First Experimental Evidence for Chaos-Assisted Tunneling in a Microwave Annular Billiard

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We report on first experimental signatures for chaos-assisted tunneling in a two-dimensional annular billiard. Measurements of microwave spectra from a superconducting cavity with high frequency resolution are combined with electromagnetic field distributions experimentally determined from a normal conducting twin cavity with high spatial resolution to resolve eigenmodes with properly identified quantum numbers. Distributions of quasidoublet splittings serve as basic observables for the tunneling between whispering gallery-type modes localized to congruent, but distinct tori which are coupled weakly to irregular eigenstates associated with the chaotic region in phase space.

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For two decades a new kind of tunneling mechanism has produced great interest, since it demonstrates how the dynamical features of a classical Hamiltonian system effect the behavior of its quantum counterpart [1–3]. This so-called "dynamical tunneling" occurs whenever a discrete symmetry of the system leads to distinct but symmetry related parts of the underlying classical phase space. In contrast to the well-known barrier tunneling, dynamical tunneling depends only upon the probability for a quantum particle, although classically forbidden, to leave certain regions of phase space and travel into others. This basically involves the coupling strength between distinct phase space regions. In the special case of *two* symmetry related *regular* regions separated by a *chaotic* area in a mixed phase space, semiclassical quantization yields pairs of quantum states which are localized to the corresponding sets of congruent, but distinct tori. These so-called quasidoublets show a very sensitive splitting behavior which depends upon the coupling to irregular eigenstates associated with the intermediate chaotic sea. This nondirect, enhanced coupling of regular eigenmodes via chaotic ones is what defines chaos-assisted tunneling [4–6] in its original sense [7].

The aim here is to demonstrate for the first time that chaos-assisted tunneling can be observed experimentally, even for a case where the size of the splitting is several orders of magnitude below the typical mean level spacing of the system. For this purpose we performed-measurements on superconducting as well as normal conducting microwave cavities constituting a special family of Bohigas' annular billiard [8–10]. This system has been proven in very extensive computer simulations, especially in Refs. [8,9], to be a paradigm for chaos-assisted tunneling and provides access for experimental investigation.

The two-dimensional geometry of the annular billiard is defined by two circles of radius r, respectively, R, the latter being set to unity, and the center displacement or eccentricity δ; see the left side of Fig. 1. In the following, only the special one-parameter family $r + \delta = 0.75$ will be considered, since it provides all the features which are relevant for chaos-assisted tunneling: From the classical point of view the system shows a transition from integrable ($\delta = 0$) to mixed behavior ($\delta > 0$), thus developing a growing chaoticity with increasing δ. Furthermore, the discrete reflection symmetry leads to congruent but classically distinct regions in phase space.

To demonstrate this, Fig. 1 (right side) shows a typical Poincaré surface of section for the configuration ($\delta = 0.20/r = 0.55$). Here the area preserving Birkhoff coordinates L (point of impact on the outer circle) and $S = \sin\alpha$ (angular momentum of the billiard particle) have been used. Besides a large chaotic sea with chains of stable islands in the center, both of which are influenced by a change of the eccentricity δ, two symmetry related but distinct neutrally stable coastal regions for $|S| > 0.75$ can be observed. Per construction of the family $r + \delta = 0.75$ these regular regions are invariant under variations of δ, since the corresponding trajectories do not hit the inner circle. As a consequence S is conserved, indicated by horizontal lines in the surface of section. Those lines correspond to two so-called whispering gallery trajectories [8]. From this, the only difference between the two distinct regular regions is the sign of S, i.e., the sense of motion for the propagating particle.

