Probing Strange Stars and Color Superconductivity by *r*-Mode Instabilities in Millisecond Pulsars

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r-mode instabilities in rapidly rotating quark matter stars (strange stars) lead to specific signatures in the evolution of pulsars with periods below 2.5 msec, and may explain the apparent lack of very rapid pulsars. Existing data seem consistent with pulsars being strange stars with a normal quark matter phase surrounded by an insulating nuclear crust. In contrast, quark stars in a color-flavor-locked phase are ruled out. Two-flavor color superconductivity is marginally inconsistent with pulsar data.

Starting with Andersson's realization, that rotating relativistic stars are generically unstable against the rotational-mode (*r*-mode) instability [1], a series of papers has investigated the many implications for gravitational radiation detection and the evolution of pulsars [2–20]. Originally it appeared that young, hot neutron stars would spin down to rotation periods of the order of 10 msec within their first year of existence. In contrast, thousands of years would be required for any *r*-mode driven spin-down of a hot strange star or a neutron star with significant quark matter content, and the rotation period would not increase above 3 msec in this case, making young, rapid pulsars potential "smoking guns" for quark matter (meta)stability [5]. Decisive for the localization of the instability regimes were the viscosities damping the modes, with strange matter characterized by a huge bulk viscosity relative to nuclear matter.

Recently Bildsten and Ushomirsky [18] pointed out that a very important effect damping *r* modes in neutron stars had been overlooked. This is damping due to viscosity in the boundary layer between the oscillating fluid and the nearly static crust, which is more than 10^5 times stronger than that from the shear in the interior. Matching of boundary conditions at the crust is particularly important for *r* modes, since these are characterized by significant horizontal flows. As a result, *r* modes in neutron stars are important only for rotation periods faster than 2 msec, and only for very high core temperatures (with the possible exception of the very brief time span before the crust forms). Unless it turns out that neutron stars are able to spin down significantly before their crust solidifies, this means that young, rapid pulsars could be neutron stars after all, not just strange stars.

But other, perhaps even better, probes of strange stars result from the *r*-mode instability as demonstrated below. Furthermore, pulsar data turn out to be very sensitive probes of color superconductivity in quark matter [21]. These prospects are pursued in the following.

In contrast to neutron stars, the quark matter fluid in a strange star need not be at rest at the base of the crust, and therefore the *r* modes in a quark matter star are not damped significantly by "surface rubbing." If strange quark matter is absolutely stable (having lower energy per baryon than nuclear matter), strange stars may be bare, consisting of quark matter fluid all the way to the surface. In this case, no surface rubbing takes place. But even in the more likely case, where gas from the supernova explosion or later accretion reaches the surface, the crust formed floats on top of a huge electrostatic potential, separated from the quark surface. This is a result of the strong interactions confining the quarks much tighter than the electrostatic forces holding the electrons, so that electrons create an atmosphere of thickness a few hundred fm [22], effectively separating quark matter from the nuclear crust. Some viscosity results from the interaction between the outer part of the electron atmosphere and the base of the crust, but as illustrated later, due to the low density in the strange star crust relative to that of a neutron star, this effect is only dominant when other sources of viscosity are exponentially suppressed in the case of a color-flavor-locked (CFL) quark phase.

Rapidly rotating millisecond pulsars (periods below 2.5–3 msec) are unstable to the *r*-mode instability for core temperatures in the range of (a few $\times$ 10^5)–(a few $\times$ 10^7) K, if they are quark stars with a normal fluid quark phase. Interestingly, the fastest pulsars known may be just outside this window of instability, reaching it within 10^4 yr due to cooling. When a pulsar reaches the instability window it will slow down by gravitational wave emission on a time scale of 10^4–10^5 yr to a period near 2.5 msec, slowing down further on a much longer time scale due to magnetic dipole braking. As demonstrated below, the *r*-mode spin-down would be characterized by a so-called braking index with an unusually high value of $N \approx 9$, a clear observational indication of the process. Some pileup of pulsar periods near 3 msec, and an underrepresentation of short periods would be expected in this scenario, apparently consistent with the data.

