Order $\alpha^3 \ln(1/\alpha)$ Corrections to Positronium Decays

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The logarithmically enhanced $\alpha^3 \ln(1/\alpha)$ corrections to the para- and orthopositronium decay widths are calculated in the framework of dimensionally regularized nonrelativistic quantum electrodynamics. In the case of parapositronium, the correction is negative, approximately doubles the effect of the leading logarithmic $\alpha^3 \ln^2(1/\alpha)$ one, and is comparable to the nonlogarithmic $O(\alpha^2)$ one. As for orthopositronium, the correction is positive and almost cancels the $\alpha^3 \ln^2(1/\alpha)$ one. The uncertainty in the theoretical prediction for the parapositronium decay width is reduced to 10^{-2} μs^{-1}.

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Positronium (Ps), which is an electromagnetic bound state of the electron e^- and the positron e^+, is the lightest known atom. Since its theoretical description is not plagued by strong-interaction uncertainties, thanks to the smallness of the electron mass m_e relative to typical hadronic mass scales, its properties can be calculated perturbatively in quantum electrodynamics (QED), as an expansion in Sommerfeld's fine-structure constant α, with very high precision. Ps is thus a unique laboratory for testing the QED theory of weakly bound systems.

The decay widths of the ${}^1S_0$ parapositronium (p-Ps) and ${}^3S_1$ orthopositronium (o-Ps) ground states to two and three photons, respectively, have been the subject of a vast number of theoretical and experimental investigations. The present theoretical knowledge may be summarized as

$$\Gamma_p^{\text{th}} = \Gamma_p^{(0)}\biggl[1 + A_p \frac{\alpha}{\pi} + 2\alpha^2 \ln\frac{1}{\alpha} + B_p\left(\frac{\alpha}{\pi}\right)^2 - \frac{3\alpha^3}{2\pi}\ln^2\frac{1}{\alpha} + C_p \frac{\alpha^3}{\pi}\ln\frac{1}{\alpha} + D_p\left(\frac{\alpha}{\pi}\right)^3 + \ldots\biggr], \tag{1}$$

$$\Gamma_o^{\text{th}} = \Gamma_o^{(0)}\biggl[1 + A_o \frac{\alpha}{\pi} - \frac{\alpha^2}{3} \ln\frac{1}{\alpha} + B_o\left(\frac{\alpha}{\pi}\right)^2 - \frac{3\alpha^3}{2\pi}\ln^2\frac{1}{\alpha} + C_o \frac{\alpha^3}{\pi}\ln\frac{1}{\alpha} + D_o\left(\frac{\alpha}{\pi}\right)^3 + \ldots\biggr], \tag{2}$$

where

$$\Gamma_p^{(0)} = \frac{\alpha^5 m_e}{2}, \qquad \Gamma_o^{(0)} = \frac{2(\pi^2 - 9)\alpha^6 m_e}{9\pi}, \tag{3}$$

are the lowest-order results. The $O(\alpha)$ coefficients in Eqs. (1) [1] and (2) [2] read

$$A_p = \frac{\pi^2}{4} - 5, \qquad A_o = -10.286\,606(10). \tag{4}$$

The logarithmically enhanced $\alpha^2 \ln(1/\alpha)$ terms in Eqs. (1) and (2) have been obtained in Refs. [3,4], respectively. Recently, the nonlogarithmic $O(\alpha^2)$ coefficients in Eqs. (1) [5] and (2) [6] have been found to be

$$B_p = 1.75(30), \qquad B_o = 44.52(26). \tag{5}$$

Note that the light-by-light-scattering diagrams have been omitted in Ref. [6]. Such diagrams contribute -2.11 to B_p [5]. The missing contribution to B_o is likely to be of similar magnitude and thus relatively suppressed. The p-Ps (o-Ps) decays into four (five) photons, which are not included in Eq. (5), lead to an increase of the coefficient B_p (B_o) by 0.274(1) [0.19(1)] [7]. In $O(\alpha^3)$, only the leading logarithmic $\alpha^3 \ln^2(1/\alpha)$ terms are known [8]. Including all the terms known so far, we obtain for the p-Ps and o-Ps total decay widths

$$\Gamma_p^{\text{th}} = 7989.512(13)\ \mu\text{s}^{-1}, \tag{6}$$

$$\Gamma_o^{\text{th}} = 7.039\,943(10)\ \mu\text{s}^{-1}, \tag{7}$$

where the errors stem from the coefficients B_p and B_o, respectively. The total uncertainties in Γ_p^{th} and Γ_o^{th} will be estimated later on. The purpose of this Letter is to complete our knowledge of the logarithmically enhanced terms of $O(\alpha^3)$ by providing the coefficients C_p and C_o, in analytic form. We also give order-of-magnitude estimates of the unknown coefficients D_p and D_o.

