Quark Dispersion Relation and Dilepton Production in the Quark-Gluon Plasma

André Peshier[1] and Markus H. Thoma[2]
[1]*Forschungszentrum Rossendorf, PF 510119, 01314 Dresden, Germany*
[2]*Institut für Theoretische Physik, Universität Giessen, 35392 Giessen, Germany*
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Under very general assumptions we show that the quark dispersion relation in the quark-gluon plasma is given by two collective branches, of which one has a minimum at a nonvanishing momentum. This general feature of the quark dispersion relation leads to structures (van Hove singularities, gaps) in the low mass dilepton production rate, which might provide a unique signature for the quark-gluon plasma formation in relativistic heavy ion collisions.

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Relativistic heavy ion experiments at SPS, RHIC, and LHC are and will be undertaken in order to produce a new state of matter, the so-called quark-gluon plasma (QGP). This deconfined phase of quarks and gluons in or close to thermal equilibrium might be present in the hot and dense fireball of such nucleus-nucleus collisions for about 10 fm$/c$ [1]. The biggest problem in the discovery of the QGP is to find a clear signature for its formation [2]. Hadrons arriving in the detectors carry mainly information about the hadronic phase following the QGP in the expansion of the fireball. Thermal photons, on the other hand, are emitted from the fireball without any further interaction [3]. Therefore they are a unique tool to test the QGP phase. In particular, virtual photons, which decay into lepton pairs (e^+e^-, $\mu^+\mu^-$), could serve as a promising signal for the QGP formation [4]. For this purpose one has to calculate the dilepton spectrum from a fireball with and without phase transition and to compare it with the experiments. Unfortunately, the dilepton production rate from the QGP seems to be similar to the one from a hadron gas at the same temperature [5]. However, this observation (duality) is based on the assumption that the dilepton production in the QGP arises solely from annihilation of bare quarks (Born term) [6]. As we will argue in the following, medium effects will change the quark dispersion relation in the QGP in a way that sharp structures (singularities, gaps) arise in the production rate of low mass dileptons. These structures might provide a clear and unique signature for the presence of deconfined, collective quarks in the fireball.

The starting point of our investigation is the most general expression for the self-energy of fermions in the chiral limit of vanishing current mass [7], which in the rest frame of the medium has the form [8]

$$\Sigma(P) = -a\not{P} - b\gamma_0 = -(ap^0 + b)\gamma_0 + a\mathbf{p}\cdot\gamma\,. \tag{1}$$

The scalar quantities a and b are functions of the energy p^0 and the magnitude p of the three-momentum, with certain properties to be studied below. The fermion dispersion relation is given by the position of the poles of the exact propagator $S = (S_0^{-1} - \Sigma)^{-1}$. Decomposed into helicity eigenstates, with $\hat{\mathbf{p}} = \mathbf{p}/p$, the propagator reads [9]

$$S(P) = \frac{\gamma_0 - \hat{\mathbf{p}}\cdot\gamma}{2D_+(P)} + \frac{\gamma_0 + \hat{\mathbf{p}}\cdot\gamma}{2D_-(P)}\,. \tag{2}$$

The zeros $\omega_\pm(p)$ of

$$D_\pm = -p^0 + T_0 \pm (1 + a)p\,, \qquad T_0 = -(ap^0 + b)\,, \tag{3}$$

describe the propagation of particle excitations q_+ with energy ω_+ and of a mode q_- called plasmino [9] with energy ω_- and negative ratio of chirality to helicity. The latter is a consequence of the medium, breaking the Lorentz invariance of the vacuum. Invariance under charge conjugation implies that $-\omega_\pm(p)$ are also solutions of (3) [10]. In the following, we focus on the solutions with positive energy.

Considering quark excitations in the QGP, we emphasize the existence of a real part of the solutions of the exact (implicit) dispersion relations

$$\omega_\pm(p) = T_0 \pm (1 + a)p \tag{4}$$

according to the assumption of a deconfined phase. Furthermore we note that at large momentum the breaking of the Lorentz symmetry is less manifest, hence $|b| \ll |ap^0|$ in the self-energy (1), and the dispersion relations are expected to approach the light cone. While this is evident for the particle branch ω_+, we speculate that this behavior also holds for the plasmino branch. In the two different approximations for the in-medium quark propagator, discussed below, the dispersion relations (4) in the form $\omega_\pm = f(\omega_\pm, p) \pm p$ are determined by the function $f = -b/(1 + a)$, having a spacelike pole close to the light cone. Taking this as a general feature, the plasmino branch indeed approaches the light cone for large momenta as well.

