Probing Possible Decoherence Effects in Atmospheric Neutrino Oscillations

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(Received 14 February 2000)

It is shown that the results of the Super-Kamiokande atmospheric neutrino experiment, interpreted in terms of $\nu_\mu \leftrightarrow \nu_\tau$ flavor transitions, can probe possible decoherence effects induced by new physics (e.g., by quantum gravity) with high sensitivity, supplementing current laboratory tests based on kaon oscillations and on neutron interferometry. By varying the (unknown) energy dependence of such effects, one can either obtain strong limits on their amplitude or use them to find an unconventional solution to the atmospheric ν anomaly based solely on decoherence.

PACS numbers: 14.60.Pq, 03.65.Bz, 04.60.–m

The Super-Kamiokande (SK) atmospheric neutrino experiment has found convincing evidence [1] for the quantum-mechanical phenomenon of ν flavor oscillations [2] in the $\nu_\mu \leftrightarrow \nu_\tau$ channel. Such evidence consistently emerges from different SK data samples (sub-GeV leptons, multi-GeV leptons, and upgoing muons [3]), as well as from other atmospheric ν experiments [4].

The simplest model for $\nu_\mu \leftrightarrow \nu_\tau$ oscillations involves two neutrino states $\nu_1 = (1,0)^T$ and $\nu_2 = (0,1)^T$ with masses m_1 and m_2, and two flavor states $\nu_\mu = (c_\theta, s_\theta)^T$ and $\nu_\tau = (-s_\theta, c_\theta)^T$, where θ is the neutrino mixing angle, $c = \cos$, $s = \sin$, and T denotes the transpose. The Liouville equation for the ν density matrix ρ,

$$\dot{\rho} = -i[H, \rho], \tag{1}$$

is then governed (in the mass basis) by the Hamiltonian $H = \frac{1}{2}\mathrm{diag}(-k, +k)$, where $k = \Delta m^2/2E$, $\Delta m^2 = m_2^2 - m_1^2$, and $E(\gg m_{1,2})$ is the ν energy (in natural units). The solution $\rho(t)$ of Eq. (1), with initial conditions $\rho(0) = \Pi_{\nu_\mu}$ (where $\Pi_{\nu_\mu} = \nu_\mu \otimes \nu_\mu^\dagger$ is the ν_μ state projector), gives the ν_μ survival probability after a length $x(\simeq t)$,

$$P(\nu_\mu \leftrightarrow \nu_\mu) = \mathrm{Tr}[\Pi_{\nu_\mu}\rho(t)] = 1 - \tfrac{1}{2}s_{2\theta}^2(1 - \cos kx), \tag{2}$$

which is the well-known oscillation formula [2].

Equation (2) beautifully fits the SK data [5] over a wide range of ν energies ($E \sim 10^{-1}$–10^3 GeV) and flight lengths ($x \sim 10^1$–10^4 km), provided that $\Delta m^2 \simeq 3 \times 10^{-3}$ eV^2 and $s_{2\theta}^2 \simeq 1$ [5,6]. Such striking agreement severely constrains possible deviations from the standard Hamiltonian H [6,7]. In this work we show that the SK data can also be used to probe deviations from the standard Liouville dynamics in Eq. (1), which might be induced by new physics beyond the standard electroweak model.

In general, modifications of Eq. (1) emerge from dissipative interactions with an environment [8] and can be parametrized by introducing an extra term $\mathcal{D}[\rho]$,

$$\dot{\rho} = -i[H, \rho] - \mathcal{D}[\rho], \tag{3}$$

which violates the conservation of $\mathrm{Tr}(\rho^2)$ and allows transitions from pure to mixed states. The operator $\mathcal{D}$ has the dimension of an energy, and its inverse defines the typical (coherence) length after which the system gets mixed [9].