On the experimental side, the present situation is not entirely clear. Recently, the Ann Arbor group measured the p-Ps width to be [9]

$$\Gamma_p^{\text{exp}} = 7990.9(1.7)\ \mu\text{s}^{-1}, \tag{8}$$

which agrees with Eq. (6) within the experimental error. However, in the case of o-Ps, their measurements [10,11],

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$$\Gamma_o^{\exp}(\text{gas}) = 7.0514(14)\ \mu\text{s}^{-1},$$
$$\Gamma_o^{\exp}(\text{vacuum}) = 7.0482(16)\ \mu\text{s}^{-1}, \tag{9}$$

exceed Eq. (7) by 8 and 5 experimental standard deviations, respectively. This apparent contradiction is known as the *o-Ps lifetime puzzle.* On the other hand, the Tokyo group found [12]

$$\Gamma_o^{\exp}(\text{SiO}_2) = 7.0398(29)\ \mu\text{s}^{-1}, \tag{10}$$

which agrees with Eq. (7) within the experimental error. Leaving this aside, the o-Ps results from Ann Arbor could be considered as a signal of new physics beyond the standard model. However, a large number of exotic decay modes have already been ruled out [13]. No conclusion on the o-Ps lifetime puzzle can be drawn until the experimental precision increases and the data become unambiguous.

On the theoretical side, it is an urgent matter to improve the predictions of the Ps lifetimes as much as possible. Thus, one is faced with the task of analyzing the $O(\alpha^3)$ corrections, which is extremely difficult, especially for o-Ps. However, there is a special subclass of the $O(\alpha^3)$ corrections which can be analyzed separately, namely those which are enhanced by powers of $\ln(1/\alpha) \approx 5$. They may reasonably be expected to make a substantial contribution to the $O(\alpha^3)$ corrections. This may be substantiated by considering Eqs. (1) and (2) in $O(\alpha^2)$, where logarithmic terms enter for the first time. In the case of p-Ps (o-Ps), the magnitude of the logarithmic term amounts to 98% (57%) of the $O(\alpha^2)$ correction. The origin of the logarithmic corrections is the presence of several scales in the bound-state problem. The dynamics of the nonrelativistic (NR) e^+e^- pair near threshold involves four different scales [14]: (i) the hard scale (energy and momentum scale like m_e); (ii) the soft scale (energy and momentum scale like βm_e); (iii) the potential scale (energy scales like $\beta^2 m_e$, while momentum scales like βm_e); and (iv) the ultrasoft (US) scale (energy and momentum scale like $\beta^2 m_e$). Here β denotes the electron velocity in the center-of-mass frame. The logarithmic integration over a loop momentum between different scales yields a power of $\ln(1/\beta)$. Since Ps is approximately a Coulomb system, we have $\beta \propto \alpha$. This explains the appearance of powers of $\ln(1/\alpha)$ in Eqs. (1) and (2). The leading logarithmic corrections may be obtained straightforwardly by identifying the regions of logarithmic integration [3,4,8]. The calculation of the subleading logarithms is much more involved because certain loop integrations must be performed exactly beyond the logarithmic accuracy.