At vanishing p, due to isotropy, the self-energy (1) is independent of the direction $\hat{\mathbf{p}}$ of the momentum, thus $ap \to 0$ for $p \to 0$. Consequently, both excitations have the same rest energy [11], which we will refer to as an effective mass, $\omega_\pm(0) = m_q^*$. According to (4), the slopes of the branches of the dispersion relation are implicitly given by

$$\omega'_\pm = \dot{T}_0\omega'_\pm + T'_0 \pm [1 + a + (\dot{a}\omega'_\pm + a')p], \quad (5)$$

where the derivatives $\partial/\partial p^0$ and $\partial/\partial p$ were denoted by dots and primes, respectively. As we will show in the following, several terms in (5) vanish for small momenta due to isotropy properties of Green's functions, similar to the argument applied above for the self-energy. Using the differential Ward identity in a ghost-free gauge [12], the quark-gluon vertex is related to the quark propagator by $\Gamma_\mu(P, P' \to P) = \partial S^{-1}/\partial P^\mu$ [13]. The isotropy of the temporal and spatial components of Γ_μ at vanishing momentum results from

$$\dot{a}p \to 0, \qquad a'p^0 + b' \to 0, \qquad a'p \to 0 \quad \text{for } p \to 0\,, \quad (6)$$

which leads to

$$\omega'_\pm(0) = \pm\frac{1 + a(p_0 = m_q^*, p = 0)}{1 - \dot{T}_0(p_0 = m_q^*, p = 0)}. \quad (7)$$

Evidently, the initial slopes of both branches are opposite [14] and, as can also be derived from the regularity properties (6), neither vanishing nor infinite. From (7) we conclude that one branch of the chiral quark dispersion relation (the plasmino mode, as it turns out) has a minimum before it approaches the light cone.

This behavior has been found in perturbative as well as nonperturbative approximations of the quark propagator. In the perturbative calculation [9] (hard-thermal-loop approximation) the effective quark mass is given by $m_q^* = gT/\sqrt{6}$ and the plasmino branch shows a minimum at $p_{\min} = 0.408m_q^*$ before approaching the light cone. The spectral strength of the plasmino branch (i.e., the residue of the pole of the propagator) vanishes exponentially close to the light cone, which shows that plasminos are purely collective excitations. In the case of a finite current quark mass there is a splitting of the two dispersion relations at zero momentum and the minimum of the plasmino branch vanishes for large current masses [15]. Similar dispersion relations were found in a nonperturbative calculation [16] of the quark propagator based on the presence of a gluon condensate, which has been measured in lattice QCD simulations, in the QGP. They differ from the perturbative results by an effective quark mass $m_q^* = 1.15T$ between the critical temperature T_c and $4T_c$ and a powerlike vanishing spectral strength of the plasmino mode close to the light cone. As an example, the quark dispersion relation following from the nonperturbative approach is shown at $T = 2T_c$ in Fig. 1.

Now we will discuss the consequence of this quark dispersion relation on the production of lepton pairs from the QGP. Virtual photons with invariant mass $M = \sqrt{k_0^2 - k^2}$ are emitted either by an electromagnetic transition from the upper to the lower branch, $q_+ \to q_-\gamma^*$, or by annihilation, e.g., $q_-\bar{q}_- \to \gamma^*$. At vanishing momentum of the virtual photon, $k = 0$, the first process yields a contribution to the dilepton production rate, which starts at $M = 0$ and terminates at the maximum difference of $E(p) = \omega_+(p) - \omega_-(p)$ of the two branches. There we encounter a van Hove singularity which is caused by the divergence of the density of states, which is proportional to $[dE(p)/dp]^{-1}$ [17]. For larger M, there is a gap in the dilepton rate before the channel from plasmino annihilation opens at the threshold $M = 2\omega_-(p_{\min})$ with another van Hove singularity originating from the minimum in the plasmino dispersion. At $M = 2m_q^*$, the annihilation of collective quarks, $q_+\bar{q}_+ \to \gamma^*$, sets in. This contribution dominates at large invariant mass, where the plasmino contribution is suppressed, and finally approaches the Born term.

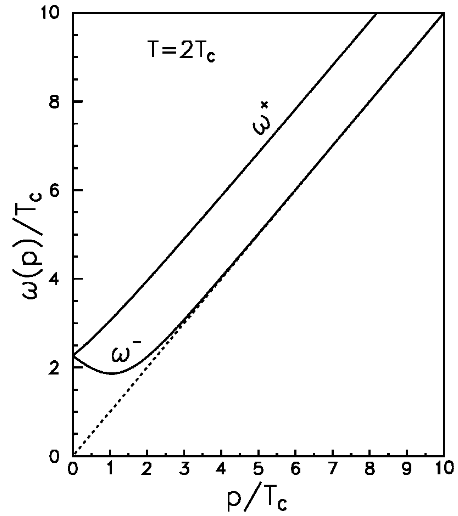

FIG. 1. Example for the quark dispersion relation in the QGP based on a quark propagator containing the gluon condensate [16].

As an illustration, the dilepton rate following from the imaginary part of the photon self-energy using the effective quark propagators that contain the gluon condensate as calculated in [16], corresponding to the dispersion relation of Fig. 1, is shown for $T = 2T_c$ in Fig. 2. The same structures of the spectrum are observed in perturbative calculations based on the hard-thermal-loop resummation technique [9,18].