In contrast, quark matter with diquark pairing into a CFL phase [21] would have an exponential reduction in the viscosities, expanding the *r*-mode instability region to encompass low-mass x-ray binaries (LMXB's) as well as many known pulsars, which should then rapidly spin down,

in disagreement with observations. Unless an as yet unconsidered viscous effect could prevent this [23], it seems that these pulsars cannot be quark stars with properties expected for a CFL phase. A 2-flavor color superconducting (2SC) phase [21] has less dramatic consequences, but still seems marginally ruled out by the data. Metastable quark matter in CFL or 2SC phases cannot be ruled out, however, since a hybrid star with quark matter in the interior and nuclear matter in the outer layers, probably with a mixed phase in between, must obey the crust boundary condition as an ordinary neutron star leading to surface rubbing.

The critical rotation frequency for a given stellar model as a function of temperature follows from

$$\frac{1}{\tau_{gw}} + \frac{1}{\tau_{sv}} + \frac{1}{\tau_{bv}} + \frac{1}{\tau_{sr}} = 0, \qquad (1)$$

where $\tau_{gw} < 0$ is the characteristic time scale for energy loss due to gravity wave emission, τ_{sv} and τ_{bv} are the damping times due to shear and bulk viscosities, and τ_{sr} is the surface rubbing time scale. Surface rubbing is decisive for neutron stars [18], whereas $1/\tau_{sr} = 0$ for bare quark stars, and is suppressed by more than 5 orders of magnitude even for strange stars with maximal crust.

Reference [5] used an analytic description of r-mode instability in uniform stars [11] to derive the characteristic time scales for strange stars. A strange star has nearly constant density except for masses very close to the gravitational instability limit, so a polytropic equation of state with a low index, n, provides a very good approximation. The case $n = 0$ corresponding to constant density was discussed in [11], whereas $n = 1$ was studied in [12]. The time scale for gravity wave emission is

$$\tau_{gw} = -[3.26(1.57) \text{ s}](\pi G\bar\rho/\Omega^2)^3, \qquad (2)$$

where prefactors outside (inside) parentheses correspond to $n = 1$ (0), G is the gravitational constant, Ω is the angular rotation frequency, and $\bar\rho$ is the mean density. With shear viscosity coefficient taken from [24], the time scale for shear viscous damping is

$$\tau_{sv} = [5.37(2.40) \times 10^8 \text{ s}](\alpha_S/0.1)^{5/3} T_9^{5/3}. \qquad (3)$$

Here, T_9 denotes temperature in units of 10^9 K, and α_S is the strong coupling.

The bulk viscosity of strange quark matter depends mainly on the rate of $u + d \leftrightarrow s + u$, which is the fastest of the reactions trying to reestablish weak equilibrium between massive strange quarks and the much lighter up and down quarks. To very good approximation the bulk viscosity is given by [25] $\zeta = \alpha T^2/[(\kappa\Omega)^2 + \beta T^4]$, with α and β given in [25]. For the dominant r mode, $\kappa = 2/3$. A low- (high-)T limit is relevant when the first (second) term in the denominator dominates. In cgs units, the low-T limit is [25] $\zeta^{low} \approx 3.2 \times 10^3 m_{100}^4 \rho T^2 (\kappa\Omega)^{-2}$, where m_{100} is the strange quark mass in units of 100 MeV. The high-T limit takes over for $T > 10^9$ K. Here [25],

$\zeta^{high} \approx 3.8 \times 10^{61} m_{100}^4 \rho^{-1} T^{-2}$. For the bulk viscous damping time the approximation in [11] used in [5] has turned out to be too crude, since bulk viscosity coupling to the r modes happens at second order. Lindblom *et al.* [12] reevaluated τ_{bv} for a strange star in the low-T limit and found

$$\tau_{bv}^{low} = (0.886 \text{ s})(\pi G\bar\rho/\Omega^2)T_9^{-2} m_{100}^{-4}. \qquad (4)$$

