton experiments. The success of this program would clearly shed new light on our understanding of the internal dynamics of the proton and at the same time serve as a check on the theoretical explanations of the polarizabilities. The new challenge to theorists will then be to explain all six of the leading electromagnetic response functions simultaneously. At present there exists only one experimental analysis that has shed some light on the magnitude of the (essentially) unknown spin polarizabilities $\gamma_i^{(p)}$ of the proton: The LEGS group has reported [7] a result for a linear combination involving three of the γ_i, namely, $\gamma_\pi^{(p)}|_{\mathrm{exp}} = \gamma_1^{(p)} + \gamma_2^{(p)} + 2\gamma_4^{(p)} = (17.3 \pm 3.4) \times 10^{-4}\ \mathrm{fm}^4$. (Note that we have subtracted the contribution of the pion-pole diagram in order to be consistent with the definition of the spin polarizabilities given in [8].) We note that this pioneering result was obtained from an analysis of an *unpolarized* Compton experiment in the backward direction, where the spin polarizabilities come in as one contribution in a whole class of subleading order nucleon structure effects in the differential cross section. Given these structure subtleties and the fact that most theoretical calculations [5,6,8–10] have predicted this particular linear combination of spin polarizabilities to be a factor of 2 smaller than the number given in [7], we can only reemphasize the need for the upcoming polarized Compton scattering experiments.

In this paper we take up the challenge on the theory side within the context of heavy baryon chiral perturbation theory (HBCHPT), extending previous efforts [8,11–13] in a significant way. The active degrees of freedom in HBCHPT are the asymptotically observable pion and nucleon fields. The various contributions from tree and loop diagrams are organized according to power counting rules, i.e., one expands in small momenta and pion masses (m_π), collectively denoted by p. Previously an order $\mathcal{O}(p^3)$ SU(2) HBCHPT calculation [11] was performed, which showed that the leading (i.e., long-range) structure effects in the spin polarizabilities are given by eight different πN loop diagrams, giving rise to a $1/m_\pi^2$ behavior in the γ_i. Subsequently, it was shown in a third order SU(2) calculation [8], in which the first nucleon resonance, the $\Delta(1232)$, was included as an explicit degree of freedom [14], that two (γ_2, γ_4) of the four spin polarizabilities receive large corrections due to $\Delta(1232)$ related effects, resulting in a big correction to the leading $1/m_\pi^2$ behavior [15]. In that phenomenological extension of HBCHPT, one also counts the nucleon-delta mass splitting as an additional small parameter and collectively denotes all small parameters as

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ϵ. The corresponding expansion, which also has a consistent power counting, is called the "small scale expansion" (SSE) (because it differs from a chiral expansion due to the nonvanishing of the $N\Delta$ mass splitting in the chiral limit). Another important conclusion of [8] was that any HBCHPT calculation that wants to calculate γ_2, γ_4 would have to be extended to $\mathcal{O}(p^5)$ before it can incorporate the large $\Delta(1232)$ related corrections found already at $\mathcal{O}(\epsilon^3)$ in [8]. Recently, two $\mathcal{O}(p^4)$ SU(2) HBCHPT calculations [12,13] of polarized Compton scattering in the forward direction appeared, from which one can extract one particular linear combination of three of the four γ_i, usually called γ_0, $\gamma_0 = \gamma_1 - (\gamma_2 + 2\gamma_4)\cos\theta|_{\theta\to 0}$. [$\gamma_0$ can also be calculated from the absorption cross sections of polarized photons on polarized nucleons via the Gell-Mann–Goldberger–Thirring sum rule [16], as pointed out in [17]. In the absence of such data several groups have tried to extract the required cross sections via a partial wave analysis of unpolarized absorption cross sections.] The authors of [12,13] claimed to have found a huge correction to γ_0 at $\mathcal{O}(p^4)$ relative to the $\mathcal{O}(p^3)$ result already found in [17], casting doubt on the usefulness/convergence of HBCHPT for spin polarizabilities. Given that γ_0 involves the very two polarizabilities γ_2, γ_4, the (known) poor convergence for γ_0 found in [12,13] should not have come as a surprise. We will return to this point later.