The fundamental question now accounts for the quantum counterpart of the classically forbidden transport between the distinct coastal regions: dynamical tunneling. Since the coupling between both regions crucially depends upon the topology and the size of the chaotic sea, the system is, in particular, adequate to study chaos-assisted tunneling. But what is the basic observable for the tunneling strength in the corresponding quantum system? To answer this it is very instructive to start with the integrable case ($\delta = 0$). Solving the Schrödinger equation with Dirichlet boundary conditions leads to eigenvalues $k_{n,m}$ and eigenstates $\Psi_{n,m}$, with the angular momentum quantum number n and the radial quantum number m. Because of Einstein-Brillouin-Keller quantization [3,8] the property $S = n/k_{n,m}$ is the quantum angular momentum which has to be compared with the classical $S = \sin\alpha$ in

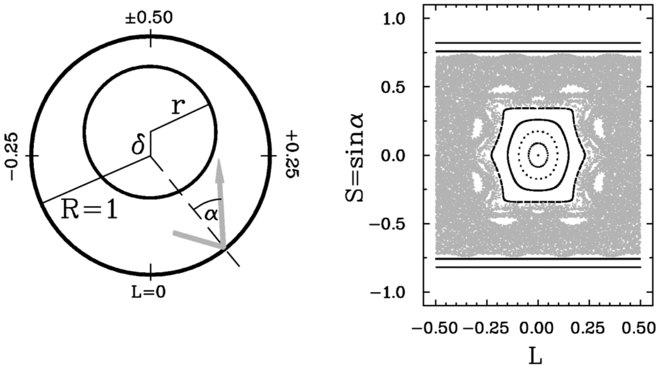

FIG. 1. The annular billiard for $\delta = 0.20$ and $r = 0.55$ (left side) together with the corresponding Poincaré surface of section (right side). Besides a large chaotic sea with stable islands in the center the phase space clearly displays two symmetry related but dynamically distinct regular coastal regions for $|S| > 0.75$. Two examples of horizontal lines corresponding to whispering gallery trajectories (clockwise as well as counterclockwise) are shown.

order to find the location of a certain quantum state in the phase space. While in the classical system the reflection symmetry of the billiard leads to *two* distinct but related regular regions with opposed sense of motion for the propagating particle (i.e., the whispering gallery trajectories clockwise and counterclockwise); the corresponding quantum eigenstates are organized in doublets for $\delta = 0$ with *two* parities, even and odd, respectively. However, continuously increasing the eccentricity δ systematically destroys this doublet structure, yielding singulets for states with $S = n/k$ right within the chaotic sea ($S < 0.75$) and quasidoublets on the remaining regular coast ($S > 0.75$). As in the case of the well-known double-well potential [1,2,5] the very small splitting of those quasidoublets is directly determined by the classically forbidden tunneling, thus presenting a very effective observable for the hardly accessible tunneling strength. The location of a certain quasidoublet on the regular coast (defined by $S = n/k$), as well as the transport features of the chaotic sea (defined by δ), has a direct impact on the splitting and is investigated systematically.

As in earlier studies (for an overview, see [11]), we simulated the quantum billiard by means of a two-dimensional electromagnetic microwave resonator of the same shape [left-hand side (lhs) of Fig. 1]. The measurements were divided into two parts: Taking in total three different configurations of the family $r + \delta = 0.75$ (i.e., $\delta = 0.10$, 0.15, and 0.20), we performed on one hand experiments with a superconducting niobium resonator (scaled to $R = \frac{1}{8}$ m) at 4.2 K in order to measure quasidoublet splittings within the frequency range up to $f = 20$ GHz. The very high quality factor of up to $Q \approx 10^6$ allows a resolution of $\Gamma/f \approx 10^{-6}$, where Γ is equal to the full width at half maximum of a resonance. For demonstration, Fig. 2 shows transmission spectra of the resonator in the vicinity of 9 GHz. Besides several singulets exactly one quasidoublet can be observed which shows a small but systematic displacement with δ. In all cases the very small splitting of the quasidoublet is clearly detectable. This is due only to the high resolution of the superconducting resonator. It is important to note that the position of the exciting antennas has to be chosen carefully in order to minimize the perturbation on the whispering gallery modes in the coastal region (Fig. 1) and thus not to influence the size of their physical quasidoublet splitting. Using antennas right within the whispering gallery region of the billiard (sketch at the top of Fig. 2) always produces "false" splittings even for the concentric system ($\delta = 0$) with twofold degenerate states. We therefore used antennas in the shadow region of the inner circle also preserving the symmetry of the whole geometry; cf. Fig. 2.

