Classical and Quantum Interaction of the Dipole

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A unified and fully relativistic treatment of the interaction of the electric and magnetic dipole moments of a particle with the electromagnetic field is given. New forces on the particle due to the combined effect of electric and magnetic dipoles are obtained. Several new experiments are proposed, which include observation of topological phase shifts.

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An aspect of electromagnetism, which is becoming increasingly fundamental, is the duality between electric and magnetic fields. It is therefore natural to extend the many interesting facets of the electromagnetic interaction of the magnetic dipole [1–7] to the dual situations [5,6,8,9]. I give here a model independent and fully relativistic derivation of the low energy Lagrangian for the interaction of electric and magnetic dipoles with an external electromagnetic field. This treatment applies to elementary particles, nuclei, atoms, and molecules that have electric and/or magnetic dipole moments. New forces on the dipole will be obtained using the non-Abelian nature of the interaction [4–6]. I propose new experiments to test this interaction, including topological aspects analogous to the Aharonov-Bohm (AB) effect [10].

The simplest classical Lorentz invariant action for a neutral particle of mass m interacting with the electromagnetic field strength $F_{\mu\nu}$ is

$$I = -\int mc\, ds - \frac{1}{2c}\int F_{\mu\nu}D^{\mu\nu}\, ds\,, \qquad (1)$$

where s is the proper time along the particle's world line. $D^{\mu\nu}$ may be taken to be antisymmetric without loss of generality, and is called the dipole moment tensor. In any frame, D^{0i} and D^{ij} that couple, respectively, to the electric field components F_{0i} and the magnetic field components F_{ij} are called the components of the electric and magnetic dipole moments. In the rest frame of the particle, D^{0i} and D^{ij} represent the intrinsic electric and magnetic dipole moments. These may be covariantly represented, respectively, by the electric and magnetic dipole moment 4-vectors

$$d^\mu \equiv D^\mu{}_\nu v^\nu, \qquad m^\mu \equiv \frac{1}{2}\,\epsilon^{\mu\nu\rho\sigma}D_{\nu\rho}v_\sigma\,, \qquad (2)$$

where the 4-velocity $v^\mu = \frac{dx^\mu}{ds}$. Then $d_\mu v^\mu = 0$, $m_\mu v^\mu = 0$. So, only three components of each 4-vector are independent. In the rest frame, in which $v^\mu = (1,0,0,0)$, each vector has only three (spatial) components.

Now the identity $\epsilon^{\mu\nu\rho\sigma}\epsilon_{\mu\alpha\beta\gamma} = -6\delta^\nu_{[\alpha}\delta^\rho_\beta\delta^\sigma_{\gamma]}$ implies [11]

$$\frac{1}{2}\,F_{\mu\nu}D^{\mu\nu} = d^\mu F_{\mu\nu}v^\nu + m^\mu F{*}_{\mu\nu}v^\nu. \qquad (3)$$

The electric and magnetic fields in the rest frame may also be covariantly defined by

$$E^\mu = F^\mu{}_\nu v^\nu, \qquad B^\mu = \frac{1}{2}\,\epsilon^{\mu\nu\rho\sigma}F_{\nu\rho}v_\sigma = F{*}^\mu{}_\nu v^\nu. \qquad (4)$$

Then $E^\mu v_\mu = 0 = B^\mu v_\mu$. In the rest frame, $E^\mu = (0,\mathbf{E})$ and $B^\mu = (0,\mathbf{B})$, where $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic (spatial) vectors, in the usual notation. On using (4), (3) reads

$$\frac{1}{2}\,F_{\mu\nu}D^{\mu\nu} = d^\mu E_\mu + m^\mu B_\mu\,. \qquad (5)$$

On defining the 4-vector potentials,

$$D_\nu = d^\mu F_{\mu\nu}, \qquad M_\nu = m^\mu F{*}_{\mu\nu}\,, \qquad (6)$$

(3) may also be rewritten as

$$\frac{1}{2}\,F_{\mu\nu}D^{\mu\nu} = D_\mu v^\mu + M_\mu v^\mu = a_\mu v^\mu, \qquad (7)$$

where $a_\mu = D_\mu + M_\mu$.

