Fuzzy Cold Dark Matter: The Wave Properties of Ultralight Particles

Wayne Hu, Rennan Barkana, and Andrei Gruzinov
Institute for Advanced Study, Princeton, New Jersey 08540
(Received 27 March 2000)

Cold dark matter (CDM) models predict small-scale structure in excess of observations of the cores and abundance of dwarf galaxies. These problems might be solved, and the virtues of CDM models retained, even without postulating *ad hoc* dark matter particle or field interactions, if the dark matter is composed of ultralight scalar particles ($m \sim 10^{-22}$ eV), initially in a (cold) Bose-Einstein condensate, similar to axion dark matter models. The wave properties of the dark matter stabilize gravitational collapse, providing halo cores and sharply suppressing small-scale linear power.

PACS numbers: 95.35.+d, 98.80.Cq

Introduction.—Recently, the small-scale shortcomings of the otherwise widely successful cold dark matter (CDM) models for structure formation have received much attention (see [1–4] and references therein). CDM models predict cuspy dark matter halo profiles and an abundance of low mass halos not seen in the rotation curves and local population of dwarf galaxies, respectively. Although the significance of the discrepancies is still disputed and solutions involving astrophysical processes in the baryonic gas may still be possible (e.g., [5]), recent attention has focused mostly on solutions involving the dark matter sector.

In the simplest modification, warm dark matter ($m \sim$ keV) replaces CDM and suppresses small-scale structure by free streaming out of potential wells [3], but this modification may adversely affect structure at somewhat larger scales [6]. Small-scale power could be suppressed more cleanly in the initial fluctuations, perhaps originating from a kink in the inflaton potential [2], but its regeneration through nonlinear gravitational collapse would likely still produce halo cusps [7].

More radical suggestions include strong self-interactions either between dark matter particles [1] or in the potential of axionlike scalar field dark matter [4]. While interesting, these solutions require self-interactions wildly in excess of those expected for weakly interacting massive particles or axions.

In this Letter, we propose a solution involving free particles only. The catch is that the particles must be extraordinarily light ($m \sim 10^{-22}$ eV) so that their wave nature is manifest on astrophysical scales. Under this proposal, dark matter halos are stable on small scales for the same reason that the hydrogen atom is stable: the uncertainty principle in wave mechanics. We call this dark matter candidate fuzzy cold dark matter (FCDM).

Equations of motion.—If the dark matter is composed of ultralight particles $m \ll 1$ eV, the occupation numbers in galactic halos are so high [8] that the dark matter behaves as a classical field obeying the wave equation

$$\Box\phi = m^2\phi\,, \tag{1}$$

where we have set $\hbar = c = 1$. On scales much larger than the Compton wavelength m^{-1} but much smaller than the particle horizon, one can employ a Newtonian approximation to the gravitational interaction embedded in the covariant derivatives of the field equation and a nonrelativistic approximation to the dispersion relation. It is then convenient to define the wave function $\psi \equiv Ae^{i\alpha}$, out of the amplitude and phase of the field $\phi = A\cos(mt - \alpha)$, which obeys

$$i\left(\partial_t + \frac{3}{2}\frac{\dot{a}}{a}\right)\psi = \left(-\frac{1}{2m}\nabla^2 + m\Psi\right)\psi\,, \tag{2}$$

where Ψ is the Newtonian gravitational potential. For the unperturbed background, the right-hand side vanishes and the energy density in the field, $\rho = m^2|\psi|^2/2$, redshifts like matter $\rho \propto a^{-3}$.

On time scales short compared with the expansion time, the evolution equations become

$$i\partial_t\psi = \left(-\frac{1}{2m}\nabla^2 + m\Psi\right)\psi, \qquad \nabla^2\Psi = 4\pi G\delta\rho\,. \tag{3}$$

Assuming the dark matter also dominates the energy density, we have $\delta\rho = m^2\delta|\psi|^2/2$. This is simply the nonlinear Schrödinger equation for a self-gravitating particle in a potential well. In the particle description, ψ is proportional to the wave function of each particle in the condensate.