For a proper identification of the quantum numbers $(n\,|\,m)$ of the modes associated with the quasidoublets on the other hand we used a normal conducting copper twin of the niobium billiard cavity. There we measured the corresponding wave function, respectively, electromagnetic field distributions, from which the relevant quantum numbers n and m could be deduced, even if they are far from being "good quantum numbers" in the given eccentric systems which are nonintegrable. This second part of the experiment was based on a field perturbation method originally introduced in accelerator physics and used successfully in billiard research before [12,13]. According to Slater's theorem [14] a small metallic body inside the cavity locally interacts with the electromagnetic field in such a way that a frequency shift of the excited mode results from the compensation of the nonequilibrium between the totally stored electric and magnetic field energy. This shift

$$\partial f = f_0 - f = f_0(a\vec{E}_0^{\,2} - b\vec{H}_0^{\,2}) \qquad (1)$$

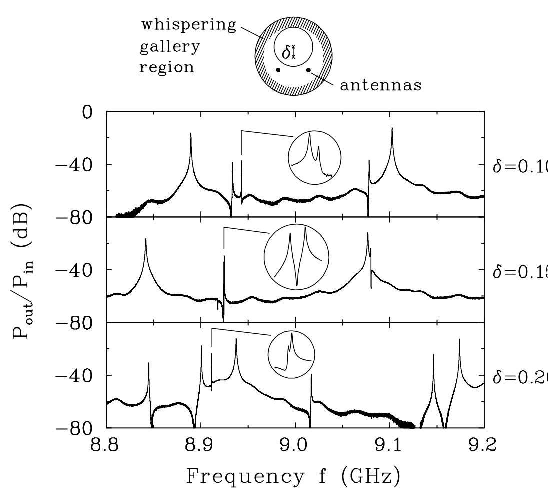

FIG. 2. Transmission spectra around 9 GHz with varying δ. Among several singulets exactly one quasidoublet, slightly moving with δ, can be observed. In the zooming circles the abscissa is stretched by a factor of 50 in order to visualize the quasidoublet splitting. Spectra at $\delta < 0.10$ could not be realized with the present setup of antenna locations.

with respect to the unperturbed mode (index 0) directly depends upon the superposition of the squared electric and magnetic fields, $\vec{E}_0$ and $\vec{H}_0$, respectively. Since the quantum wave function is related to the electric field only, the magnetic component has to be removed by a proper choice of the geometry constants a and b in Eq. (1) by choosing needlelike bodies (1.84 mm in length and 1.00 mm in diameter). Moving the body across the whole two-dimensional surface of the billiard with a spatial resolution of about one-tenth of a wavelength by means of a guiding magnet and detecting ∂f at each position finally provides the complete field distribution. Examples of those for the configuration $\delta = 0.20$ in the vicinity of 9 GHz are plotted in the upper part of Fig. 3. Of the three distributions only the middle one with the quantum numbers $n = 18$ (36 field maxima in the polar direction) and $m = 1$ (one field maximum in the radial direction) is characteristic for the modes which are localized in the whispering gallery region (Fig. 2). Contrary to this the distributions on the lhs and on the right-hand side (rhs) show a totally different pattern. The parity of the distributions is determined as follows: If there is maximum field strength on the line which defines the reflection symmetry of the billiard, positive parity can be assigned to the mode, likewise negative parity for zero field strength on this line.

Underneath the squared electric field strength distributions in Fig. 3, the corresponding transmission spectrum taken at 300 K with the normal conducting copper cavity is shown. The three broad resonances associated with the field distributions are much better resolved in the measurement at 4.2 K of the superconducting niobium twin cavity. The small displacement in frequency of the resonances in the two measurements is due to mechanical imperfections of each individual cavity and positioning errors of the respective inner circles within the resonators. Mechanical uncertainties of order ± 100 μm relative to the radius of the outer circle $R = 125$ mm are sufficient to account for the observed displacements. The spectrum at 4.2 K in Fig. 3 shows that the resonance magnified in the inset is one of the expected quasidoublets characteristic for chaos-assisted tunneling. Naturally, this quasidoublet is not resolved in the spectrum taken at room temperature, and the field distribution in the upper part of Fig. 3 proves that of the two modes corresponding to the doublet in the particular case considered the one with negative parity is excited stronger than the other.