Among the possible sources of decoherence, a particularly intriguing one might be provided by quantum gravity, as suggested by Hawking in the context of black-hole thermodynamics [10]. From such a viewpoint, any physical system is inherently "open," due to its unavoidable, decohering interactions with a pervasive "environment" (the spacetime and its Planck-scale dynamics [11]). Following the pioneering paper [12], quantum gravity decoherence effects have been investigated in oscillating systems which propagate over macroscopic distances (see [13] for reviews). Analyses have been mainly focused on $K\overline{K}$ oscillations [12,14,15] and on neutron interferometry [12,16], by assuming reasonable phenomenological forms for $\mathcal{D}$. In both systems, no evidence has been found for $\mathcal{D} \neq 0$, and strong limits have been derived on the quantities parametrizing $\mathcal{D}$ [13]:

$$\|\mathcal{D}\| \lesssim 10^{-21}\ \mathrm{GeV}\ (K\overline{K},\ n\ \text{systems}). \tag{4}$$

Theoretical estimates for $\|\mathcal{D}\|$ are very uncertain [12] and can range from unobservably small values up to the limits in (4). Therefore, it is wise to adopt a phenomenological viewpoint, trying to learn from experiments and to improve the laboratory limits (4) with novel approaches, such as those provided by ν oscillations. Indeed, attempts have been made to explain the solar ν deficit through decoherence [17–19]. It has also been suggested that decoherence might play a role in interpreting the atmospheric ν data [19,20] although, to our knowledge, no detailed analysis of the SK results has been attempted so far. The crucial point is that, for typical atmospheric ν energies ($10^{0\pm1}$ GeV), the oscillation length $\lambda = 2\pi/k$ spans the range $\sim 10^{3\pm1}$ km; then, if the (de)coherence length ℓ is of comparable size, terms as small as $\|D\| \sim \ell^{-1} \sim 10^{-22\pm1}$ GeV can be probed.

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In order to fix a well-defined framework, we specialize Eq. (3) under reasonable (although not compelling) phenomenological assumptions. The most general requirement is perhaps that of *complete positivity* [21,22], corresponding to assume a linear, Markovian, and trace-preserving map $\rho(0) \rightarrow \rho(t)$. This implies the so-called Lindblad form [23] for the decoherence term,

$$\mathcal{D}[\rho] = \sum_n \{\rho, D_n D_n^\dagger\} - 2D_n \rho D_n^\dagger\,, \tag{5}$$

where the operators D_n arise from tracing away the environment dynamics (see [24] for a recent proof). Master equations of the Lindblad form are ubiquitous in physics (see [8,25] for theorems and applications). Concerning ν oscillations, such equations describe ν propagation in dissipative media as, e.g., matter with fluctuating density [26] or thermal baths [27]. Here, however, the environment embeds possible new physics (e.g., the spacetime "foam" [11]) for which there is no established theory.

In the absence of first-principles calculations, we assume that at least the laws of thermodynamics hold in the ν system. The time increase of the von Neumann entropy $S(\rho) = -\mathrm{Tr}(\rho \ln\rho)$ can be enforced by taking $D_n = D_n^\dagger$ [28], so that Eq. (5) becomes $\mathcal{D}[\rho] = \sum_n [D_n, [D_n, \rho]]$. The conservation of the average value of the energy $[\mathrm{Tr}(\rho H)]$ requires, in addition, that $[H, D_n] = 0$ [14,29].

The Hermitian operators ρ, Π_{ν_μ}, H, and D_n can be expanded [8] onto the basis formed by the unit matrix $\mathbf{1}$ and by the Pauli matrix vector $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)^T$. We take $\rho = \frac{1}{2}(\mathbf{1} + \boldsymbol{p} \cdot \boldsymbol{\sigma})$, $\Pi_{\nu_\mu} = \frac{1}{2}(\mathbf{1} + \boldsymbol{q} \cdot \boldsymbol{\sigma})$, $H = \frac{1}{2}\boldsymbol{k} \cdot \boldsymbol{\sigma}$, and $D_n = \frac{1}{2}\boldsymbol{d}_n \cdot \boldsymbol{\sigma}$, where $\boldsymbol{q} = (s_{2\theta}, 0, c_{2\theta})^T$ and $\boldsymbol{k} = (0, 0, -k)^T$. Defining $G = \sum_n |\boldsymbol{d}_n|^2 \mathbf{1} - \boldsymbol{d}_n \otimes \boldsymbol{d}_n^T$, Eq. (3) is transformed into a Bloch equation, $\dot{\boldsymbol{p}} = \boldsymbol{k} \times \boldsymbol{p} - G \cdot \boldsymbol{p}$, which has a simple physical interpretation: the standard term $\boldsymbol{k} \times \boldsymbol{p}$ induces ν oscillations, while the decoherence term $G \cdot \boldsymbol{p}$ is responsible for their damping [8,27].