The validity of perturbative calculations as [9] assuming $g \ll 1$ is doubtful at temperatures within reach of the experiments, and the reliability of phenomenological nonperturbative estimates, as the one discussed in [16], is difficult to control. Fortunately, the appearance of a distinct structure in the dilepton rate is a general feature independent of the approximation assumed for the quark propagator, as we have argued. The effective quark mass m_q^* is expected to be of the order of 0.5 GeV in the temperature regime under consideration, as can be deduced by comparing lattice

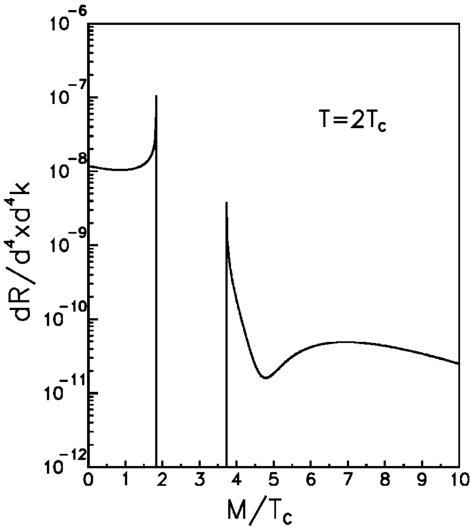

FIG. 2. Example for the dilepton production rate at zero photon momentum following from the quark dispersion of Fig. 1 in the QGP [16].

calculations for the equation of state of the QGP with an ideal gas model of quasiparticles, i.e., quarks and gluons with effective temperature dependent masses [19]. Therefore we predict that these structures manifest themselves in the dilepton spectrum for invariant masses of around 1 GeV and below.

If, contrary to the general expectation, the plasmino branch were a monotonically decreasing function of the momentum, which intersects the light cone, van Hove singularities in the dilepton rate would be absent. Nevertheless, even in this case there would be a significant enhancement of the rate at small momentum compared to the Born rate due to the transition $q_+ \rightarrow q_- \gamma^*$. The behavior of the plasmino branch, however, can possibly be studied in lattice simulations of the quark propagator at finite temperature [20]. The anticipated existence of the plasmino minimum could, hence, be corroborated by such *ab initio* calculations which also allow one to consider nonzero current quark masses.

Whether the structures in the dilepton spectrum can be experimentally observed or not is a difficult question. Definitely they will be smeared out and, at least partially, covered by various effects. First of all, there will be smooth contributions to the dilepton rate such as Compton scattering, $q\bar{q}$ annihilation with gluon emission, and bremsstrahlung, which might cover up the van Hove singularities. In the hard-thermal-loop calculation these processes are represented by cut contributions of the photon self-energy, which are due to the imaginary part of the hard-thermal-loop quark self-energy entering the resummed quark propagator. In the gluon condensate calculation, on the other hand, cut contributions are not present due to the fact that the quark self-energy is real in this case. In our general consideration, however, for covering up the van Hove peaks by smooth higher order contributions completely, these contributions also had to dominate clearly over the Born term and therefore, as well, over the hadronic contributions according to Ref. [5]. If this is not ruled out already by SPS data [21], a strong enhancement of the low mass dilepton rate caused by cut and higher order QCD contributions could serve itself as a signal for the QGP formation.

Furthermore, the sharp structures will be smeared out by contributions to the dilepton spectrum at finite photon momentum [22] and due to damping effects, which we have not touched in our discussion of the real parts of the poles. Another smoothing of these structures originates from the space-time evolution of the fireball, with which the rate has to be convoluted in order to extract the dilepton spectrum. To what extent the sharp structures in the dilepton rate, coming from the in-medium quark dispersion relation, will survive is an open question. After all it will be worthwhile looking for new structures in the spectrum of low mass dileptons with small transverse momentum. Such structures could not be seen at SPS [21] because the contributions from the QGP phase to the dilepton spectrum are estimated by hydrodynamical calculations [23] to be 1 or 2 orders of magnitude below the data due to the small lifetime of the QGP. At RHIC and LHC, however, where the QGP phase is expected to dominate the dilepton spectrum, these structures could show up.

Summarizing, we have argued that in general quarks in the QGP possess a dispersion relation corresponding, at small momentum, to two massive collective modes, and that one branch has a minimum at finite momentum. As a consequence of this general feature, the dilepton production rate exhibits sharp structures (peaks and gaps) at invariant masses below about 1 GeV. Dilepton rates from a hadron gas, on the other hand, seem to be rather flat due to collisional broadening of the resonances [5]. Therefore the observation of new structures in the low mass dilepton spectrum at RHIC and LHC would be a strong indication for the presence of collective excitations of deconfined quarks in the fireball. But even if van Hove peaks cannot be seen, we expect in the case of a QGP formation a significant enhancement of the low mass dilepton rate due to the existence of a plasmino branch and due to higher order contributions compared to the hadronic predictions [5]. In conclusion, we predict that there will be a nontrivial QGP contribution to the low mass dilepton spectrum at RHIC and LHC and therefore the dilepton spectrum is a promising candidate for revealing the QGP phase.

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