From (1) and (7), the relativistic Lagrangian is

$$L_R = -mc\,\frac{ds}{dt} - \frac{1}{c}\,M_\mu\frac{dx^\mu}{dt} - \frac{1}{c}\,D_\mu\frac{dx^\mu}{dt}\,. \qquad (8)$$

Clearly then, the duality between the interactions of the electric and magnetic *dipoles* is complete to all orders of v/c independently of whether or not there are magnetic monopoles. Each of the last two terms in (8) is analogous to the interaction term $-\frac{e}{c}A_\mu\frac{dx^\mu}{dt}$ in the Lagrangian of a charged particle, where A_μ is the electromagnetic 4-vector potential. This immediately suggests topological effects analogous to the AB effect [10].

When the Lagrangian is quantized, the phase shift that the particle experiences due to the field is given, using the action (1), and (7), by the phase factor

$$P\exp\!\left(-\frac{i}{2c\hbar}\int_\gamma F_{\mu\nu}D^{\mu\nu}\,ds\right) = P\exp\!\left(-\frac{i}{\hbar c}\times\int_\gamma a_\mu\,dx^\mu\right), \qquad (9)$$

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where P represents path ordering because $D^{\mu\nu}$, D_μ, M_μ, and therefore a_μ are now operators which need not commute. For a topological phase, the effect of (9) on the wave function should not change when γ is deformed in a suitable region. Such a deformation may be obtained by infinitesimal deformations. To see the effect of an infinitesimal deformation, consider (9) around an infinitesimal closed curve spanning a surface element $d\sigma^{\mu\nu}$:

$$P\exp\left(-\frac{i}{\hbar c}\oint_\gamma a_\mu\, dx^\mu\right) = 1 - \frac{i}{2\hbar c}G_{\mu\nu}d\sigma^{\mu\nu}, \quad (10)$$

where the "Yang-Mills field strength" $G_{\mu\nu}$ is defined by

$$G_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu - \frac{i}{\hbar c}[a_\mu, a_\nu]. \quad (11)$$

An external field modifies the motion of a wave function, via Huyghen's principle, by the phase shift that it causes in the interference of secondary wavelets. This gives a relativistic correspondence principle that enables the classical equation of motion to be determined from the phase shift around an infinitesimal closed curve and vice versa [12]. Let $\xi^\mu(t) = (t, \langle\psi(t)|x^i|\psi(t)\rangle)$ represent the world line of a wave packet $|\psi(t)\rangle$ of the particle, where x^i are the position operators. Then for a suitably localized wave packet, the 4-force

$$f^\mu \equiv m\frac{d^2\xi^\mu}{ds^2} = \langle\psi|G^\mu{}_\nu|\psi\rangle\frac{d\xi^\mu}{ds} \quad (12)$$

to a good approximation. The low energy limit of (12), to $O(1/c)$, was obtained by writing $d^2\langle x^i\rangle/dt^2$ in terms of the commutators of the Hamiltonian [5]. Then Lorentz covariance implies that (12) is valid to all orders in $1/c$. A classical derivation of (12), with the expectation values replaced by the corresponding classical quantities, may be obtained by replacing the above quantum commutators by appropriate Poisson brackets of the relativistic classical Hamiltonian. The 4-force (12) is also analogous to the classical 4-force on a Yang-Mills particle [13].

Equations (10) and (12) show that $G_{\mu\nu}$ determines *both* whether the phase shift is topological [whether (9) is unchanged when γ is varied] and nonlocal (the force on the interfering beams is zero). If

$$G_{\mu\nu} = 0 \quad (13)$$

everywhere along the interfering beams, the phase shift will be called *strongly topological or nonlocal.* The phase shift will be called simply *topological or nonlocal* if

$$G_{\mu\nu}|\psi\rangle = 0 \quad (14)$$

in this region. The phase shift will be called *weakly topological or nonlocal* if

$$\langle\psi|G_{\mu\nu}|\psi\rangle = 0. \quad (15)$$

Clearly, (13) implies (14) and (14) implies (15), but the converses are not true.