In the following we report on the results of an $\mathcal{O}(p^4)$ calculation in HBCHPT of all four spin polarizabilities γ_i, which allows one to study the issue of convergence in chiral effective field theories for these important new spin-structure parameters of the nucleon. The pertinent results of our investigation can be summarized as follows:

(1) We first want to comment on the extraction of polarizabilities from nucleon Compton scattering amplitudes. In previous analyses [8,11] it has always been stated that, in order to obtain the spin polarizabilities from the calculated Compton amplitudes, one only has to subtract the nucleon tree-level (Born) graphs from the fully calculated amplitudes. The remainder in each (spin amplitude) then started with a factor of ω^3 and the associated Taylor coefficient was related to the spin polarizabilities. Because of the (relatively) simple structure of the spin amplitudes at this order, this prescription gives the correct result in the $\mathcal{O}(p^3)$ HBCHPT [11] and the $\mathcal{O}(\epsilon^3)$ SSE [8] calculations. However, at $\mathcal{O}(p^4)$ [and also at $\mathcal{O}(\epsilon^4)$ [18]], one has to resort to a definition of the (spin) polarizabilities that is soundly based on field theory, in order to make sure that one picks up only those contributions at ω^3 that are really connected with (spin) polarizabilities. In fact, at $\mathcal{O}(p^4)$ [$\mathcal{O}(\epsilon^4)$] the prescription given in [8,11] leads to an admixture of effects resulting from two successive, uncorrelated γNN interactions with a one-nucleon intermediate state. In order to avoid these problems, we advocate the following definition for the *spin-dependent* polarizabilities in (chiral) effective field theories: Given a complete set of spin-structure amplitudes for Compton scattering to a certain order in perturbation theory, one first removes all one-particle (i.e., one-nucleon or one-pion) reducible (1PR) contributions from the full spin-structure amplitudes. To be more precise, at order $\mathcal{O}(p^4)$, one removes $F(\omega)/\omega$ terms from the amplitude, where $F(\omega)$ denotes the energy dependence of the γNN vertex function.

Specifically, starting from the general form of the T matrix for real Compton scattering assuming invariance under parity, charge conjugation, and time reversal symmetry, we utilize the following six structure amplitudes $A_i(\omega,\theta)$ [8,11] in the Coulomb gauge, $\epsilon_0 = \epsilon_0' = 0$,

$$
\begin{aligned}
T = {} & A_1(\omega,\theta)\vec{\epsilon}^{*\prime}\cdot\vec{\epsilon} + A_2(\omega,\theta)\vec{\epsilon}^{*\prime}\cdot\hat{k}\vec{\epsilon}\cdot\hat{k}' + iA_3(\omega,\theta)\vec{\sigma}\cdot(\vec{\epsilon}^{*\prime}\times\vec{\epsilon}) + iA_4(\omega,\theta)\vec{\sigma}\cdot(\hat{k}'\times\hat{k})\vec{\epsilon}^{*\prime}\cdot\vec{\epsilon} \\
& + iA_5(\omega,\theta)\vec{\sigma}\cdot[(\vec{\epsilon}^{*\prime}\times\hat{k})\vec{\epsilon}\cdot\hat{k}' - (\vec{\epsilon}\times\hat{k}')\vec{\epsilon}^{*\prime}\cdot\hat{k}] + iA_6(\omega,\theta)\vec{\sigma}\cdot[(\vec{\epsilon}^{*\prime}\times\hat{k}')\hat{\epsilon}\cdot\hat{k}' - (\vec{\epsilon}\times\hat{k})\vec{\epsilon}^{*\prime}\cdot\hat{k}], \quad (1)
\end{aligned}
$$

where θ corresponds to the c.m. scattering angle, $\vec{\epsilon}, \hat{k}$ ($\vec{\epsilon}', \hat{k}'$) denote the polarization vector, direction of the incident (final) photon, while $\vec{\sigma}$ represents the (spin-) polarization vector of the nucleon. Each (spin-)structure amplitude is now separated into 1PR contributions and a remainder, which contains the response of the nucleon's excitation structure to two photons:

$$
A_i(\omega,\theta) = A_i(\omega,\theta)^{\rm 1PR} + A_i(\omega,\theta)^{\rm exc}, \quad i = 3,\ldots,6. \quad (2)
$$