Calculating the quantum angular momentum $S = n/k$ (with $k = 2\pi R f / c_0$, where f denotes the centroid frequency of the quasidoublet and c_0 denotes the speed of light) finally yields the position of the whispering gallery-type mode on the corresponding classical surface of section; see the rhs of Fig. 1. In the case of mode $(18 \mid 1)$ one obtains $S \approx 0.77$ characteristic for a mode in the so-called "beach region" [9] defined by the borderline $S = 0.75$ between the chaotic sea and the regular coast, respectively.

This comparison demonstrates that the measurements combine the high spatial resolution of about $\lambda/10$ for the normal conducting billiard with the high frequency resolution of about $1/Q$ for the superconducting one, thus allowing a very effective classification of regular quasidoublets as well as chaotic singulets in the range up to approximately 14 GHz, where the splittings become smaller than the resonance widths of the superconducting resonator. The difference in frequency between the peaks of each quasidoublet was estimated through a nonlinear fitting to "skew Lorentzians" [see Eq. (4) in [15]].

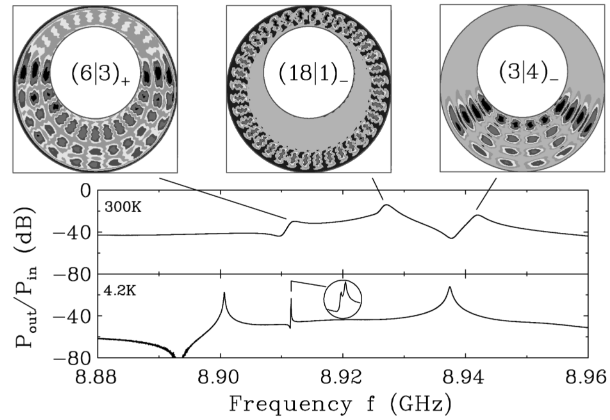

FIG. 3. Matching of field strength plots taken at 300 K and microwave spectrum taken at 4.2 K. The combined normal/superconducting setups allow one to measure highly resolved quasidoublets including quantum numbers $(n \mid m)$ and parity.

Now, we consider only the family with quantum numbers $(n \mid 1)$, since it consists of some 30 undoubtedly identified quasidoublets. To uncover effects due to chaos-assisted tunneling, the splitting of a certain quasidoublet has to be analyzed as a function of its corresponding position in the classical phase space. As mentioned above, this position might be expressed in terms of $S = n/k$ as the quantized analog of the classical $S = \sin\alpha$, directly representing the very location in the underlying surface of section. The resulting curve, again for the configuration $\delta = 0.20$, is shown in the lower part of Fig. 4.

Here, the distribution of normalized splittings $|\Delta f / f|$ shows a smooth transition from chaotic states defined by large splittings right within the chaotic sea ($S < 0.75$) to regular quasidoublets with very small splittings in the classical coastal region ($S > 0.75$). Also the measured field distributions show an increasing regularity with growing S, as can be seen from the examples in the upper part of Fig. 4. Especially mode $(29 \mid 1)$ is hardly distinguishable from the corresponding concentric mode ($|\Delta f / f| = 0$) although the system is strongly eccentric.