The requirement $[H, D_n] = 0$ implies that each vector $\boldsymbol{d}_n$ is parallel to $\boldsymbol{k}$ [29]. Therefore, the tensor G takes the form $G = \mathrm{diag}(\gamma, \gamma, 0)$ with $\gamma = \sum_n |\boldsymbol{d}_n|^2 \geq 0$ [30]. The general solution $[\boldsymbol{p}(t) = V \cdot \boldsymbol{p}(0)]$ of the Bloch equation is then given by the evolution operator

$$V = \begin{pmatrix} +e^{-\gamma t}\cos kt & +e^{-\gamma t}\sin kt & 0 \\ -e^{-\gamma t}\sin kt & +e^{-\gamma t}\cos kt & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{6}$$

If the system is prepared in the pure (zero entropy) ν_μ state $[\boldsymbol{p}(0) = \boldsymbol{q}]$, the asymptotic final state is $\boldsymbol{p}(\infty) = (0, 0, c_{2\theta})$. Since $\mathrm{Tr}[\rho^2(\infty)] = (1 + c_{2\theta}^2)/2 < 1$ and $S[\rho(\infty)] = -c_\theta^2 \ln c_\theta^2 - s_\theta^2 \ln s_\theta^2 > 0$, the system evolves indeed into a mixed state with positive entropy. Maximal entropy ($S = \ln 2$) corresponds to maximal ν mixing ($s_{2\theta}^2 = 1$). Purity and entropy are conserved only if ρ is prepared in a pure mass eigenstate $[\boldsymbol{p}(0) = (0, 0, \pm 1)^T]$.

The survival probability $P_{\mu\mu} = \frac{1}{2}(1 + \boldsymbol{q}^T \cdot V \cdot \boldsymbol{q})$ reads

$$P_{\mu\mu} = 1 - \tfrac{1}{2}s_{2\theta}^2(1 - e^{-\gamma x}\cos kx)\,, \tag{7}$$

which reduces to the standard expression (2) in the limit $\gamma \rightarrow 0$. For $\gamma x \sim O(1)$, one expects significant deviations from the usual oscillation fit to the SK data.

We make a quantitative study of the effects of $P_{\mu\mu}$ in (7), by computing the theoretical SK lepton distributions in zenith angle (ϑ), and by fitting them to the SK data through a χ^2 statistics, as extensively discussed in [6]. The main difference from [6] is (i) the 30 data bins for the SK distributions refer to a longer detector exposure (52 kton yr [5]); (ii) the oscillation probability is here taken from Eq. (7). In the fit, we study both the case with $(\Delta m^2, s_{2\theta}^2, \gamma)$ unconstrained (oscillations plus decoherence) and the case with $\Delta m^2 = 0$ and $(s_{2\theta}^2, \gamma)$ unconstrained (decoherence only). We find significant differences in the results, depending on the energy variation assumed for γ (which is not necessarily a constant parameter). For definiteness, we discuss only three scenarios, corresponding to a possible power-law dependence of the kind $\gamma = \gamma_0(E/\mathrm{GeV})^n$ with $n = 0$, 2, and -1.

For $n = 0$ ($\gamma = \gamma_0 = \mathrm{const}$) the best fit with oscillations plus decoherence ($\chi^2_{\min} = 22.6$) is reached for $\Delta m^2 = 3 \times 10^{-3}\ \mathrm{eV}^2$, $s_{2\theta}^2 = 1$, *and* $\gamma_0 = 0$, which corresponds to the case of pure $\nu_\mu \leftrightarrow \nu_\tau$ oscillations. Since no evidence is seen to emerge for decoherence effects, meaningful upper bounds on the parameter γ can be placed. By taking $\chi^2 - \chi^2_{\min} = 6.25$ (corresponding to 90% C.L. for three degrees of freedom), we get

$$\gamma_0 < 3.5 \times 10^{-23}\ \mathrm{GeV} \qquad (n = 0)\,. \tag{8}$$

The limits at 95% and 99% C.L. are found to be 4.1×10^{-23} GeV and 5.5×10^{-23} GeV, respectively. The bound (8) shows that (i) if decoherence effects have the same origin (e.g., quantum gravity) and similar size in the different K, n, and ν systems, then atmospheric ν observations can improve the current laboratory limits (4); and (ii) decoherence effects, if any, can develop only over a typical length scale $\ell = \gamma_0^{-1} \gtrsim 5600$ km.