The prefactor here is 7 times smaller than used in [5], and the scaling with Ω is opposite, resulting in some changes in the results, though not in the conclusions of [5]. But now also the high-T limit becomes important. In this limit (not considered in [5,12])

$$\tau_{bv}^{high} = (0.268 \text{ s})(\pi G\bar\rho/\Omega^2)^2 T_9^2 m_{100}^{-4}. \qquad (5)$$

Because τ_{bv}^{high} increases with temperature, it can lead to r-mode instability for very high T.

Figure 1 shows the regions of r-mode (in)stability in a plot of pulsar spin frequency [$\nu \equiv \Omega/(2\pi)$] versus temperature for a strange star with mass $M = 1.4M_\odot$ and radius $R = 10$ km. An $n = 1$ polytrope was assumed, but conclusions are not changed for uniform density. Also indicated are the positions of LMXB's, presumed to be old pulsars being spun up by accretion to eventually become rapid millisecond pulsars, as well as the positions of the two most rapidly spinning pulsars known, with periods of 1.5578 and 1.6074 msec ($\nu \approx 642$ and 622 Hz). It should be stressed that the core temperatures are uncertain upper limits derived from x-ray limits on the surface temperatures, increased by roughly 2 orders of magnitude to include effects of an insulating crust. The actual numbers are taken from [18], valid for a neutron star model, but
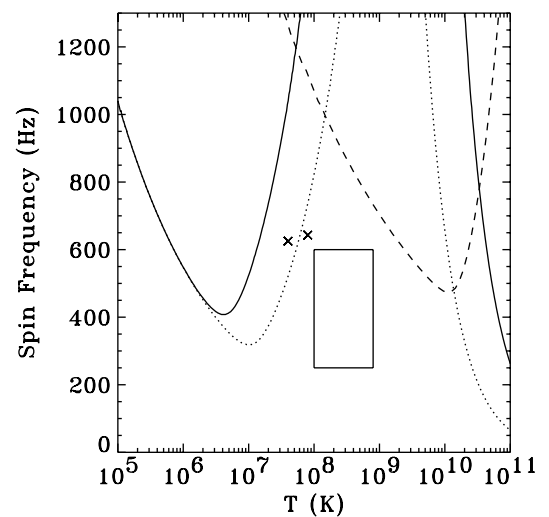

FIG. 1. Critical spin frequencies in Hz for strange stars as functions of temperature. Solid (dotted) curves assume $m_{100} = 2$ (1). The r-mode instability is active above the curves. The box shows characteristic positions of LMXB's, and crosses upper bounds on core temperatures for the two fastest known pulsars. Dashed curve marks the r mode instability domain for neutron stars [18].

similar limits apply to strange stars with a significant crust. Bare strange stars or stars with a thin crust would have a core temperature close to the surface temperature, moving them closer to or even inside the region of r-mode instability. Completely bare strange stars are very poor emitters of radiation below the quark matter plasma frequency of 20 MeV [22], but even a thin crust/atmosphere would allow normal thermal radiation. For simplicity both categories are denoted "bare," but conclusions based on surface temperatures relate only to strange stars with a tiny layer of surface pollution.

