Comment on "Energy of a Plasma in the Classical Limit"

In a recent Letter [1], Opher and Opher (O2) find through approximate (model) calculations of energy in fluctuations with finite frequencies (ω) in a plasma that "the energy density of the fluctuations is appreciably larger than previously thought" and derive a certain cosmological implication therefrom. In this Comment, I point out through a rigorous formulation that their principal claims are ill-founded and plagued by logical lapses; their alluded cosmological consequence is groundless.

Equations (1) and (2) in [1] may be expressed in a generalized form [2] as

$$E_\alpha(\mathbf{k},\omega) = \frac{i\hbar}{4\pi}\coth\left(\frac{\hbar\omega\beta}{2}\right)[\chi_\alpha(\mathbf{k},\omega) - \chi_\alpha(-\mathbf{k},-\omega)], \tag{1}$$

where β denotes the inverse temperature in energy units and $\chi_\alpha(\mathbf{k},\omega)$ refer to generalized susceptibilities [α equals (1) longitudinal, (2) transverse E, and (3) transverse B] under the *retarded* boundary conditions [3]. Integration of (1) over the frequencies can be carried out precisely by a contour-integration technique [2] as

$$\int_{-\infty}^{\infty} d\omega\, E_\alpha(\mathbf{k},\omega) \equiv E_\alpha(\mathbf{k}) = -\frac{1}{\beta}\sum_{\nu=-\infty}^{\infty}\chi_\alpha^c(\mathbf{k},z_\nu) \quad (\nu = 0, \pm1, \pm2, \ldots), \tag{2}$$

where $z_\nu = (2\pi i/\hbar\beta)\nu$ and $\chi_\alpha^c(\mathbf{k},\omega)$ refer to the generalized susceptibilities under the *causal* boundary conditions [2]. In the classical limit, i.e., $\hbar \to 0$, only the $\nu = 0$ contributions remain in Eq. (2). In the quantum (ground state) limit, i.e., $\beta \to \infty$, the summation (2) turns into a frequency integration along the entire imaginary-frequency axis.

The quantity $E_1(\mathbf{k})$ is closely related to the exchange-correlation energy [2]; O2 interpreted it erroneously as a fluctuation energy. Causality requirements [4] prove that $1/\varepsilon_L(\mathbf{k},0) < 1$, so that $E_1(\mathbf{k}) < 0$ in the classical limit. The inequality, $E_1(\mathbf{k}) < 0$, is true not only in the classical limit ($\hbar\omega\beta \ll 1$) but also generally in the entire temperature regime, including the ground state, where *all* of the imaginary frequencies z_ν contribute to the evaluation (2); negative exchange-correlation energies are a character *universal* for Coulombic systems [4]. It is straightforward to see in (2) that the electromagnetic (transverse) contributions, $E_2(\mathbf{k}) + E_3(\mathbf{k})$, remain positive definite, irrespective of the system being in a classical or in a quantum state; these are related to the electromagnetic fluctuation energies that may legitimately enter a cosmological argument.

In their paper, O2 noted the inequality $E_1(\mathbf{k}) < 0$ in the specific calculations based on a classic Debye-Hückel approximation and modifications therefrom. Identifying them fallaciously as fluctuation energies, they equated those model calculations to what were "previously thought" and concluded that "the energy density of the fluctuations is appreciably larger than previously thought" based simply on the observation, $E_2(\mathbf{k}) + E_3(\mathbf{k}) > 0 > E_1(\mathbf{k})$. Smallness in magnitude (and negatives) of $E_1(\mathbf{k})$ as compared with the Planckian contributions, $E_2(\mathbf{k}) + E_3(\mathbf{k})$, in fact, have been well recognized and documented [2]; these features have been correctly taken into consideration in the existing cosmological applications.

I thank the Aspen Center for Physics, where this Comment was prepared.

Setsuo Ichimaru
The University of Tokyo
Hongo, Tokyo 113-0033, Japan

Received 13 July 1999
PACS numbers: 52.25.Dg, 95.30.Qd

[1] M. Opher and R. Opher, Phys. Rev. Lett. **82**, 4835 (1999).
[2] S. Ichimaru, *Statistical Plasma Physics,* Vol. I (Addison-Wesley, Redwood City, CA, 1992) [cf. Sect. 3.1C and Fig. 3.1 for "retarted" and "causal" responses and deformation of the integration contours leading to (2)].
[3] In the notation of [1], $\chi_1(\mathbf{k},\omega) = 1 - \varepsilon_L^{-1}$; $\chi_2(\mathbf{k},\omega) = 2/[\varepsilon_T - (kc/\omega)^2] - 2$; $\chi_3(\mathbf{k},\omega) = -2(kc/\omega)^2/[\varepsilon_T - (kc/\omega)^2]$.
[4] S. Ichimaru, Rev. Mod. Phys. **54**, 1017 (1982) [cf. Sect. II.B.6 on "causality"].

0031-9007/00/85(5)/1139(1)$15.00