Jeans/de Broglie scale.—The usual Jeans analysis tells us that when gravity dominates there exists a growing mode $e^{\gamma t}$ where $\gamma^2 = 4\pi G\rho$; however, a free field oscillates as e^{-iEt} or $\gamma^2 = -(k^2/2m)^2$. In fact, $\gamma^2 = 4\pi G\rho - (k^2/2m)^2$ and therefore there is a Jeans scale

$$\begin{aligned} r_J &= 2\pi/k_J = \pi^{3/4}(G\rho)^{-1/4}m^{-1/2}, \\ &= 55 m_{22}^{-1/2}(\rho/\rho_b)^{-1/4}(\Omega_m h^2)^{-1/4}\ \text{kpc}\,, \end{aligned} \tag{4}$$

below which perturbations are stable and above which they behave as ordinary CDM. Here $m_{22} = m/10^{-22}$ eV and $\rho_b = 2.8 \times 10^{11}\Omega_m h^2 M_\odot$ Mpc^{-3} is the background density. The Jeans scale is the geometric mean between the dynamical scale and the Compton scale (cf. [9–11]), as originally shown in a more convoluted manner by [12].

 0031-9007/00/85(6)/1158(4)$15.00

The existence of the Jeans scale has a natural interpretation: it is the de Broglie wavelength of the ground state of a particle in the potential well. To see this, note that the velocity scales as $v \sim (G\rho)^{1/2} r$ so that the de Broglie wavelength $\lambda \sim (mv)^{-1} \sim m^{-1}(G\rho)^{-1/2} r^{-1}$. Setting $r_J = \lambda = r$ returns the Jeans scale. Stability below the Jeans scale is thus guaranteed by the uncertainty principle: an increase in momentum opposes any attempt to confine the particle further.

The physical scale depends weakly on the density, but in a dark matter halo ρ will be much larger than the background density ρ_b. Consider the density profile of a halo of mass M [$\equiv (4\pi r_v^3/3)200\rho_b$ in terms of the virial radius r_v] found in CDM simulations [13],

$$\rho(r, M) \sim \frac{200}{3} \frac{f\rho_b}{(cr/r_v)(1 + cr/r_v)^2}, \tag{5}$$

where $f(c) = c^3/[\ln(1 + c) - c/(1 + c)]$ and the concentration parameter c depends weakly on mass. This profile implies an r^{-1} cusp for $r < r_v/c$ which will be altered by the presence of the Jeans scale. Solving for the Jeans scale in the halo $r_{J\mathrm{h}}$ as a function of its mass using the enclosed mean density yields

$$r_{J\mathrm{h}} \sim 3.4(c_{10}/f_{10})^{1/3} m_{22}^{-2/3} M_{10}^{-1/9} (\Omega_m h^2)^{-2/9}\ \mathrm{kpc}, \tag{6}$$

where we have scaled the mass dependent factors to the regime of interest $c_{10} = c/10$, $f_{10} = f(c)/f(10)$, and $M_{10} = M/10^{10} M_\odot$. For estimation purposes, we have assumed $r_{J\mathrm{h}} \ll r_v/c$ which is technically violated for $M_{10} \lesssim 1$ and $m = 10^{-22}$ eV, but with only a mild effect. In the smallest halos, the Jeans scale is above the turnover radius r_v/c, and there is no region where the density scales as r^{-1}. The maximum circular velocity will then be lower than that implied by Eq. (5). More massive halos will have their cuspy r^{-1} behavior extend from $r = r_v/c$ down to $r_{J\mathrm{h}}$.

These simple scalings show that the wave nature of dark matter can prevent the formation of the kpc scale cusps and substructure in dark matter halos if $m \sim 10^{-22}$ eV. However, alone they do not determine what does form instead. To answer this question, cosmological simulations will be required [14], and this lies beyond the scope of this paper. Instead we provide here the tools necessary to perform such a study, a discussion of possible astrophysical implications, and illustrative one-dimensional simulations comparing FCDM and CDM.

