

Primordial Adiabatic Fluctuations from Cosmic Defects

P. P. Avelino^{1,2,*} and C. J. A. P. Martins^{3,†}

¹*Centro de Astrofísica, Universidade do Porto, Rua das Estrelas s/n, 4150-762 Porto, Portugal*

²*Departamento de Física da Faculdade de Ciências da Universidade do Porto, Rua do Campo Alegre 687, 4169-007 Porto, Portugal*

³*Department of Applied Mathematics and Theoretical Physics, Centre for Mathematical Sciences, University of Cambridge, Wilberforce Road, Cambridge CB3 0WA, United Kingdom*

(Received 22 February 2000)

In the context of “two-metric” theories of gravity there is the possibility that cosmic defects will produce a spectrum of primordial adiabatic density perturbations, when the speed characterizing the defect-producing scalar field is much larger than the speed characterizing gravity and standard model particles. This model will exactly mimic the standard predictions of inflationary models, with the exception of a small non-Gaussian signal which could be detected by future experiments.

PACS numbers: 98.80.Cq, 04.50.+h, 98.65.Dx, 98.70.Vc

I. Introduction.—Cosmology is entering a crucial stage, where a growing body of high-precision data will allow us to determine a number of cosmological parameters, and to identify the mechanism that produced the “seeds” for the structures we observe today [1]. There are currently two classes of models that could be responsible for these—topological defect [2] and inflationary [3] models. The main difference between them is related to causality. Initial conditions for the defect network are set up on a Cauchy surface that is part of the standard history of the universe. Hence, there will not be any correlations between quantities defined at any two spacetime points whose backward light cones do not intersect on that surface. Inflation pushes this surface to much earlier times, and if the inflationary epoch is long enough there will be essentially no causality constraints. This can also be seen by noting that inflation can be defined as an epoch when the comoving Hubble length decreases. It starts out very large, and perturbations can be generated causally. Then inflation forces this length to decrease enough so that, even though it grows again after inflation ends, it is never as large (by today) as the preinflationary era value. Once primordial fluctuations are produced they can simply freeze in comoving coordinates and let the Hubble length shrink and then (for small enough scales) grow past them.

As a step towards identifying the specific model that operated in the early universe, one would like to determine which of the two mechanisms above was involved. The presence of super-horizon perturbations might seem a good enough test, but this is not the case: in defect models (as well as open or Λ models) significant contributions are generated after the epoch of last scattering due to the integrated Sachs-Wolfe effect. The presence of the “Doppler peaks” on small angular scales [4] is also not ideal: Turok [5] has shown that a causal scaling source can be constructed so as to mimic inflation and reproduce its contribution to the cosmic microwave background (CMB) anisotropies. This source is constructed “by hand,” and there is no attempt to provide a framework in which it

could be realized. In any case, it shows that inflationary predictions are not as unique as one might think. We should also mention, however, that a nice argument due to Liddle [6] (see also [7]) shows that the existence of adiabatic perturbations on scales much larger than the Hubble radius implies that either inflation occurred in the past, the perturbations were there as initial conditions, or causality (or Lorentz invariance) is violated. On the other hand, it is also possible to construct “designer inflation” models [8] that would have no secondary Doppler peaks, although these suffer from analogous caveats and they would still be identifiable by other means [9,10].

Finally, there are Gaussianity tests. There have been recent claims of a non-Gaussian component in the CMB [11] (but see also [12]). Defects will generally produce non-Gaussian fluctuations on small enough scales [13], whereas the simplest inflationary models produce Gaussian ones. It is possible to build inflationary models that produce, e.g., non-Gaussianity with a chi-squared distribution [14], but if one found non-Gaussianity in the form of line discontinuities, then it is hard to see how cosmic strings could fail to be involved.

This discussion shows that although defect and inflationary models have of course a number of distinguishing characteristics, there is a greater overlap between them than most people would care to admit. It is also easy to obtain models where both defects and inflation generate density fluctuations [15,16]. The aim of this Letter is to present a further example of this overlap. We discuss a model where the primordial fluctuations are generated by a defect network, but are nevertheless very similar to a standard inflationary model. The only difference between these models and the standard inflationary scenario will be a small non-Gaussian component. A detailed discussion will be presented in a forthcoming publication [17].