By Taylor expanding the spin-dependent $A_i(\omega,\theta)^{\rm 1PR}$ for the case of a proton target in the c.m. frame into a power series in ω, the leading terms are linear in ω and are given by the venerable low energy theorems of Low, Gell-Mann and Goldberger [19]:

$$
\begin{aligned}
A_3(\omega,\theta)^{\rm 1PR} &= \frac{[1 + 2\kappa^{(p)} - (1+\kappa^{(p)})^2\cos\theta]e^2}{2M_N^2}\,\omega + \mathcal{O}(\omega^2), \\
A_4(\omega,\theta)^{\rm 1PR} &= -\frac{(1+\kappa^{(p)})^2 e^2}{2M_N^2}\,\omega + \mathcal{O}(\omega^2), \quad (3)\\
A_5(\omega,\theta)^{\rm 1PR} &= \frac{(1+\kappa^{(p)})^2 e^2}{2M_N^2}\,\omega + \mathcal{O}(\omega^2), \\
A_6(\omega,\theta)^{\rm 1PR} &= -\frac{(1+\kappa^{(p)}) e^2}{2M_N^2}\,\omega + \mathcal{O}(\omega^2).
\end{aligned}
$$

While it is not advisable to really perform this Taylor expansion for the spin-dependent $A_i(\omega,\theta)^{\rm 1PR}$ due to the complex pole structure, one can do so without problems for the $A_i(\omega,\theta)^{\rm exc}$ as long as $\omega \ll m_\pi$. For the case of a proton, one then finds

$$A_3(\omega,\theta)^{\text{exc}} = 4\pi[\gamma_1^{(p)} - (\gamma_2^{(p)} + 2\gamma_4^{(p)})\cos\theta]\omega^3 + \mathcal{O}(\omega^4),$$
$$A_{(4,5,6)}(\omega,\theta)^{\text{exc}} = 4\pi\gamma_{(2,4,3)}^{(p)}\omega^3 + \mathcal{O}(\omega^4). \quad (4)$$

We therefore take Eq. (4) as the starting point for the calculation of the spin polarizabilities, which are related to the ω^3 Taylor coefficients of $A_i(\omega,\theta)^{\text{exc}}$. As noted above, both the $\mathcal{O}(p^3)$ HBCHPT [11] and the $\mathcal{O}(\epsilon^3)$ SSE [8] results are consistent with this definition.

(2) Utilizing Eqs. (2) and (4), we have calculated the first subleading correction, $\mathcal{O}(p^4)$, to the four isoscalar spin polarizabilities $\gamma_i^{(s)}$ already determined to $\mathcal{O}(p^3)$ in [11] in SU(2) HBCHPT. We employ here the convention [8]

$$\gamma_i^{(p)} = \gamma_i^{(s)} + \gamma_i^{(v)}, \qquad \gamma_i^{(n)} = \gamma_i^{(s)} - \gamma_i^{(v)}. \quad (5)$$

Contrary to popular opinion, we show that even at subleading order all four spin polarizabilities can be given in closed form expressions which are free of any unknown chiral counterterms. The only parameters appearing in the results are the axial-vector nucleon coupling constant $g_A = 1.26$, the pion decay constant $F_\pi = 92.4$ MeV, the pion mass $m_\pi = 138$ MeV, the mass of the nucleon $M_N = 938$ MeV as well as its isoscalar, $\kappa^{(s)} = -0.12$, and isovector, $\kappa^{(v)} = 3.7$, anomalous magnetic moments. All $\mathcal{O}(p^4)$ corrections arise from 25 one-loop πN continuum diagrams, with the relevant vertices obtained from the well-known SU(2) HBCHPT $\mathcal{O}(p)$ and $\mathcal{O}(p^2)$ Lagrangians given in detail in Ref. [11]. To $\mathcal{O}(p^4)$ we find

$$\gamma_1^{(s)} = +\frac{e^2 g_A^2}{96\pi^3 F_\pi^2 m_\pi^2}[1 - \mu\pi], \quad (6)$$

$$\gamma_2^{(s)} = +\frac{e^2 g_A^2}{192\pi^3 F_\pi^2 m_\pi^2}\left[1 + \mu\,\frac{(-6 + \kappa^{(v)})\pi}{4}\right], \quad (7)$$