In the following, we briefly outline the main features of our analysis. We work in NR QED (NRQED) [15], which is the effective field theory that emerges by expanding the QED Lagrangian in β and integrating out the hard modes. If we also integrate out the soft modes and the potential photons, we arrive at the effective theory of potential NRQED (pNRQED) [16], which contains potential electrons and US photons as active particles. Thus, the dynamics of the NR e^+e^- pair is governed by the effective Schrödinger equation and by its multipole interaction with the US photons. The corrections from harder scales are contained in the higher-dimensional operators of the NR Hamiltonian, corresponding to an expansion in β, and in the Wilson coefficients, which are expanded in α. In the process of scale separation, spurious infrared (IR) and ultraviolet (UV) divergences arise, which endow the operators in the NR Hamiltonian with anomalous dimensions. We use dimensional regularization (DR), with $d = 4 - 2\epsilon$ space-time dimensions, to handle these divergences [16–18]. This has the advantage that contributions from different scales are matched automatically. The logarithmic corrections are closely related to the anomalous dimensions and can be found by analyzing the divergences of the NR effective theory. In this way, we have obtained the leading logarithmic third-order corrections to the energy levels and wave functions at the origin of heavy quark-antiquark bound states [19], which includes the QED result [3,4,8] as a special case. Here, we extend this approach to the subleading logarithms in QED. Note that the NRQED approach, endowed with an explicit momentum cutoff and a fictitious photon mass to regulate the UV and IR divergences, has also been applied to find the third-order correction, including subleading logarithms, to the hyperfine splitting in muonium [20].

The annihilation of Ps is the hard process which gives rise to imaginary parts in the local operators of the NR Hamiltonian [21]. The decay width can be obtained by averaging these operators over the bound-state wave function. The hard-scale corrections, which require fully relativistic QED calculations and are most difficult to find, do not depend on β and do not lead to logarithmic contributions by themselves. However, they can interfere with the logarithmic corrections from the softer scales. Thus, the only results from relativistic perturbation theory that enter our analysis are (i) the one-loop hard renormalizations of the imaginary parts of the leading four-fermion operators, i.e., the Born decay amplitudes, which are given by the coefficients A_p and A_o, and (ii) the hard parts of the one-loop $O(\alpha\beta^2)$ operators [22]. The missing ingredients can all be obtained in the NR approximation. These include (i) the correction to the Ps ground-state wave function at the origin due to the $O(\alpha\beta^2)$ terms in the NR Hamiltonian, (ii) the $O(\alpha\beta^2)$ corrections to the leading four-fermion operators, and (iii) the correction due to the emission and absorption of US photons by the Ps bound state.

The value of the ground-state ($n = 1$) wave function at the origin $\psi_1(0)$ may be extracted from the NR Green function $G(\mathbf{x}, \mathbf{y}, E)$, which satisfies the equation

$$(\mathcal{H}_C + \Delta\mathcal{H} - E)G(\mathbf{x}, \mathbf{y}, E) = \delta^{(3)}(\mathbf{x} - \mathbf{y}), \tag{11}$$

where $\mathcal{H}_C$ is the Coulomb Hamiltonian and $\Delta\mathcal{H}$ stands for the terms of higher orders in α and β. The solution of Eq. (11) can be found in time-independent perturbation theory as an expansion in α around the leading-order

Coulomb Green function. We thus obtain the correction $\Delta\psi_1^2$ in the relationship $|\psi_1(0)|^2 = |\psi_1^C(0)|^2(1 + \Delta\psi_1^2)$, where $\psi_1^C(0)$ is the ground-state wave function at the origin in the Coulomb approximation. As mentioned above, this analysis may be enormously simplified by the use of DR. Proceeding along the lines of Ref. [19], we thus recover with ease the well-known $\alpha^2\ln(1/\alpha)$ terms in Eqs. (1) and (2) [3,4],

$$\Delta'\psi_1^2 = \alpha^2 \ln\frac{1}{\alpha}\left[2 - \frac{7}{6}S(S+1)\right], \qquad (12)$$

where S is the eigenvalue of the total-spin operator $\mathbf{S}$. As mentioned above, $\Delta'\psi_1^2$ interferes with the one-loop hard renormalization of the Born amplitudes, A_p and A_o, to produce $\alpha^3\ln(1/\alpha)$ terms. The resulting contributions to the coefficients C_p and C_o read $2A_p$ and $-A_o/3$, respectively.

The generic logarithmic $O(\alpha^3)$ correction $\Delta''\psi_1^2$ to $|\psi_1(0)|^2$ is generated by the following one-loop operators, given in the momentum representation with $O(\epsilon)$ accuracy,

$$\Delta_{\mathrm{h}}\mathcal{H} = -\frac{1}{3}\frac{\pi\alpha}{m_e^2}\left[\frac{1}{\hat\epsilon}\left(\frac{\mu^2}{m_e^2}\right)^\epsilon + \frac{39}{5} - 12\ln 2 + \left(\frac{32}{3} + 6\ln 2\right)\mathbf{S}^2\right], \qquad (13)$$