II. The model.—Our model follows the recent work on so-called “varying speed of light” theories [18–24], and more particularly the spirit of “two-metric” theories [18,21,23,24], having two natural speed parameters, say

c_ϕ and c ; the first is relevant for the dynamics of the scalar field which will produce topological defects, while the second is the ordinary speed of light that is relevant for gravity and all standard model interactions.

We assume that $c_\phi \gg c$ so that the correlation length of the network of topological defects will be much greater than the horizon size. We could, in analogy with [23,24], define our effective theory by means of an action, and postulate a relation between the two metrics. However, this is not needed for the basic point we are discussing, so we leave it for a future publication [17]. We concentrate on the case of cosmic strings, whose dynamics and evolution are better known than those of other defects [2,25–27] although much of what we will discuss will apply to them as well. Note that c_ϕ could either be a constant {say $[g_\phi]_{00} = (c_\phi^2/c^2)g_{00}$ } or, as in [23] one could set up a model such that the two speeds are equal at very early and at recent times, and between these two epochs there is a period, limited by two phase transitions, where $c_\phi \gg c$. As will become clear, the basic mechanism will work in both cases, although the observational constraints on it will be different for each specific realization.

The string network evolution is qualitatively analogous to the standard case [2,25–27], and, in particular, a “scaling” solution will be reached after a relatively short transient period. The long-string characteristic length (or “correlation length”) L will evolve as $L = \gamma c_\phi t$, with $\gamma = \mathcal{O}(1)$, while the string rms velocity will obey $v_\phi = \beta c_\phi$, with $\beta < 1$. Note, however, that there are some differences relative to the standard scenario. The first one is obvious: if $c_\phi \gg c$, the string network will be outside the horizon, measured in the usual way. Hence these defects will induce fluctuations when they are well outside the horizon, thus avoiding causality constraints. Note that compensation now acts outside the “ c_ϕ horizon.” We expect the effect of gravitational backreaction to be much stronger than in the standard case [23,28]. The general effect of the backreaction is to reduce the scaling density and velocity of the network relative to the standard value [28]. Thus we should expect fewer defects per c_ϕ horizon, than in the standard case. However, despite this strong backreaction, strings will still move relativistically. It can be shown [28] that although backreaction can slow strings down by a measurable amount, only friction forces [26,27] can force the network into a strong nonrelativistic regime. Thus we expect v_ϕ to be somewhat lower than c_ϕ , but still larger than c . Only in the case of monopoles, which are pointlike, one would expect the defect velocities to drop below c due to graviton radiation [23]. This does not happen for extended objects, since their tension naturally tends to make the dynamics take place with a characteristic speed c_ϕ [29]. This is actually crucial: if the network was completely frozen while it was outside the horizon (as in standard scenarios [15]) then no significant perturbations would be generated.

A third important aspect is that the symmetry breaking scale, say Σ , which produces the defects can be significantly lower than the grand unified theory scale, as density perturbations can grow for a longer time than usual. The earlier the defects are formed, the lighter they could be. Proper normalization of the model will produce a further constraint on Σ . Finally, in the case where c_ϕ is a time-varying quantity which only departs from c for a limited period, the defects will become frozen and start to fall inside the horizon after the second phase transition. Here we require that the defects are sufficiently outside the horizon and are relativistic when density fluctuations in the observable scales are generated. This will introduce additional constraints on model parameters, notably on the epochs at which the phase transitions take place.

III. Cosmological consequences.—In the synchronous gauge, the linear evolution equations for radiation and cold dark matter perturbations, δ_r and δ_m , in a flat universe with zero cosmological constant are

$$\ddot{\delta}_m + \frac{\dot{a}}{a} \dot{\delta}_m - \frac{3}{2} \left(\frac{\dot{a}}{a} \right)^2 \left(\frac{a\delta_m + 2a_{\text{eq}}\delta_r}{a + a_{\text{eq}}} \right) = 4\pi G\Theta_+, \quad (1)$$

$$\ddot{\delta}_r - \frac{1}{3} \nabla^2 \delta_r - \frac{4}{3} \dot{\delta}_m = 0, \quad (2)$$

where $\Theta_{\alpha\beta}$ is the energy-momentum tensor of the external source, $\Theta_+ = \Theta_{00} + \Theta_{ii}$, a is the scale factor, “eq” denotes the epoch of radiation-matter equality, and a dot represents a derivative with respect to conformal time. We will consider the growth of superhorizon perturbations with $ck\eta \ll 1$. Then Eq. (1) becomes