$$\gamma_3^{(s)} = +\frac{e^2 g_A^2}{384\pi^3 F_\pi^2 m_\pi^2}[1 - \mu\pi], \quad (8)$$

$$\gamma_4^{(s)} = -\frac{e^2 g_A^2}{384\pi^3 F_\pi^2 m_\pi^2}\left[1 - \mu\,\frac{11}{4}\,\pi\right], \quad (9)$$

with $\mu = m_\pi/M_N \simeq 1/7$ and the numerical values given in Table I. The leading $1/m_\pi^2$ behavior of the isoscalar spin polarizabilities is not touched by the $\mathcal{O}(p^4)$ correction, as expected. With the notable exception of $\gamma_4^{(s)}$, which even changes its sign due to a large $\mathcal{O}(p^4)$ correction, we show that this first subleading order of $\gamma_1^{(s)}, \gamma_2^{(s)}, \gamma_3^{(s)}$ amounts to a 25%–45% correction to the leading order result. This does not quite correspond to the expected m_π/M_N correction of (naive) dimensional analysis, but can be considered acceptable. The large correction in $\gamma_4^{(s)}$ should be considered accidental. It is not related to the large Δ effects found in the SSE calculation of [8], because these will only show up at $\mathcal{O}(p^5)$ in the HBCHPT framework.

(3) We further report the first results for the four *isovector* spin polarizabilities $\gamma_i^{(v)}$ obtained in the framework of chiral effective field theories. Previous calculations at $\mathcal{O}(p^3)$ [11] and $\mathcal{O}(\epsilon^3)$ [8] were only sensitive to the isoscalar spin polarizabilities $\gamma_i^{(s)}$, therefore this calculation gives the first indication from a chiral effective field theory about the magnitude of the difference in the low energy spin structure between proton and neutron. As in the case of the isoscalar spin polarizabilities there are again no unknown counterterm contributions to this order in the $\gamma_i^{(v)}$. All $\mathcal{O}(p^4)$ contributions arise from 16 one-loop πN continuum diagrams with the relevant $\mathcal{O}(p)$ and $\mathcal{O}(p^2)$ vertices again obtained from the Lagrangians given in Ref. [11]. To $\mathcal{O}(p^4)$, one finds

$$\gamma_1^{(v)} = \frac{e^2 g_A^2}{96\pi^3 F_\pi^2 m_\pi^2}\left[0 - \mu\,\frac{5\pi}{8}\right], \quad (10)$$

$$\gamma_2^{(v)} = \frac{e^2 g_A^2}{192\pi^3 F_\pi^2 m_\pi^2}\left[0 - \mu\,\frac{(1 + \kappa^{(s)})\pi}{4}\right], \quad (11)$$

TABLE I. Predictions for the spin polarizabilities in HBCHPT in comparison with the dispersion analyses of Refs. [5,6,9] (Mainz1, Mainz2, BGLMN) and the $\mathcal{O}(\epsilon^3)$ results of the small scale expansion [8] (SSE1). All results are given in units of 10^{-4} fm^4.

$\gamma_i^{(N)}$	$\mathcal{O}(p^3)$	$\mathcal{O}(p^4)$	Sum	Mainz1	Mainz2	BGLMN	SSE1
$\gamma_1^{(s)}$	+4.6	−2.1	+2.5	+5.6	+5.7	+4.7	+4.4
$\gamma_2^{(s)}$	+2.3	−0.6	+1.7	−1.0	−0.7	−0.9	−0.4
$\gamma_3^{(s)}$	+1.1	−0.5	+0.6	−0.6	−0.5	−0.2	+1.0
$\gamma_4^{(s)}$	−1.1	+1.5	+0.4	+3.4	+3.4	+3.3	+1.4
$\gamma_1^{(v)}$	⋯	−1.3	−1.3	−0.5	−1.3	−1.6	⋯
$\gamma_2^{(v)}$	⋯	−0.2	−0.2	−0.2	+0.0	+0.1	⋯
$\gamma_3^{(v)}$	⋯	+0.1	+0.1	−0.0	+0.5	+0.5	⋯
$\gamma_4^{(v)}$	⋯	+0.0	+0.0	+0.0	−0.5	−0.6	⋯