Linear perturbations.—The evolution of fluctuations in the linear regime provides the initial conditions for cosmological simulations and also directly affects the abundance of dark matter halos. Because the initial conditions are set while the fluctuations are outside the horizon, we must generalize the Newtonian treatment above to include relativistic effects.

Following the "generalized dark matter" (GDM) approach of [15], we remap the equations of motion for the scalar field in Eq. (1) onto the continuity and Euler equations of a relativistic imperfect fluid. Note that in the Newtonian approximation the current density $\mathbf{j} \propto \psi^*\nabla\psi - \psi\nabla\psi^*$ plays the role of momentum density so that "probability conservation" becomes the continuity equation. The dynamical aspect of Eq. (2) then becomes the Euler equation for a fluid with an effective sound speed $c_{\mathrm{eff}}^2 = k^2/4a^2m^2$, where k is the comoving wave number.

This Newtonian relation breaks down below the Compton scale which for any mode will occur when $a < k/2m$. In this regime, the scalar field is slowly rolling in its potential rather than oscillating, and it behaves like a fluid with an effective sound speed $c_{\mathrm{eff}}^2 = 1$ [15]. For our purposes, it suffices to simply join these asymptotic solutions and treat the FCDM as GDM with

$$c_{\mathrm{eff}}^2 = \begin{cases} 1, & a \le k/2m, \\ k^2/4a^2m^2, & a > k/2m, \end{cases} \tag{7}$$

with no anisotropic stresses in linear theory. We have verified that the details of this matching have a negligible effect on the results. Since the underlying treatment is relativistic, this prescription yields a consistent, covariant treatment of the dark matter inside and outside the horizon. The linear theory equations including radiation and baryons are then solved in the usual way but with initial curvature perturbations in the radiation and no perturbations in the FCDM [16].

The qualitative features of the solutions are easily understood. The comoving Jeans wave number scales with the expansion as $k_J \propto a\rho_b^{1/4}(a)$ or $\propto a^{1/4}$ during matter domination and is constant during radiation domination. Because the comoving Jeans scale is nearly constant, perturbation growth above this scale generates a sharp break in the spectrum. More precisely, the critical scale is k_J at matter-radiation equality $k_{J\mathrm{eq}} = 9 m_{22}^{1/2}$ Mpc^{-1}. Numerically, we find that the linear density power spectrum of FCDM is suppressed relative to the CDM case by

$$P_{\mathrm{FCDM}}(k) = T_{\mathrm{F}}^2(k) P_{\mathrm{CDM}}(k), \qquad T_{\mathrm{F}}(k) \approx \frac{\cos x^3}{1 + x^8}, \tag{8}$$

where $x = 1.61 m_{22}^{1/18} k/k_{J\mathrm{eq}}$. The power drops by a factor of 2 at

$$k_{1/2} \approx \frac{1}{2} k_{J\mathrm{eq}} m_{22}^{-1/18} = 4.5 m_{22}^{4/9}\ \mathrm{Mpc}^{-1}. \tag{9}$$

The break in k is much sharper than those expected from inflation [2] or quartic self-interaction of a scalar field. In the latter, the Jeans scale is fixed in physical coordinates so that the suppression is spread over 3–4 orders of magnitude in scale [4]. In warm dark matter scenarios, the break is comparably sharp, but its relationship to the halo core radius [see Eq. (6)] differs.

Low mass halos.—In the CDM model, the abundance of low mass halos is too high when compared with the luminosity function of dwarf galaxies in the local group [17,18].

Based on analytic scalings, Kamionkowski and Liddle [2] argued that a sharp cutoff in the initial power spectrum at $k = 4.5h$ Mpc^{-1} might solve this problem. Thus, the FCDM cutoff at $k \sim 4.5$ Mpc^{-1}, produced if $m \sim 10^{-22}$ eV is chosen to remove kpc scale cusps, may solve the low mass halo abundance problem as well. Whether the required masses actually coincide in detail can only be addressed by simulations.