Figure 1 shows (for $n = 0$) the zenith distributions of SK events for best-fit standard oscillations ($\gamma_0 = 0$) and in the presence of an additional decoherence term ($\gamma_0 = 10^{-22}$ GeV). The electron (e) distributions are unaffected ($P_{ee} = 1$). In the sub-GeV μ sample, decoherence is almost unobservable, due to the large intrinsic smearing [6] of both energy and angle. In the multi-GeV μ sample, the transition from no oscillation ($P_{\mu\mu} \sim 1$ for $\cos\vartheta \sim 1$) to averaged oscillations ($P_{\mu\mu} \sim 1/2$ for $\cos\vartheta \sim -1$) is made only slightly faster by decoherence effects. Such effects are instead dominant in the higher-energy sample of upgoing μ, where the oscillation phase kx is small, and decoherence generates a much faster suppression of vertical muons ($\cos\vartheta \sim -1$), corresponding to the longest ν flight lengths. Finally, we find a bad fit ($\chi^2 \gtrsim 49$) when oscillations are switched off [$H = 0$, corresponding to $k = 0$ in Eq. (7)]. Therefore, in the case $n = 0$, the SK data cannot be explained solely by decoherence.

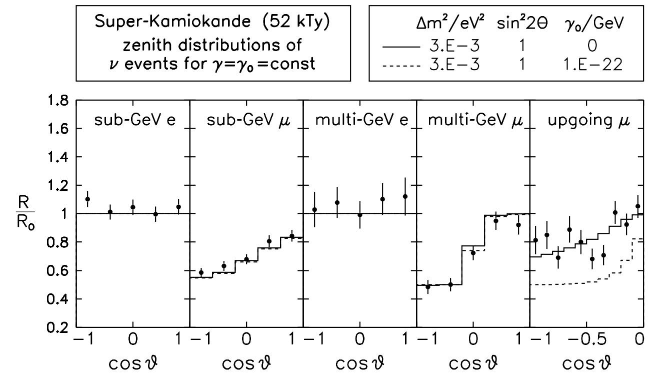

FIG. 1. Effects of decoherence ($\gamma_0 \neq 0$) on the distributions of lepton events as a function of the zenith angle (ϑ). The SK data are shown as dots with $\pm 1\sigma$ error bars. The histograms represent our theoretical calculations. In each bin, the electron (e) and muon (μ) rates R are normalized to standard (no oscillation, no decoherence) expectations R_0.

The case $n = 2$ may also be of phenomenological interest, in the light of a possible dimensional guess of the form $\gamma \propto E^2/M_P$ [31]. In this case, decoherence effects are even more disfavored than for $n = 0$, since they produce a faster suppression of muons with increasing energy, contrary to observations. We find an upper limit $\gamma_0 < 0.9 \times 10^{-27}$ GeV at 90% C.L. [to be compared with the limit (8)]. For $k = 0$ (decoherence without oscillations) the fit is also very bad ($\chi^2 \gtrsim 70$).

From the previous cases ($n = 0$ and $n = 2$) we learn that decoherence effects can be strongly constrained, more the faster they increase with energy. Conversely, we expect weaker constraints for a *decreasing* energy dependence, such as for $\gamma \propto E^{-1}$ ($n = -1$).