$$\ddot{\delta}_m + \frac{\dot{a}}{a} \dot{\delta}_m - \frac{1}{2} \left(\frac{\dot{a}}{a} \right)^2 \left(\frac{3a + 8a_{\text{eq}}}{a + a_{\text{eq}}} \right) \delta_m = 4\pi G\Theta_+, \quad (3)$$

and $\delta_r = 4\delta_m/3$. Its solution, with initial conditions $\delta_m = 0$, $\dot{\delta}_m = 0$ can be written as

$$\delta_m^S(\mathbf{x}, \eta) = 4\pi G \int_{\eta_i}^{\eta} d\eta' \int d^3x' \times \mathcal{G}(X; \eta, \eta') \Theta_+(\mathbf{x}', \eta'), \quad (4)$$

$$\mathcal{G}(X; \eta, \eta') = \frac{1}{2\pi^2} \int_0^\infty \tilde{\mathcal{G}}(k; \eta, \eta') \frac{\sin kX}{kX} k^2 dk. \quad (5)$$

Here $X = |\mathbf{x} - \mathbf{x}'|$ and “S” indicates that these are the “subsequent” fluctuations, according to the notation of [30], to be distinguished from “initial” ones.

We are interested in computing the inhomogeneities at late times in the matter era. When $\eta_0 \gg \eta_{\text{eq}}$, the Green functions are dominated by the growing mode, $\propto a_0/a_{\text{eq}}$, so the function we would like to solve for is [30]

$$T(k; \eta) = \lim_{\eta_0/\eta_{\text{eq}} \rightarrow \infty} \frac{a_{\text{eq}}}{a_0} \tilde{\mathcal{G}}(k, \eta_0, \eta). \quad (6)$$

Consider the growth of superhorizon perturbations, for which the transfer function can be written [30]

$$T(0; \eta) = \frac{\eta_{\text{eq}}}{10(3 - 2\sqrt{2})\eta}. \quad (7)$$

Linear perturbations induced by defects are the sum of initial and subsequent perturbations:

$$\begin{aligned} \delta_m(k; \eta_0) &= \delta_m^I(k; \eta_0) + \delta_m^S(k; \eta_0) \\ &= 4\pi G(1 + z_{\text{eq}}) \int_{\eta_i}^{\eta_0} d\eta T_c(k; \eta) \tilde{\Theta}_+(k; \eta), \end{aligned} \quad (8)$$

where η_i is the epoch of defect formation. The transfer function for the subsequent perturbations, those generated actively, was obtained in Eq. (7) for superhorizon perturbations with $ck\eta_0 \ll 1$. To include compensation for the initial perturbations, δ_m^I , we make the substitution $T_c(k; \eta) = [1 + (k_c/k)^2]^{-1} T(k; \eta)$, where $k_c \propto (c_\phi \eta)^{-1}$ is a long-wavelength cutoff at the compensation scale. This results from the fact that defect perturbations cannot propagate with a velocity greater than c_ϕ . For $(c_\phi \eta_0)^{-1} \ll k \ll (c_\phi \eta_i)^{-1}$ the analytic expression for the power spectrum of density perturbations induced by defects is

$$P(k) = 16\pi^2 G^2 (1 + z_{\text{eq}})^2 \int_0^\infty d\eta \mathcal{F}(k, \eta) |T_c(k, \eta)|^2, \quad (9)$$

where $\mathcal{F}(k, \eta)$ is the structure function which can be obtained directly from the unequal time correlators [30–32]. It can be shown [31] that for a scaling network $\mathcal{F}(k, \eta) = \mathcal{F}(k\eta)$ which, combined with the above relations gives

$$P(k) \propto \int_0^\infty d\eta S(k\eta)/\eta^2 \propto k, \quad (10)$$

where the function S is just the structure function, \mathcal{F} , times the compensation cutoff function. Up until now we considered only the spectrum of primordial (i.e., generated at very early times) fluctuations induced by cosmic defects. In our model a Harrison-Zel'dovich spectrum is predicted just as in the simplest inflationary models. The final processed spectrum will also be the same as for the simplest inflationary models.