The topological and geometrical phases implied by the fully relativistic Lagrangian (8) are valid to all orders in $1/c$, unlike in earlier treatments. We need not enter the complexities of the relativistic quantum wave equation [5] if we first make a low energy approximation of the above theory, by neglecting terms of $O(1/c^2)$, and then quantize it. From (6) and (8), the parts of D_μ and M_μ that contribute to L_R in this approximation are

$$\begin{aligned} D_\mu &= (-\mathbf{d}\cdot\mathbf{E}, \mathbf{d}\times\mathbf{B}), \\ M_\mu &= (-\boldsymbol{\mu}\cdot\mathbf{B}, -\boldsymbol{\mu}\times\mathbf{E}), \\ a_\mu &= M_\mu + D_\mu \\ &= (-\boldsymbol{\mu}\cdot\mathbf{B} - \mathbf{d}\cdot\mathbf{E}, \mathbf{d}\times\mathbf{B} - \boldsymbol{\mu}\times\mathbf{E}), \end{aligned} \quad (16)$$

where $\mu^i = m^i$. The Lagrangian from (8), in this approximation after subtracting the rest mass energy, is [14]

$$L = \frac{1}{2}mv^2 + \boldsymbol{\mu}\cdot\mathbf{B} + \frac{1}{c}\mathbf{v}\cdot\boldsymbol{\mu}\times\mathbf{E} + \mathbf{d}\cdot\mathbf{E} - \frac{1}{c}\mathbf{v}\cdot\mathbf{d}\times\mathbf{B}. \quad (17)$$

The Hamiltonian obtained from (17) is

$$\begin{aligned} H &= \frac{1}{2m}\left(\mathbf{p} - \frac{1}{c}\boldsymbol{\mu}\times\mathbf{E} + \frac{1}{c}\mathbf{d}\times\mathbf{B}\right)^2 - \boldsymbol{\mu}\cdot\mathbf{B} - \mathbf{d}\cdot\mathbf{E} \\ &\equiv \frac{1}{2m}\left(\mathbf{p} - \frac{1}{c}\mathbf{a}\right)^2 + a_0. \end{aligned} \quad (18)$$

The velocity of the center of the wave packet, using Schrödinger's equation, is

$$\begin{aligned} v^i &\equiv \frac{d}{dt}\langle\psi|x^i|\psi\rangle = \frac{i}{\hbar}\langle\psi|[H, x^i]|\psi\rangle \\ &= \frac{1}{m}\langle\psi|p^i - \frac{a^i}{c}|\psi\rangle. \end{aligned} \quad (19)$$

The force is

$$m\frac{dv^i}{dt} = \langle\psi|\frac{\partial}{\partial t}\left(p^i - \frac{a^i}{c}\right)|\psi\rangle + \frac{i}{\hbar}\langle\psi|\left[H, p^i - \frac{a^i}{c}\right]|\psi\rangle. \quad (20)$$

This gives the force to be the same as the spatial components of (12), in the present limit.

Suppose now that the dipole is made of two particles with charges $q, -q$, masses m_1, m_2, and coordinates $\mathbf{x}_1, \mathbf{x}_2$. It can be shown that the orbital angular momentum of the system about an arbitrary origin,

$$\mathbf{x}_1\times\mathbf{p}_1 + \mathbf{x}_2\times\mathbf{p}_2 = \mathbf{x}\times(\mathbf{p}_1 + \mathbf{p}_2) + \mathbf{r}\times\mathbf{p}, \quad (21)$$

where $\mathbf{x} = (m_1 + m_2)^{-1}(m_1\mathbf{x}_1 + m_2\mathbf{x}_2)$ is the center of mass, $\mathbf{r} = \mathbf{x}_1 - \mathbf{x}_2$ the relative coordinates, and

$$\mathbf{p} = \frac{1}{m_1 + m_2}(m_2\mathbf{p}_1 - m_1\mathbf{p}_2). \quad (22)$$

Then, $[p_i, r_j] = -i\hbar\delta_{ij}$ and $[p_i, x_j] = 0$. The last term in (21), $\mathbf{L} \equiv \mathbf{r}\times\mathbf{p}$, is the total orbital angular momentum about the center of mass. Since $\mathbf{p}$ is invariant under Galilei boosts, $\mathbf{p}_1 \to \mathbf{p}_1 - m_1\mathbf{u}, \mathbf{p}_2 \to \mathbf{p}_2 - m_2\mathbf{u}$, so is $\mathbf{L}$. Also, $\mathbf{L}$ satisfies

$$[L_i, L_j] = i\hbar\epsilon_{ijk}L_k,\ [L_i, r_j] = i\hbar\epsilon_{ijk}r_k,\ [L_i, x_j] = 0.$$