The case $n = -1$ may also be motivated by assuming that the exponent in Eq. (7) behaves as a Lorentz scalar. A boost from the ν rest frame to the laboratory frame would then introduce a factor m_ν/E (just as for the oscillation phase), giving a decoherence parameter of the form $\gamma = \gamma_0(E/\mathrm{GeV})^{-1}$. Of course, this ansatz should be taken with a grain of salt, since dissipative equations are known to entail problems with Lorentz invariance [14,32] (however, see [29,33]). In any case, assuming $\gamma = \gamma_0(E/\mathrm{GeV})^{-1}$, we have performed a fit to the SK data with $(\Delta m^2, s^2_{2\theta}, \gamma_0)$ unconstrained. The best fit is reached, once again, for $\gamma_0 = 0$, but the upper limit on γ_0 is now relatively weak, $\gamma_0 < 2 \times 10^{-21}$ GeV at 90% C.L. Therefore, for $n = -1$, one may add sizable decoherence effects to oscillations, without destroying the agreement with SK data. Can one switch off completely oscillations and explain the data as a pure decoherence effect? The answer is surprisingly positive. For $\Delta m^2 = 0$, the best agreement with the data is reached at $s^2_{2\theta} = 1$ and $\gamma_0 = 1.2 \times 10^{-21}$ GeV, with $\chi^2_{\min}/N_{\mathrm{DF}} = 27.1/(30 - 2)$, giving a good fit. This case represents a novel solution to the atmospheric ν anomaly, based solely on decoherence.

Figure 2 shows such an "exotic" best fit (decoherence without oscillations) as compared to the "canonical" best fit (oscillations without decoherence). The two cases appear to be almost indistinguishable within errors, although they entail completely different physics. It is amusing to notice that, for the two best-fit cases of Fig. 2, the ν_μ survival probability approximately read

$$P_{\mu\mu} \simeq \tfrac{1}{2}[1 + \cos(+\beta L/E)] \quad \text{(pure oscillations)}, \tag{9}$$

$$P_{\mu\mu} \simeq \tfrac{1}{2}[1 + \exp(-\beta L/E)] \quad \text{(pure decoherence)}, \tag{10}$$

where E is in GeV, L is the ν path length (km), and $\beta \sim 7 \times 10^{-3}$ GeV/km. Both cases have the same asymptotic behavior, namely, $\langle P_{\mu\mu}\rangle \simeq 1$ $(\frac{1}{2})$ for small (large) L/E. For intermediate values of L/E, the strong differences between the oscillating cosine factor and the monotonic exponential damping appear to be effectively suppressed by the large smearing in the ν energy and angle, due to the interaction and detection processes in SK. Therefore, future long-baseline accelerator experiments (such as K2K, MINOS, and the CERN-to-Gran Sasso project [34]) will be crucial in discriminating the above two functional forms for $P_{\mu\mu}$, by revealing the oscillation (or damping) pattern now hidden by smearing effects.

Finally, we test the best-fit decoherence case of Fig. 2 against the negative results of current $\nu_\mu \to \nu_\tau$ appearance searches [34]. In the CHORUS and NOMAD experiments [35] one has $\langle L/E\rangle \simeq 0.025$ km/GeV and $P_{\mu\tau} = 1 - P_{\mu\mu} \simeq \frac{1}{2}\beta\langle L/E\rangle$ (for $\Delta m^2 = 0$ and $s^2_{2\theta} = 1$). Then the experimental limit $P_{\mu\tau} \lesssim 1.3 \times 10^{-4}$ [34] implies the upper bound $\beta \lesssim 1.1 \times 10^{-2}$ GeV/km, which is compatible with the best-fit value $\beta \sim 7 \times 10^{-3}$ GeV/km.

In conclusion, we have performed a phenomenological analysis of modifications of the Liouville dynamics, in the context of atmospheric $\nu_\mu \leftrightarrow \nu_\tau$ transitions. Within a simple model embedding the relevant physics (oscillations plus decoherence), we have found that the Super-Kamiokande data can be a sensitive probe of decoherence effects (e.g., originated by quantum gravity), supplementing current laboratory tests based on K and n interferometry. Depending on the energy behavior assumed for such