We investigate the Gaussianity of the string-induced fluctuations as in [13]. The conclusions can easily be extended for other defect models. In the standard cosmic string scenario the structure function $\mathcal{F}(k, \eta)$ has a turnover scale at the network correlation length, $k_\xi = 20(c_\phi \eta)^{-1}$ [31,33]. At a particular time, perturbations induced on scales larger than the correlation length are generated by many string elements and are expected to have a nearly Gaussian. On the other hand, perturbations induced on smaller scales are very non-Gaussian because they can be either very large within the regions where a string has passed by or else very small outside these. This allows us to roughly divide the power spectrum of cosmic-string-seeded density perturbations into a nearly Gaussian component generated when the string cor-

relation length was smaller than the scale under consideration, and a strongly skewed non-Gaussian component generated when the string correlation length was larger (we call these the ‘‘Gaussian’’ and ‘‘non-Gaussian’’ contributions, respectively). The ratio of these two components may be easily computed by splitting the structure function in (9), in two parts: a Gaussian part $\mathcal{F}_g(k, \eta) = \mathcal{F}(k, \eta)$ for $k < k_\xi$ ($\mathcal{F}_g = 0$ for $k > k_\xi$) and a non-Gaussian part $\mathcal{F}_{ng}(k, \eta) = \mathcal{F}(k, \eta)$ for $k > k_\xi$ ($\mathcal{F}_{ng} = 0$ for $k < k_\xi$). We can then integrate (9) with this Gaussian/non-Gaussian split, to compute the relative contributions to the total power spectrum. The final result will depend on the choice of compensation scale k_c . If we take the maximum compensation scale allowed by causality [34] [$k_c \sim 2(c_\phi \eta)^{-1}$] the Gaussian contribution to the total power spectrum will be less than 5%. In any case, the non-Gaussian contribution will always be smaller than the Gaussian one if, as expected, the compensation scale is larger or equal to the correlation length of the string network ($k_c \leq k_\xi$). Departures from a Gaussian distribution are scale independent and analogous to those of standard defect models on large scales.

By allowing for a characteristic velocity for the scalar field c_ϕ much larger than the velocity of light (and gravity), we were able to construct a model with primordial, adiabatic ($\delta_r = 4\delta_m/3$), nearly Gaussian fluctuations whose primordial spectrum is of the Harrison-Zel'dovich form. This is almost indistinguishable from the simplest inflationary models (as far as structure formation is concerned) except for the small non-Gaussian component which could be detected with future CMB experiments. The C_l spectrum and the polarization curves of the CMB radiation predicted by this model should also be identical to the ones predicted in the simplest inflationary models as the perturbations in the CMB are not generated ‘‘directly’’ by the defects.

IV. Discussion and conclusions.—We presented further evidence of the non-negligible overlap between topological defect and inflationary structure formation models. The key ingredient is having the speed of the defect-producing scalar field much larger than the speed of gravity and standard model particles. This provides a ‘‘violation of causality,’’ as required by [6]. The only distinguishing characteristic of this model, by comparison with the simplest inflationary models, will be a small non-Gaussian signal.

Admittedly our model could be considered ‘‘unnatural’’ in the context of our present theoretical prejudices, and the same can certainly be said about other examples such as ‘‘mimic inflation’’ [5] and ‘‘designer inflation’’ [8]. Be that as it may, however, the fact that these examples can be constructed (and one wonders how many more are possible) highlights the fact that extracting robust predictions from cosmological observations is a much more difficult and subtle task than many experimentalists (and theorists) believe.

We thank Paul Shellard for useful discussions and comments. C. M. is funded by FCT (Portugal) under “Programa PRAXIS XXI,” Grant No. PRAXIS XXI/BPD/11769/97. We thank CAUP for the facilities provided.

*Electronic address: pedro@astro.up.pt

†Also at C.A.U.P., Rua das Estrelas s/n, 4150 Porto, Portugal.