Numerical simulations of CDM with a smooth cutoff in the initial power spectrum qualitatively confirm the analytic estimates but suggest that a somewhat larger scale may be necessary: half power at $k = 2h$ Mpc^{-1} reduces the $z = 3$ abundance of $10^{10}h^{-1}M_\odot$ halos by a factor of $\sim$5 and the abundance of $10^{11}h^{-1}M_\odot$ halos by a factor of $\sim$3 [19]. Note, however, that our model produces a much sharper cutoff in the power spectrum than in the model tested. Furthermore, astrophysical influences such as feedback or photoionization may have prevented dwarf galaxy halos from accumulating much gas or stars [20].

First objects and reionization.—At very high redshift, much of the star formation in a CDM model is predicted to occur in low mass halos which are not present in the FCDM model. In a CDM model the first round of star formation is thought to occur in objects of mass $\sim 10^5 M_\odot$ (see [21], and references therein) due to molecular hydrogen cooling. The consequent destruction of molecular hydrogen [22,23] implies that it is larger mass objects $\gtrsim 10^8 M_\odot$, where atomic cooling is possible, that are responsible for reionization. In our scenario, if the cutoff scale in Eq. (9) were set to reduce the abundance of $M \lesssim 10^9 M_\odot$ halos, reionization could be delayed and the number of detectable galaxies prior to reionization reduced by a factor of $\sim$5 [24].

Live halos.—Precisely what effect the Jeans (de Broglie) scale has on the structure and abundance of low mass halos is best answered through simulations. To provide some insight into these issues, we conclude with simulations of the effects in one dimension.

We solve the wave equation (3) in an interval $0 < x < L$ with boundary $\psi(0) = \psi(L) = 0$. At $t = 0$, the density perturbation is $\delta\rho = \rho_0 \sin(\pi x/L) \gg \rho_b$, with ψ real. We define the Jeans length r_J by Eq. (4) with the density ρ_0, then the choice of r_J/L specifies m. It is also convenient to define the dynamical time scale $t_{\rm dyn} = (4\pi G\rho_0)^{-1/2}$. A one-dimensional CDM simulation with the same initial density and zero initial velocity was run for comparison.

For $r_J \gg L$, the field model does not form a gravitating halo. For $r_J \sim L$ a gravitationally bound halo is formed, but the cusp, which is clearly seen in the CDM simulation, is not observed (see Fig. 1). Interference effects cause continuous evolution on the dynamical time scale $t_{\rm dyn}$ (cf. [10]). Note, however, that the gravitational acceleration is much smoother so that the trajectories of test particles (i.e., visible matter) will be less affected by these wiggles.