One notes that the LMXB's are well within the region stable against r-mode instabilities, allowing them to accrete and speed up unhindered by the instability. The rapid pulsars are also apparently in the stable regime (at least for $m_{100} = 2$), but as the time scale for cooling to 10^7 K is only around 10^4 yr [26], they should soon enter the unstable region and start spinning down. Since $\tau_{gw} \ll \tau_{cool}$, the pulsars should follow a track indistinguishable from the curve marking the instability region, corresponding to an unusually high braking index of $N \approx 9$. This value follows because $\Omega \propto T^{1/2} \propto t^{-1/8}$ (the latter comes from $t \propto \tau_{cool} \approx (10^{-4}$ yr$)T_9^{-4}$ for standard neutrino cooling of a quark star [26]). Thus $\dot{\Omega} \propto -\frac{1}{8}t^{-9/8}$, $\ddot{\Omega} \propto \frac{1}{8}\frac{9}{8}t^{-17/8}$, and the braking index $N \equiv \Omega\ddot{\Omega}/\dot{\Omega}^2 = 9$. The star will reach a spin frequency of 400 Hz (2.5 msec rotation period) within a cooling time scale of 10^5 yr. Notice that pulsars with the highest spin frequencies need less time to reach the region of instability. This may explain why no frequencies above 642 Hz have been observed, and the spin-down to 2.5–3 msec by the r-mode instability may lead to some clustering of observed rotation periods around this value (not inconsistent with data, but statistics are not overwhelming due to a low number of objects). No similar effects arise in the case of ordinary neutron stars, where r-mode instabilities only seem to work at frequencies above 500 Hz, and then mainly for $T \approx 10^{10}$ K (dashed curve in Fig. 1).

Notice that the agreement with pulsar data remains valid only if an insulating crust allows the bulk temperature of the pulsar to be some 2 orders of magnitude higher than the observed upper limits on the surface temperature (about 6×10^5 K and 9×10^5 K for the pulsars plotted) to locate the pulsars to the right of the r-mode instability range. A position inside or to the left of this regime seems ruled out (the latter because pulsars spun up in the LMXB domain would have to cross the instability regime before reaching a position to the left). Therefore, strange stars without a significant crust (having comparable surface and bulk temperatures) are ruled out as models for these rapid pulsars unless they are completely bare and therefore hidden in x rays.

Superfluidity in the quark phase completely changes the behavior. If quark pairing is characterized by an energy gap, Δ, reaction rates involving two quarks (as relevant for bulk as well as shear viscosities) are suppressed by a factor $\exp(-2\Delta/T)$, assuming equal behavior for all quark flavors, as expected in a high density color-flavor-locked phase. This increases the bulk viscous time scale by $\exp(2\Delta/T)$ and τ_{sv} (including screening) by $\exp[\Delta/(3T)]$, significantly increasing the parameter space where the r-mode instabilities are active. In fact at low T the viscosity is now determined by shear due to electron-electron scattering or by surface rubbing. The time scale for electron shear is $\tau_{sv}^{ee} \approx (2.95 \times 10^7 \text{ s})(\mu_e/\mu_q)^{-14/3}T_9^{5/3}$. In Fig. 2 (dashed curve) the effect of electron shear is maximized, using a very high $\mu_e/\mu_q = 0.1$. Surface rubbing due to the electron atmosphere being carried along by the r modes in the quark phase, scattering mainly on phonons in the nuclear crust, corresponds to a viscous time scale $\tau_{sr} \approx (1.42 \times 10^8 \text{ s})T_9(\nu/1 \text{ kHz})^{-1/2}$ for a crust with maximal density [27] (dash-dotted curve). For lower crust base density, the effect of surface rubbing is reduced further. Figure 2 shows r-mode instabilities in strange stars dominated by the CFL phase with $\Delta = 1$ MeV. Much higher energy gaps (50–100 MeV) are expected in recent studies of color superconductivity [21], but as seen from Fig. 2, even a value of 1 MeV is incompatible with pulsar data, since basically all rapid pulsars are located in the unstable regime, and therefore should spin down by gravitational wave emission in a matter of hours, cf. τ_{gw}, clearly in contradiction with the facts.