In general, the magnetic moment

$$\boldsymbol{\mu} = \gamma_L \mathbf{L} + \gamma_S \mathbf{S}, \tag{23}$$

where $\mathbf{S}$ is the total spin and γ_L, γ_S are constants. The electric dipole moment $\mathbf{d} = q\mathbf{r}$. It follows that

$$[\mu_i, \mu_j] = i\hbar\epsilon_{ijk}(\gamma_L^2 L_k + \gamma_S^2 S_k), \qquad [d_i, d_j] = 0,$$
$$[\mu_i, d_j] = i\hbar\gamma_L\epsilon_{ijk}d_k. \tag{24}$$

In the rest frame of the dipole, i.e., a frame in which (19) is zero, from (12), $f^0 = 0$ and $f^i = \langle\psi|G^i{}_0|\psi\rangle$. In the present low energy limit, the latter force is

$$\mathbf{f} = \nabla(\boldsymbol{\mu}\cdot\mathbf{B} + \mathbf{d}\cdot\mathbf{E}) + \frac{1}{c}\left(\mathbf{d}\times\frac{\partial\mathbf{B}}{\partial t} - \boldsymbol{\mu}\times\frac{\partial\mathbf{E}}{\partial t}\right) - \frac{\gamma_S^2}{c}(\mathbf{S}\times\mathbf{B})\times\mathbf{E} - \frac{\gamma_L^2}{c}(\mathbf{L}\times\mathbf{B})\times\mathbf{E} - \frac{\gamma_L}{c}(\mathbf{B}\times\mathbf{d})\times\mathbf{B} + \frac{\gamma_L}{c}(\mathbf{E}\times\mathbf{d})\times\mathbf{E}. \tag{25}$$

Equation (25) may also be obtained from (20). Quantum mechanically, the right-hand side of (25) is the expectation value of the corresponding operator expression. The force in an arbitrary inertial frame may be obtained by Lorentz transforming the above f^μ to this frame. Of the nonlinear terms in (25), $-(\gamma_S^2/c)(\mathbf{S}\times\mathbf{B})\times\mathbf{E}$ was previously obtained by the author [5,6] and may be experimentally detectable [15]. The remaining three nonlinear terms are new. Particularly interesting are the last two terms in (25), which are due to the combined effect of the electric and magnetic dipole moments. This is because they are due to the commutator of $\boldsymbol{\mu}$ with $\mathbf{d}$, given by (24), and depend on $\mathbf{d}$ and γ_L that determines $\boldsymbol{\mu}$ via (23). They may be observed in principle by subjecting a beam of identical molecules that have an electric dipole moment and a magnetic dipole moment due to orbital angular momentum to an electric or magnetic field and observing the acceleration.

For an elementary particle, the only intrinsic direction is provided by the spin $\mathbf{S}$. Then its intrinsic $\boldsymbol{\mu} = \gamma_S\mathbf{S}$ and its intrinsic $\mathbf{d} = \delta_S\mathbf{S}$, where δ_S is a constant. The force on the particle is obtained from (12) or (20) to be

$$\mathbf{f}_S = \nabla(\boldsymbol{\mu}\cdot\mathbf{B} + \mathbf{d}\cdot\mathbf{E}) + \frac{1}{c}\left(\mathbf{d}\times\frac{\partial\mathbf{B}}{\partial t} - \boldsymbol{\mu}\times\frac{\partial\mathbf{E}}{\partial t}\right) - \frac{1}{c}(\boldsymbol{\mu}\times\mathbf{B} + \mathbf{d}\times\mathbf{E})\times(\gamma_S\mathbf{E} - \delta_S\mathbf{B}). \tag{26}$$

It is emphasized that the general expression (12) is model independent.

For any particular model, all that needs to be done is to evaluate the commutator $[a_i, a_0]$, where a_μ is given by (16). This determines the new nonlinear terms in the force, which have been worked out for the above two cases in (25) and (26).