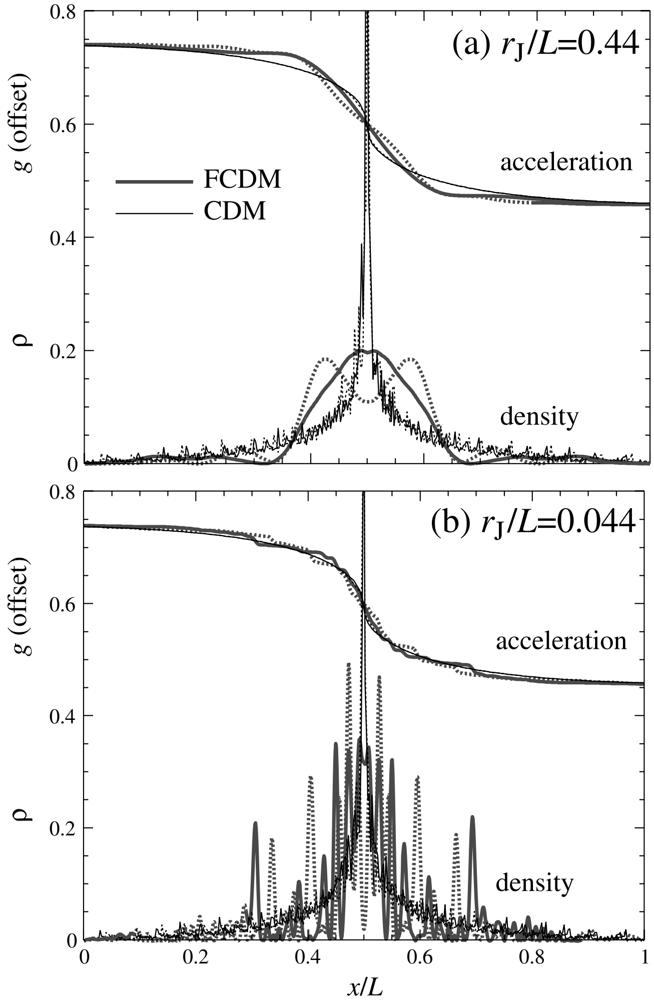

FIG. 1. One-dimensional simulations (a) large Jeans scale $r_J/L = 0.44$ and (b) small Jeans scale $r_J/L = 0.044$. Two snapshots, $t/t_{\rm dyn} = 99$ (solid lines) and 100 (dotted lines), are shown for the density profile (units of $15\rho_0$) and gravitational acceleration (units of $3L/2t_{\rm dyn}^2$, offset for clarity).

For $r_J \ll L$, the density computed from the field follows the CDM simulation when smoothed over many Jeans scales. This is to be expected because in this limit the Schrödinger equation can be solved in the geometrical optics approximation. Nonetheless, interference features localized in space ($\delta x \sim r_J$) and time ($\delta t \lesssim t_{\rm dyn}$) are quite strong, of order 1. Again these small-scale features make only a small contribution to the gravitational acceleration.

Discussion.—We have shown that the wave properties of ultralight ($m \sim 10^{-22}$ eV) dark matter can suppress kpc scale cusps in dark matter halos and reduce the abundance of low mass halos. While such a mass may seem unnatural from a theoretical standpoint, the observational and experimental evidence allows even substantially lighter scalar fields $m \lesssim 10^{-33}$ eV [25], where the wave nature is manifest across the whole particle horizon so as to provide a smooth energy component to explain observations of accelerated expansion [26].

Three-dimensional numerical simulations are required to determine whether our proposal works in detail. Our one-dimensional simulations suggest that the small-scale cutoff appears at $r \sim r_J$ and that the density profile on these scales not only is not universal but also evolves continuously on the dynamical time scale (or faster) due to interference effects. The observable rotation curves are smoother than the density profile so that this prediction, while testable with high-resolution data, is not obviously in conflict with the data today. Likewise, the time variation of the potential is smaller than that of the density but can, in principle, transfer energy from the FCDM to the baryons in the halo. This could puff up the baryons in dwarf galaxies while bringing the FCDM closer to a stationary ground state, but the precise evolution requires detailed calculations.

Our Jeans scale is a weak function of density, $r_J \propto \rho^{-1/4}$. This has two testable consequences. The first is a sharp cutoff at $k \sim 4.5 m_{22}^{1/2}$ Mpc^{-1} in the linear power. Quantities related to the abundance of low mass halos, e.g., dwarf galaxies in the local group, the first objects, faint galaxies at very high redshifts, and reionization can be seriously affected by the cutoff. Counterintuitively, quantities related to the *nonlinear* power spectrum of the dark matter are only weakly affected due to the gravitational regeneration of small-scale power [19]. The second consequence is that choosing the mass to set the core radius in one class of dark matter dominated objects sets the core radius in another set, given the ratio of characteristic densities. This relation can, in principle, be tested by comparing the local dwarf spheroidals [27] to higher mass systems.

While the detailed implications remain to be worked out, fuzzy cold dark matter provides an interesting means of suppressing the excess small-scale power that plagues the cold dark matter scenario.

We thank J. P. Ostriker, P. J. E. Peebles, D. N. Spergel, M. Srednicki, and M. White for useful discussions. W. H. and A. G. are supported by the Keck Foundation and NSF-9513835, and R. B. is supported by Institute Funds and the NSF under Grant No. PHY94-07194.