$$\gamma_3^{(v)} = \frac{e^2 g_A^2}{384\pi^3 F_\pi^2 m_\pi^2}\left[0 + \mu\,\frac{\pi}{4}\right], \qquad (12)$$

$$\gamma_4^{(v)} = 0\,, \qquad (13)$$

with the numerical values again given in Table I. The result of our investigation is that the size of the $\gamma_i^{(v)}$ really tends to be an order of magnitude smaller than the one of the $\gamma_i^{(s)}$ (with the possible exception of $\gamma_1^{(v)}$), supporting the scaling expectation, $\gamma_i^{(v)} \sim (m_\pi/M_N)\gamma_i^{(s)}$, from (naive) dimensional analysis. This is reminiscent of the situation in the spin-independent electromagnetic polarizabilities $\bar{\alpha}^{(v)}, \bar{\beta}^{(v)}$ [2], which are also suppressed by one chiral power relative to their isoscalar partners $\bar{\alpha}^{(s)}, \bar{\beta}^{(s)}$.

(4) Finally, we want to comment on the comparison between our results and existing calculations using dispersion analyses. Given our comments on the convergence of the chiral expansion for the (isoscalar) spin polarizabilities [8], we reiterate that we do not believe our $\mathcal{O}(p^4)$ HBCHPT result for $\gamma_2^{(s)}, \gamma_4^{(s)}$ to be meaningful. Their large inherent $\Delta(1232)$ related contribution just cannot be included (via a counterterm) before $\mathcal{O}(p^5)$ in HBCHPT that only deals with pion and nucleon degrees of freedom. In Table I it is therefore interesting to note that, by adding ("by hand") the delta-pole contribution of $\sim -2.5 \times 10^{-4}$ fm^4 found in [8] to $\gamma_2^{(s)}$, one could get quite close to the range for this spin polarizability as suggested by the dispersion analyses [5,6,9]. Similarly, adding $\sim +2.5 \times 10^{-4}$ fm^4 to $\gamma_4^{(s)}$ as suggested by [8] also leads quite close to the range advocated by the dispersion results [5,6,9]. However, such a procedure is of course not legitimate in an effective field theory, but it raises the hope that an extension of the $\mathcal{O}(\epsilon^3)$ SSE calculation of [8] that includes explicit delta degrees of freedom could lead to a much better behaved perturbative expansion for the isoscalar spin polarizabilities. Whether this expectation holds true will be known quite soon [18]. For the isovector spin polarizabilities we have given the first predictions available from effective field theory. In general the agreement with the range advocated by the dispersion analyses is quite good. Furthermore, we now discuss our results for those linear combinations of the γ_i (reconstructed from the numbers given in Table I) that typically are the main focus of attention in the literature, i.e. γ_0 and γ_π. However, we reemphasize that we do not consider our $\mathcal{O}(p^4)$ HBCHPT predictions for $\gamma_0^{(s)}, \gamma_\pi^{(s)}$ to be meaningful, because they involve $\gamma_2^{(s)}, \gamma_4^{(s)}$. The corresponding isovector combinations, however, again seem to agree quite well with the dispersive results and so far we have no reason to suspect that they might be affected by the poor convergence behavior of some of their isoscalar counterparts. We further note that our $\mathcal{O}(p^4)$ HBCHPT predictions for $\gamma_0^{(s,v)}$ differ from the ones given in two recent calculations [12,13]. As noted above, this difference arises solely from a different definition of nucleon spin polarizabilities. If we (by hand) Taylor expand our γNN vertex functions $F(\omega)$ in powers of ω and include the resulting terms into the γ_0 structure, we obtain the $\mathcal{O}(p^4)$ corrections $\gamma_0^{(s)} = -6.9$, $\gamma_0^{(v)} = -1.6$ in units of 10^{-4} fm^4, in numerical (and analytical) agreement with [12,13]. This brings us to an important point: Once the first polarized Compton asymmetries have been measured, it has to be checked very carefully whether or not the same input data fitted to the terms we define as 1PR plus the additional free γ_i parameters lead to the same numerical fit results for the spin polarizabilities as in the dispersion theoretical codes usually employed to extract polarizabilities from Compton data. Small differences, for example, in the treatment of the pion/nucleon pole could lead to quite large systematic errors in the determination of the γ_i. Such studies are under way [18].

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