An experiment that would detect a topological effect due to D_0, which is analogous to the scalar AB effect [10], is the following. Split a beam of identical neutral particles with electric dipole moment $\mathbf{d}$ into two, and send one beam between a pair of capacitor plates that are initially neutral. The beam is so weak that at most one particle is inside the capacitor at any given time. The beam is also polarized so that $\mathbf{d}$ is parallel to the electric field in the capacitor. When each particle is inside the capacitor turn on the homogeneous electric field of the capacitor and turn it off before the particle leaves the capacitor plate. Then the phase shift due to D_0 is [16]

$$-\frac{1}{\hbar}\int_0^T D_0\, dt = \frac{1}{\hbar}\int_0^T \mathbf{d}\cdot\mathbf{E}\, dt = \frac{ETd}{\hbar}. \tag{27}$$

This proposed experiment is analogous to the experiment to detect the topological phase shift due to M_0 proposed by Zeilinger [17] in analogy with the scalar AB effect [10], and by the author [6] as the scalar phase shift corresponding to the vector phase shift considered by Aharonov and Casher [2]. This phase shift has been observed [18–20].

In the above mentioned three interferometry experiments, it is the expectation value of the quantum force operator, or the classical force, that vanishes, whereas the quantum force that is $m(d^2\hat{x}^i/dt^2)$ in the Heisenberg picture, is nonzero [21]. For example, in the second of these experiments [6,17], the linear terms in the quantum force vanish because of the homogeneity of the magnetic field, on using a Maxwell's equation. But the nonlinear term $-\gamma_S^2(\hat{\mathbf{S}}\times\mathbf{B})\times\mathbf{E}$ does not vanish. It is the expectation value of the latter term that vanishes. All three experiments are weakly topological because, along the interfering beams, (15) holds whereas (13) does not hold. This is analogous to the Stern-Gerlach experiment with the spin perpendicular to the inhomogeneity of the magnetic field for which also (15) holds whereas (13) does not hold. In the latter experiment the existence of the quantum force is obvious because it splits the beam into two, even though the classical force is zero. On the other hand, the AB effect [10] is strongly topological, as defined above, because $G_{\mu\nu} = eF_{\mu\nu} = 0$ in this case.

However, in the following two proposed interferometric experiments (13) holds and hence the quantum force is zero. Consider an interferometer whose arms form a parallelogram. The interfering particles have magnetic moment, but no electric charge and zero or negligible electric dipole moment, as, for example, in a neutron interferometer. Subject the *entire* interferometer to a *homogeneous* and time-independent electric field $\mathbf{E}$ that is parallel to a pair of arms of the interferometer. Then the magnetic field $-\frac{1}{c}\mathbf{v}\times\mathbf{E}$ in the rest frame of the particle is zero along these two arms and is normal to the plane of the interferometer in the other two arms. However, since the rest frames are different for the two pairs of beams, the phases along the beams are better studied in the laboratory frame in which they are due to **M**. Along the second pair of parallel arms, at distances ℓ_1 and ℓ_2, the magnetic dipole moment is rotated about parallel axes by 180° by suitable magnetic fields. In a neutron interference experiment, if θ is the angle between these arms and **E**, the phase shifts for spin components parallel and antiparallel to the normal to the interferometer are [1] $\pm\frac{2\mu E}{\hbar c}(\ell_1 - \ell_2)\sin\theta$. Varying $\ell_1 - \ell_2$ would show that this phase shift is due to the quantum force-free interaction of the dipole and is independent of the two identical spin flips whose effects cancel each other [22].

The dual of the above proposed experiment will use an interferometer for a neutral particle, such as a molecule, with electric dipole moment **d**, but negligible magnetic moment. Instead of the electric field, a constant uniform magnetic field will be applied parallel to one pair of arms.

A sensitive method for detecting the electric dipole interaction is by the electric resonance method described by Ramsey [23]. Here a molecular beam that is polarized so that the electric dipole moments are parallel is deflected and refocused by two inhomogeneous electric fields. In the intermediate region homogeneous and oscillatory electric fields are applied. The fields are chosen so that an allowed electric dipole transition occurs due to the latter fields which makes the intensity of the refocused beam maximum. Introduce a magnetic field **B** in the intermediate region so that $\mathbf{v} \times \mathbf{B}$ is in the direction of the homogeneous electric field. This shifts the intensity from the maximum. From this shift, which may be varied by changing **B** and **v**, the $\mathbf{d} \cdot \mathbf{v} \times \mathbf{B}$ term in (17) may be confirmed. This shift may be distinguished from the shift due to the $\boldsymbol{\mu} \cdot \mathbf{B}$ interaction from the former shift's velocity dependence, and by choosing molecules for which μ is sufficiently small.

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