[1] D. N. Spergel and P. J. Steinhardt, Phys. Rev. Lett. **84**, 3760 (2000).
[2] M. Kamionkowski and A. R. Liddle, Phys. Rev. Lett. **84**, 4525 (2000).
[3] J. Sommer-Larsen and A. Dolgov, astro-ph/9912166; C. J. Hogan and J. J. Dalcanton, Phys. Rev. D (to be published).
[4] P. J. E. Peebles, astro-ph/0002495; J. Goodman, astro-ph/0003018.
[5] F. C. van den Bosch, B. E. Robertson, and J. J. Dalcanton, astro-ph/9911372.
[6] V. K. Narayanan, D. N. Spergel, R. Dave, and C-P. Ma, astro-ph/0005095.
[7] B. Moore *et al.,* astro-ph/9903164.
[8] S. Tremaine and J. E. Gunn [Phys. Rev. Lett. **42**, 407 (1979)] place a lower limit on the mass of fermionic dark matter which does not apply to our bosonic particles.
[9] V. Sahni and L. Wang (astro-ph/9910097) also considered mass effects on halo cusps but assumed stability only on the much smaller Compton wavelength m^{-1}.
[10] S.-J. Sin [Phys. Rev. D **50**, 3650 (1994)] considered halos to be stationary energy states.
[11] W. H. Press, B. S. Ryden, and D. N. Spergel [Phys. Rev. Lett. **64**, 1084 (1990)] considered $m \sim 10^{-28}$ eV and a Jeans scale of $\sim$100 Mpc.
[12] M. Yu. Khlopov, B. A. Malomed, and Ya. B. Zel'dovich, Mon. Not. R. Astron. Soc. **215**, 575 (1985).
[13] J. F. Navarro, C. S. Frenk, and S. D. M. White, Astrophys. J. **490**, 493 (1996).
[14] L. M. Widrow and N. Kaiser, Astrophys. J. **416**, L71 (1993); G. Davies and L. M. Widrow, Astrophys. J. **485**, 484 (1997).
[15] W. Hu, Astrophys. J. **506**, 485 (1998).
[16] If isocurvature perturbations from inflation are negligible, gravity rapidly generates adiabatic conditions from curvature perturbations; cf. M. S. Turner, Phys. Rev. D **33**, 889 (1986).
[17] A. Klypin, A. V. Kravtsov, O. Valenzuela, and F. Prada, Astrophys. J. **522**, 82 (1999).
[18] B. Moore *et al.,* Astrophys. J. **524**, L19 (1999).
[19] M. White and R. A. C. Croft, astro-ph/0001247.
[20] G. Efstathiou, Mon. Not. R. Astron. Soc. **256**, 43 (1992); A. A. Thoul and D. H. Weinberg, Astrophys. J. **465**, 608 (1996); J. S. Bullock, A. V. Kravtsov, and D. H. Weinberg, astro-ph/0002214.
[21] Z. Haiman, A. A. Thoul, and A. Loeb, Astrophys. J. **464**, 523 (1996).
[22] T. P. Stecher and D. A. Williams, Astrophys. J. **149**, L29 (1967); Z. Haiman, M. Rees, and A. Loeb, Astrophys. J. **476**, 458 (1996); **484**, 985(E) (1997).
[23] Quasars might have catalyzed molecular hydrogen formation leading to continued star formation in CDM (Z. Haiman, T. Abel, and M. Rees, astro-ph/9903336); in FCDM, it would still be suppressed.
[24] R. Barkana and A. Loeb, Astrophys. J. (to be published).
[25] S. M. Carroll, Phys. Rev. Lett. **81**, 3067 (1998).
[26] B. Ratra and P. J. E. Peebles, Phys. Rev. D **37**, 3406 (1988); K. Coble, S. Dodelson, and J. A. Frieman, Phys. Rev. D **55**, 1851 (1997); R. R. Caldwell, R. Dave, and P. J. Steinhardt, Phys. Rev. Lett. **80**, 1582 (1998).
[27] M. Mateo, Annu. Rev. Astron. Astrophys. **36**, 435 (1998).