$$\Delta_{\mathrm{s}}\mathcal{H} = -\frac{7}{3}\frac{\pi\alpha}{m_e^2}\left[\frac{1}{\hat\epsilon}\left(\frac{\mu^2}{\mathbf{q}^2}\right)^\epsilon - \frac{1}{7}\right], \qquad (14)$$

$$\Delta_{\mathrm{us}}\mathcal{H} = \frac{8}{3}\frac{\pi\alpha}{m_e^2}\left[\frac{1}{\hat\epsilon}\left(\frac{\mu}{\mathbf{p}^2/m_e - E_1^C}\right)^{2\epsilon} + \frac{5}{3} - 2\ln 2\right], \qquad (15)$$

where $1/\hat\epsilon = 1/\epsilon - \gamma_E + \ln(4\pi)$, with γ_E being Euler's constant, μ is the 't Hooft mass scale of DR, $\mathbf{q}$ is the three-momentum transfer, and $E_1^C = -\alpha^2 m_e/4$ is the Coulomb ground-state energy. Equations (13) and (14) give the hard [22] and soft [17] $O(\alpha\beta^2)$ contributions to the NR Hamiltonian, respectively. The US contribution given in Eq. (15) arises from the emission and absorption of an US photon, which converts the on-shell Ps ground state into some off-shell state of the Coulomb spectrum, with three-momentum $\mathbf{p}$, before it decays. It is the only US contribution which can be represented by an operator of instantaneous interaction and thus give rise to logarithmic corrections. It has been found with the help of the method developed for the more complicated case of quantum chromodynamics in Ref. [23], where it was applied to the on-shell renormalization of the heavy-quarkonium wave function at the origin. The singularities of the operators in Eqs. (13)–(15) yield the logarithmic corrections which we are interested in. Up to their logarithmic dependences on $\mathbf{q}^2$ and $\mathbf{p}^2$, these operators are of the δ-function type in coordinate space and, therefore, lead to additional singularities in the Coulomb Green function at the origin [19]. As a consequence, in the evaluation of the Green function in time-independent perturbation theory from Eq. (11) with $\Delta_{\mathrm{h}}\mathcal{H}$, $\Delta_{\mathrm{s}}\mathcal{H}$, and $\Delta_{\mathrm{us}}\mathcal{H}$, overlapping logarithmic divergences appear in the part of the first-order term which corresponds to the interference of the one-photon contribution to the Coulomb Green function and the first terms of Eqs. (13)–(15). This results in the double-logarithmic contribution, which can be directly extracted from the coefficient of the leading double-pole singularity [19]. Since we are interested in the single-logarithmic contribution, we also have to keep the subleading terms in this analysis. The logarithmic corrections which are generated by the nonoverlapping singularities can be obtained by putting $\mu = m_e$ in the Coulomb Green function at the origin and $\mathbf{q}^2 = \mathbf{p}^2 = -m_e E_1^C$ in Eqs. (13)–(15), and proceeding as in the evaluation of Eq. (12). We thus obtain

$$\Delta''\psi_1^2 = \frac{\alpha^3}{\pi}\left\{-\frac{3}{2}\ln^2\frac{1}{\alpha} + \left[-\frac{184}{45} + \frac{2}{3}\ln 2 + \left(\frac{16}{9} + \ln 2\right)S(S+1)\right]\ln\frac{1}{\alpha}\right\}. \qquad (16)$$

The first term herein agrees with the corresponding terms in Eqs. (1) and (2) [8], while the second one represents a new result.

The last source of $\alpha^3\ln(1/\alpha)$ terms is the $O(\alpha\beta^2)$ corrections to the leading four-fermion operators. Since they do not involve the singular Coulomb Green function at the origin, there are no overlapping divergences, and we may simply read off the resulting $\alpha^3\ln(1/\alpha)$ terms from the poles of their US parts, which are given by the operator

$$-\frac{1}{\epsilon}\frac{2\alpha}{3\pi}\frac{\mathbf{p}^2 + \mathbf{p}'^2}{m_e^2}V_4(\mathbf{p},\mathbf{p}',\mathbf{S}). \qquad (17)$$