Besides this global behavior a first strong signature of chaos-assisted tunneling can be observed in the particular shape of the splitting curve: In the direct vicinity of the beach at $S = 0.75$ the quasidoublets show a locally enhanced splitting amplitude, thus indicating a very effective coupling between the regular coast and the chaotic sea. As described above, this corresponds to a locally enhanced

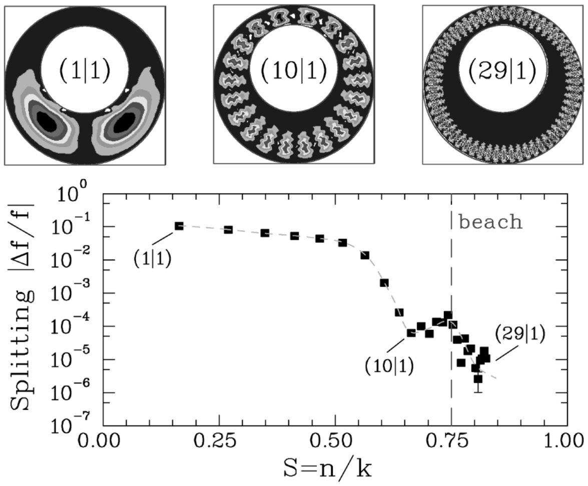

FIG. 4. Distribution of normalized splittings with respect to the position of a certain quasidoublet in classical phase space. The error on the splitting is generally smaller than the size of the squares except for the smallest splitting observed. Besides a smooth transition from states with a large splitting right within the chaotic sea ($S < 0.75$) towards states with a very small splitting on the regular coast ($S > 0.75$) a local maximum occurs in the direct vicinity of the beach, representing a very impressive signature for chaos-assisted tunneling.

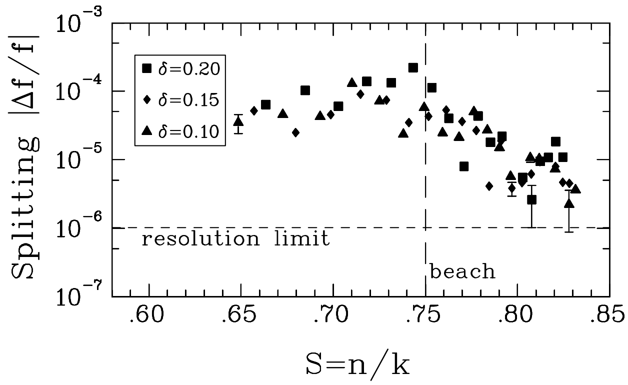

FIG. 5. Distribution of normalized splittings in the beach region for different eccentricities δ. The local maximum is rebuilt by all measured quasidoublets forming a systematically rising and a randomly falling part below and above $S = 0.75$.

tunneling strength in the beach region as theoretically predicted in [9].

To evaluate the influence of the chaoticity of the system, Fig. 5 shows a direct comparison of the splittings for all eccentricities δ in the vicinity of the beach at $S = 0.75$. Note that the splittings for different δ not only enhance the visibility of the maximum, they furthermore reveal an additional feature of chaos-assisted tunneling: While splittings on the rising left part of the maximum, i.e., within the chaotic sea ($S < 0.75$), are distributed quite systematically (e.g., the data points of $\delta = 0.20$ always correspond to the largest splittings) show the falling right part large fluctuations for a given eccentricity δ. This effect is also theoretically predicted [5,8,9] and accounts for the high rate of anticrossings with chaotic modes for high angular momenta S. Thus on the rhs of $S = 0.75$ the tunneling strength shows a very random dependence on the eccentricity δ leading to strong fluctuations in the distribution of splittings. Finally, for even larger values of S the splitting amplitudes are of the order of the inverse quality factor, $\Delta f/f \approx 1/Q \approx 10^{-6}$, defining the resolution limit of the present setup.

In summary, we have presented first experimental signatures for chaos-assisted tunneling in a billiard. As the basic observable we have investigated the splittings of quasidoublets with respect to their position in the classical phase space and their dependence on the eccentricity δ. A local maximum in the vicinity of the beach region with a systematically rising and randomly falling part has been found which directly reflects the enhanced tunneling strength at this critical location between the regular coast and the chaotic sea. In this context, a combined experimental setup using normal as well as superconducting billiards has offered a very effective tool for measuring highly resolved quasidoublets with properly identified quantum numbers.

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