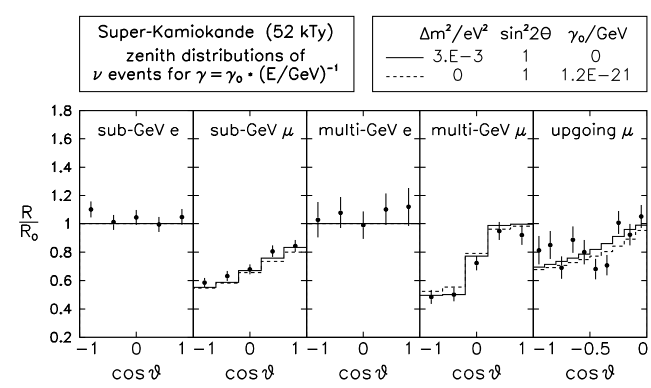

FIG. 2. Comparison of best-fit scenarios for pure oscillations (solid line, as in Fig. 1) and for pure decoherence with $\gamma \propto 1/E$ (dashed line).

effects, one can either constrain them strongly or use them to explain the atmospheric ν anomaly without oscillations.

We thank J. Ellis, R. Floreanini, S. Pakvasa, and S. Pascazio for useful comments.

Note added.—After submission of this Letter, two related works appeared [36]. We also noted a recent preprint [37] suggesting an exceedingly small theoretical estimate for γ ($\sim k^2/M_P$), which would discourage current experimental tests with neutrinos (as well as with kaons and neutrons). It seems to us that such an estimate [37], being essentially based on a dimensional guess, should be presently considered with great caution. In the absence of both a full dynamical theory and of *ab initio* calculations for decoherence effects, any current ansatz may prove to be wrong. This fact warrants phenomenological analyses as ours, whose results, inferred from experimental data, remain valid independently of (uncertain) guesses about the origin and the size of γ.

[1] Super-Kamiokande Collaboration, Y. Fukuda *et al.,* Phys. Rev. Lett. **81**, 1562 (1998).

[2] B. Pontecorvo, Zh. Eksp. Teor. Fiz. **53**, 1717 (1967) [Sov. Phys. JETP **26**, 984 (1968)]; Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962).

[3] Super-Kamiokande Collaboration, Y. Fukuda *et al.,* Phys. Lett. B **433**, 9 (1998); Phys. Lett. B **436**, 33 (1998); Phys. Rev. Lett. **82**, 2644 (1999).

[4] MACRO Collaboration, M. Ambrosio *et al.,* Phys. Lett. B **434**, 451 (1998); Soudan 2 Collaboration, W. W. M. Allison *et al.,* Phys. Lett. B **449**, 137 (1999).

[5] Y. Totsuka, in *Proceedings of the 15th Particles and Nuclei International Conference (PANIC '99), Uppsala, Sweden, 1999,* edited by G. Faldt, B. Hoistad, and S. Kullander [Nucl. Phys. **A663/A664**, 218c (2000)].

[6] G.L. Fogli, E. Lisi, A. Marrone, and G. Scioscia, Phys. Rev. D **59**, 033001 (1999).

[7] G.L. Fogli, E. Lisi, A. Marrone, and G. Scioscia, Phys. Rev. D **59**, 117303 (1999); **60**, 053006 (1999); P. Lipari and M. Lusignoli, Phys. Rev. D **60**, 013003 (1999); S. Pakvasa, in *Proceedings of the 8th International Workshop on Neutrino Telescopes, Venice, Italy, 1999,* edited by M. Baldo Ceolin (University of Padua, Padua, Italy, 1999), Vol. 1, p. 283.

[8] R. Alicki and K. Lendi, *Quantum Dynamical Semigroups and Applications,* Lecture Notes in Physics Vol. 286 (Springer-Verlag, Berlin, 1987).

[9] In this context, it is useful to remember the conversion constant $(1\ \text{km})^{-1} = 1.97 \times 10^{-19}$ GeV.

[10] S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975); **87**, 395 (1982); Phys. Rev. D **14**, 2460 (1976).

[11] S. B. Giddings and A. Strominger, Nucl. Phys. **B307**, 854 (1988); W. H. Zurek, Phys. Today **44**, No. 10, 36 (1991); G. Amelino-Camelia, J. Ellis, N.E. Mavromatos, and D.V. Nanopoulos, Int. J. Mod. Phys. A **12**, 607 (1997); L.J. Garay, Int. J. Mod. Phys. A **14**, 4079 (1999).

[12] J. Ellis, J. S. Hagelin, D. V. Nanopoulos, and M. Srednicki, Nucl. Phys. **B241**, 381 (1984).