Here $V_4(\mathbf{p},\mathbf{p}',\mathbf{S})$ is the local four-fermion operator which generates the leading-order decay widths. Taking the expectation value of Eq. (17) with respect to the ground-state wave function, one encounters power-divergent integrals [21]. They can be consistently treated within DR [5]. This leads to the substitution $\mathbf{p}^2, \mathbf{p}'^2 \to m_e E_1^C$ in the matrix element. The UV-pole contribution of Eq. (17) is then canceled by the IR pole of the hard contribution [24]. This implies that the logarithmic integration ranges from the US scale $\alpha^2 m_e$ up to the hard scale m_e, so that $1/\epsilon$ should be replaced by $4\ln(1/\alpha)$ [19]. The resulting $\alpha^3\ln(1/\alpha)$ corrections to the decay widths are spin independent and read

$$\Delta\Gamma_{p,o} = \Gamma_{p,o}^{(0)}\frac{4\alpha^3}{3\pi}\ln\frac{1}{\alpha}. \qquad (18)$$

Summing up the various $\alpha^3\ln(1/\alpha)$ terms derived above, we obtain

$$C_p = 2A_p - \frac{124}{45} + \frac{2}{3}\ln 2 \approx -7.359,$$
$$C_o = -\frac{A_o}{3} + \frac{4}{5} + \frac{8}{3}\ln 2 \approx 6.077. \qquad (19)$$

In the case of p-Ps, the new $\alpha^3 \ln(1/\alpha)$ term has the same sign and nearly the same magnitude as the $\alpha^3 \ln^2(1/\alpha)$ one. The sum of these two terms approximately compensates the positive contribution from the nonlogarithmic $O(\alpha^2)$ term. As for o-Ps, the new $\alpha^3 \ln(1/\alpha)$ term cancels approximately 4/5 of the $\alpha^3 \ln^2(1/\alpha)$ contribution. Our final predictions for the p-Ps and o-Ps total decay widths, including the multiphoton channels, read

$$\Gamma_p^{\mathrm{th}} = 7989.476(13)\ \mu\mathrm{s}^{-1}, \tag{20}$$

$$\Gamma_o^{\mathrm{th}} = 7.039\,970(10)\ \mu\mathrm{s}^{-1}, \tag{21}$$

which has to be compared with Eqs. (6) and (7). Again, we quote only the errors due to Eq. (5). As before, Eq. (20) agrees with the Ann Arbor [9] measurement (8), and Eq. (21) favors the Tokyo [12] measurement (10), while it significantly undershoots the Ann Arbor [10,11] measurements (9).

The missing nonlogarithmic $O(\alpha^3)$ corrections in Eqs. (1) and (2) receive contributions from three-loop QED diagrams with a considerable number of external lines, which are far beyond the reach of presently available computational techniques. In this sense, we expect Eqs. (20) and (21) to remain the best predictions for the forseeable future. However, we may speculate about the magnitudes of the coefficients D_p and D_o. Two powers of α in these terms can be of NR origin. Each of them should be accompanied by the characteristic factor π, which happens for the logarithmic terms. Thus, we estimate the coefficients D_p and D_o to be a few units times π^2. This rule of thumb is in reasonable agreement with the situation at $O(\alpha^2)$, where we have $B_p \approx \pi^2/6$ and $B_o \approx 4\pi^2$. If the coefficients D_p and D_o do not have magnitudes in excess of 100, then the uncertainties due to the lack of their knowledge falls within the errors quoted in Eqs. (20) and (21). In the case of p-Ps, our analysis then reduces the uncertainty in the predicted decay width to $10^{-2}\ \mu\mathrm{s}^{-1}$, while in the case of o-Ps the uncertainty is limited by the unknown light-by-light-scattering contribution to B_o. If we assume, for example, that the latter is of the same size as the one to B_p, then the resulting contribution to Γ_o^{th} amounts to $8 \times 10^{-5}\ \mu\mathrm{s}^{-1}$, which exceeds the error quoted in Eq. (21) by a factor of 8. Anyway, further progress in our understanding of the Ps lifetime problem crucially depends on the reduction of the experimental errors, which now greatly exceed the theoretical ones.

Finally, we note that the technique developed in this Letter can also be applied to the calculation of the subleading logarithmic $\alpha^7 \ln(1/\alpha)$ terms for the Ps hyperfine splitting. This problem is of special interest because of the apparent discrepancy between the latest experimental data [25] and the best theoretical predictions, which include the $O(\alpha^6)$ corrections (see Ref. [18] and references cited therein) and the leading logarithmic $\alpha^7 \ln^2\alpha$ term [8].

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