At lower density, the high mass of the s quark relative to u and d prevents creation of the CFL phase; instead two color states of u and d may pair, creating a 2SC phase that introduces energy gaps for four out of nine quark color-flavor states. If the corresponding energy gap is of any significance, the states with a gap can be safely ignored compared to the remaining unpaired s quarks and one color of u and d. This reduces the rate of the weak reaction $u + s \leftrightarrow d + u$ by a factor of 1/9, increasing τ_{bv} by a
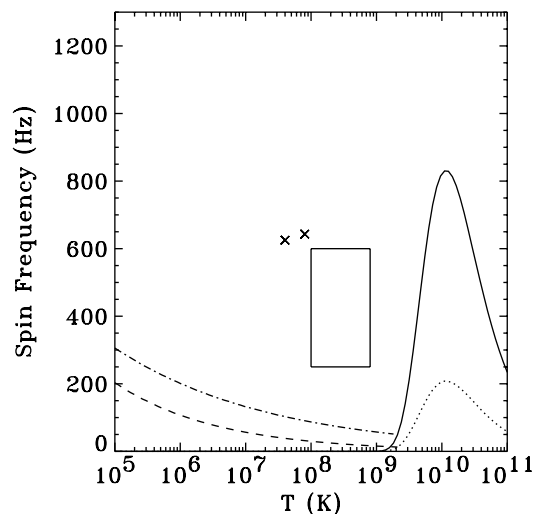

FIG. 2. As Fig. 1, but assuming quark matter in a CFL phase with energy gap $\Delta = 1$ MeV. Dashed curve results from electron shear; dash-dotted curve from surface rubbing for a maximal crust. Quark stars above the curves are r mode unstable, in clear disagreement with pulsar data.
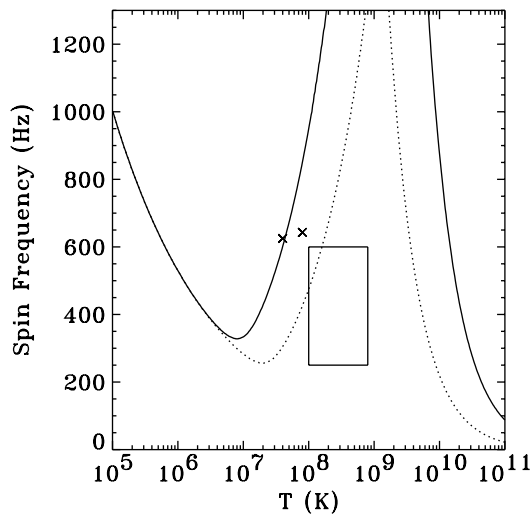
FIG. 3.   As Fig. 1, but assuming a 2SC state.

factor of 9. The strong scattering rates responsible for the shear viscosity are reduced by $(5/9)^{1/3}$, thus increasing τ_{sv} by $(9/5)^{1/3}$. This expands the domain of r-mode instability as shown in Fig. 3 to an extent where some rapid pulsars are in the unstable zone, in disagreement with data. It is fair to say, though, that the uncertainties and approximations involved may be large enough that 2SC quark matter stars may not be ruled out completely.

The r-mode instability thus provides several interesting tests of the hypothesis of stable quark matter stars (strange stars). If strange quark matter is absolutely stable, pulsars would be expected to consist of quark matter. Data on pulsar rotation properties are consistent with this if the quark matter is nonsuperfluid (but only for strange stars with a thick crust or completely bare strange stars). The lack of observed very rapid pulsars may be due to the r-mode instability, and rapid pulsars reaching the region of instability will spin down in a characteristic manner that can be tested by observations.

Strange stars in a color-flavor-locked phase are, in contrast, not permitted by pulsar data. Most rapid pulsars would be r mode unstable, and should spin down within hours, which clearly they do not. So if strange quark matter is stable, it may be concluded that a CFL phase, and probably a 2SC phase as well, is not reached at densities relevant in pulsars, i.e., up to a few times nuclear density. These arguments do not rule out a color superconducting phase at such densities if quark matter is only metastable, because then a pulsar, even if it contains quark matter, will not have the separation of the crust characteristic of a strange star. Thus, such a star is susceptible to the full surface rubbing effect, and will not be r mode unstable to a similar degree. A detailed study of r modes in such hybrid stars would be interesting.

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