Electronic address: C.J.A.P.Martins@damtp.cam.ac.uk

- [1] E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley, Reading, MA, 1994).
- [2] A. Vilenkin and E. P. S. Shellard, *Cosmic Strings and other Topological Defects* (Cambridge University Press, Cambridge, 1994).
- [3] A. D. Linde, *Particle Physics and Inflationary Cosmology* (Harwood, Chur, Switzerland, 1990).
- [4] A. Albrecht *et al.*, Phys. Rev. Lett. **76**, 1413 (1996); J. Magueijo *et al.*, Phys. Rev. Lett. **76**, 2617 (1996); W. Hu and M. White, Phys. Rev. Lett. **77**, 1687 (1996).
- [5] N. Turok, Phys. Rev. D **54**, 3686 (1996); Phys. Rev. Lett. **77**, 4138 (1996).
- [6] A. Liddle, Phys. Rev. D **51**, 5347 (1995).
- [7] Y. Hu, M. S. Turner, and E. J. Weinberg, Phys. Rev. D **49**, 3830 (1994).
- [8] D. Salopek, J. R. Bond, and J. M. Bardeen, Phys. Rev. D **40**, 1753 (1989).
- [9] A. Liddle, astro-ph/9910110; J. Barrow and A. Liddle, Gen. Relativ. Gravit. **29**, 1501 (1997).
- [10] A. Lewin and A. Albrecht, astro-ph/9908061.
- [11] P. Ferreira, J. Magueijo, and K. M. Gorski, Astrophys. J. **503**, L1 (1998); J. Pando, D. Valls-Gabaud, and L.-Z. Fang, Phys. Rev. Lett. **81**, 4568 (1998).
- [12] A. J. Banday, S. Zaroubi, and K. M. Gorski, Astrophys. J. **533**, 575 (2000).
- [13] P. P. Avelino, E. P. S. Shellard, J. H. P. Wu, and B. Allen, Astrophys. J. Lett. **507**, L101 (1998).
- [14] A. D. Linde, Phys. Lett. **158B**, 375 (1985); L. Kofman, Phys. Lett. B **173**, 400 (1986); A. D. Linde and V. Mukhanov, Phys. Rev. D **56**, 535 (1997).
- [15] P. P. Avelino, R. R. Caldwell, and C. J. A. P. Martins, Phys. Rev. D **59**, 123509 (1999).
- [16] C. Contaldi, M. Hindmarsh, and J. Magueijo, Phys. Rev. Lett. **82**, 2034 (1999); R. A. Battye and J. Weller, Phys. Rev. D **61**, 043501 (2000).
- [17] P. P. Avelino and C. J. A. P. Martins (to be published).
- [18] J. W. Moffat, Int. J. Mod. Phys. D **2**, 351 (1992).
- [19] A. Albrecht and J. Magueijo, Phys. Rev. D **59**, 043516 (1999); J. D. Barrow, Phys. Rev. D **59**, 043515 (1999); J. D. Barrow and J. Magueijo, Phys. Lett. B **443**, 104 (1998).
- [20] P. P. Avelino and C. J. A. P. Martins, Phys. Lett. B **459**, 468 (1999).
- [21] I. T. Drummond, gr-qc/9908058.
- [22] P. P. Avelino, C. J. A. P. Martins, and G. Rocha, Phys. Lett. B **483**, 210 (2000).
- [23] B. A. Bassett, S. Liberati, C. Molina-Paris, and M. Visser, astro-ph/0001441.
- [24] M. A. Clayton and J. W. Moffat, Phys. Lett. B **460**, 263 (1999); **477**, 269 (2000); gr-qc/0003070; J. W. Moffat, astro-ph/0003444.
- [25] C. J. A. P. Martins and E. P. S. Shellard, Phys. Rev. D **53**, 575 (1996).
- [26] C. J. A. P. Martins and E. P. S. Shellard, Phys. Rev. D **54**, 2535 (1996).
- [27] C. J. A. P. Martins, Ph.D. thesis, University of Cambridge, 1997.
- [28] C. J. A. P. Martins, E. P. S. Shellard, and B. Allen, “The Velocity-dependent One-scale String Evolution Model” (to be published).
- [29] B. Carter, in *Proceedings of the 2nd Mexican School on Gravitation and Mathematical Physics, Tlaxcala Lecture Notes*, edited by A. Garcia *et al.* (World Scientific, Singapore, 1996).
- [30] S. Veeraraghavan and A. Stebbins, Astrophys. J. **365**, 37 (1990).
- [31] J.-H. P. Wu, P. P. Avelino, E. P. S. Shellard, and B. Allen, astro-ph/9812156.
- [32] A. Albrecht and A. Stebbins, Phys. Rev. Lett. **68**, 2121 (1992).
- [33] P. P. Avelino, E. P. S. Shellard, J.-H. P. Wu, and B. Allen, Phys. Rev. Lett. **81**, 2008 (1998).
- [34] J. Robinson and B. D. Wandelt, Phys. Rev. D **53**, 618 (1996).