[13] J. Ellis, N.E. Mavromatos, and D.V. Nanopoulos, in *Proceedings of the 31st International School of Subnuclear Physics, Erice, Italy, 1993,* edited by A. Zichichi, Subnuclear Series Vol. 31 (World Scientific, Singapore, 1995); Chaos Solitons Fractals **10**, 345 (1999).

[14] T. Banks, L. Susskind, and M. E. Peskin, Nucl. Phys. **B244**, 125 (1984).

[15] P. Huet and M.E. Peskin, Nucl. Phys. **B488**, 335 (1997); J. Ellis, J.L. Lopez, N.E. Mavromatos, and D.V. Nanopoulos, Phys. Rev. D **53**, 3846 (1996); F. Benatti and R. Floreanini, Phys. Lett. B **389**, 100 (1996); **401**, 337 (1997).

[16] F. Benatti and R. Floreanini, Phys. Lett. B **451**, 422 (1999).

[17] Y. Liu, L. Hu, and M.L. Ge, Phys. Rev. D **56**, 6648 (1997).

[18] Y. Liu, J.L. Chen, and M.L. Ge, J. Phys. G **24**, 2289 (1998); C.P. Sun and D.L. Zhou, hep-ph/9808334.

[19] C.H. Chang, W.S. Dai, X.Q. Li, Y. Liu, F.C. Ma, and Z.J. Tao, Phys. Rev. D **60**, 033006 (1999).

[20] Y. Grossman and M.P. Worah, hep-ph/9807511.

[21] V. Gorini, A. Frigerio, M. Verri, A. Kossakowski, and E.C.G. Sudarshan, J. Math. Phys. (N.Y.) **17**, 821 (1976).

[22] For the issue of complete positivity in the $K\overline{K}$ system, see F. Benatti and R. Floreanini, Phys. Lett. B **468**, 287 (1999), and references therein.

[23] G. Lindblad, Commun. Math. Phys. **48**, 119 (1976).

[24] S.L. Adler, Phys. Lett. A **265**, 58 (2000).

[25] Book series *Quantum Probability and Related Topics,* edited by L. Accardi (World Scientific, Singapore, 1992–1994).

[26] C.P. Burgess and D. Michaud, Ann. Phys. (N.Y.) **256**, 1 (1997).

[27] R.A. Harris and L. Stodolsky, Phys. Lett. **B116**, 464 (1982); L. Stodolsky, Phys. Rev. D **36**, 2273 (1987).

[28] F. Benatti and H. Narnhofer, Lett. Math. Phys. **15**, 325 (1988).

[29] J. Liu, hep-th/9301082.

[30] In [12] the matrix G is parametrized in terms of three variables (α, β, γ), reducing to our diagonal form for $\alpha = \gamma$ and $\beta = 0$. This follows from the evolution map in [21] being simply *positive* rather than *completely positive*. Complete positivity is a safer assumption, preventing the possible occurrence of negative probabilities [22].

[31] J. Ellis, N.E. Mavromatos, and D.V. Nanopoulos, Mod. Phys. Lett. A **12**, 1759 (1997); V.A. Kostelecky and R. Potting, Nucl. Phys. **B359**, 545 (1991); F. Benatti and R. Floreanini, Ann. Phys. (N.Y.) **273**, 58 (1999).

[32] M. Srednicki, Nucl. Phys. **B410**, 143 (1993).

[33] W.G. Unruh and R.M. Wald, Phys. Rev. D **52**, 2176 (1995).

[34] L. DiLella, in Proceedings of LP '99, 19th International Symposium on Lepton and Photon Interactions at High Energies, Stanford, CA, 1999 (to be published).

[35] CHORUS Collaboration, E. Eskut *et al.,* Phys. Lett. B **434**, 205 (1998); NOMAD Collaboration, P. Astier *et al.,* Phys. Lett. B **453**, 169 (1999).

[36] F. Benatti and R. Floreanini, J. High Energy Phys. **2**, 32 (2000); H.V. Klapdor-Kleingrothaus, H. Päs, and U. Sarkar, hep-ph/0004123.

[37] S. Adler, hep-ph/0005220.