

## Decoherence via the Dynamical Casimir Effect

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We derive a master equation for a mirror interacting with the vacuum field via radiation pressure. The dynamical Casimir effect leads to decoherence of a superposition state in a time scale that depends on the degree of “macroscopicity” of the state components, and which may be much shorter than the relaxation time scale. Coherent states are selected by the interaction as pointer states.

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Within the framework of quantum mechanics, a closed system may be found in any quantum state of the Hilbert space. As pointed out by Schrödinger [1], this is in apparent contradiction with the classical behavior of macroscopic systems. However, macroscopic systems are seldom isolated, and the interaction with the environment engenders the decay of most states into a statistical mixture of “pointer states,” which are linked to classical properties of the system [2]. Coherent superpositions of pointer states decohere into a statistical mixture in a time scale which is usually of the order of the damping time divided by some parameter representing the degree of “classicality” of the states. The decoherence time scale for a microwave field in a high- $Q$  superconducting cavity was recently measured [3] to be in agreement with such prediction [4].

Several different heuristic models for the coupling with the environment have been considered [5]. In this paper, we show that the coupling with the quantum vacuum field via radiation pressure provides a more fundamental, *ab initio* model for decoherence. The Casimir effect for moving boundaries has attracted a lot of interest recently [6]. The vacuum radiation pressure force dissipates the mechanical energy of an oscillating mirror, and the associated photon emission effect could in principle be measured experimentally [7]. Usually, one assumes that the mirror follows a prescribed trajectory, thus neglecting the recoil effect. However, here we want to focus on the mirror as a dynamical quantum system, hence the need to take the full mirror-plus-field dynamics into account. Jaekel and Reynaud treated this problem by using linear response theory [8], in order to calculate the fluctuations of the position of a dispersive mirror driven by the vacuum radiation pressure. Mass corrections were also obtained in Refs. [9] and [10].

In this paper, we consider a nonrelativistic partially reflecting mirror of mass  $M$  (position  $q$  and momentum  $p$ ) in a harmonic potential of frequency  $\omega_0$ , and under the action of vacuum radiation pressure. We take a scalar field in  $1 + 1$  dimensions, and neglect third and higher order terms in  $v/c$ , where  $v$  is the mirror’s velocity (we set  $c = 1$ ). We start from the Hamiltonian formalism

developed in Ref. [10]. The Hamiltonian is given by  $H = H_M + H_F + H_{\text{int}}$ , where

$$H_M = \frac{p^2}{2M} + \frac{M\omega_0^2}{2} q^2, \quad (1)$$

$$H_F = \int \frac{dx}{2} [\Pi^2 + (\partial_x \phi)^2] + \Omega \phi^2(x=0) \quad (2)$$

is the Hamiltonian for the scalar field  $\phi$  ( $\Pi = \partial_t \phi$  is its momentum canonically conjugate) under the boundary condition corresponding to a partially reflecting mirror at rest at  $x = 0$ , where the coupling constant  $\Omega$  also plays the role of a transparency frequency, the frequency-dependent reflection amplitude being  $R(\omega) = -i\Omega/(\omega + i\Omega)$  [9,10]. Since the emitted photons have frequencies smaller than  $\omega_0$ , the perfectly reflecting limit corresponds to  $\omega_0 \ll \Omega$ . We allow in principle for arbitrary values of  $\omega_0/\Omega$ , but assume from the start that  $\hbar\omega_0/M$ , which is of the order of the recoil velocity of the mirror, is very small. The interaction Hamiltonian  $H_{\text{int}}$  describes, on one hand, the modification of the boundary condition for the field due to the motion of the mirror, and, on the other hand, the modification of the mirror’s motion engendered by the radiation pressure force. The first effect leads to the emission of photon pairs out of the vacuum state (dynamical Casimir effect), whereas the second leads to dissipation and decoherence of the mirror’s motion, as shown below. To second order in  $v/c$ , we have

$$H_{\text{int}} = -\frac{p\mathcal{P}}{M} + \frac{\mathcal{P}^2}{2M} - \frac{1}{2}\Omega\phi^2(0)\frac{p^2}{M^2}, \quad (3)$$

where  $\mathcal{P} = -\int dx \partial_x \phi \partial_t \phi$  is the field momentum operator. In the right-hand side of (3), the first term is the most important, yielding the effects of dissipation and decoherence. The second term does not depend on the mirror’s variables, and hence will be of no relevance here, whereas the third term, being already of second order in  $v/c$ , is taken only to first order in perturbation theory. As discussed in Ref. [10], it provides a contribution to the mirror’s mass shift.

We derive a master equation for the reduced density matrix of the mirror  $\rho(t)$  by assuming that at  $t = 0$  the mirror

and the field are not correlated, so that the density matrix of the combined system  $\tilde{\rho}$  is written as  $\tilde{\rho}(0) = \rho(0) \otimes \rho_F$ , where  $\rho_F$  is the density matrix of the field alone. Then we compute  $\tilde{\rho}(t)$  up to second order in the perturbation Hamiltonian  $H_{\text{int}}$ . Note that the small perturbation parameter is the mirror's velocity  $v/c$ , and not the coupling constant  $\Omega$ , which is incorporated in the field Hamiltonian  $H_F$ . Finally, the master equation for  $\rho(t)$  is obtained by tracing  $\tilde{\rho}(t)$  over the field variables, taking the field to be in the vacuum state. We find

$$i\hbar\dot{\rho} = \left[ H_M - \frac{\Delta M(t)}{M} \frac{p^2}{2M}, \rho \right] - \Gamma(t)[p, \{q, \rho\}] - \frac{i}{\hbar} D_1(t)[p, [p, \rho]] - \frac{i}{\hbar} D_2(t)[p, [q, \rho]]. \quad (4)$$

The mass shift in (4) is given by  $\Delta M(t) = \Delta M_1 + \Delta M_2(t)$ , where the cutoff dependent  $\Delta M_1 = \langle \Omega \phi^2(0) \rangle$  is the only (first order) contribution of the  $p^2$  term in Eq. (3). It was derived earlier by different methods in [9] and [10]. Except for  $\Delta M_1$ , the terms in (4) come from second order perturbation theory. The corresponding coefficients are calculated from vacuum correlation functions of the momentum operator. The mass shift  $\Delta M_2(t)$  and the damping coefficient  $\Gamma(t)$  are obtained from the antisymmetric correlation function  $\xi(t) = \langle [\mathcal{P}(t), \mathcal{P}(0)] \rangle$ , which is connected to the susceptibility describing how the field momentum is affected by the motion of the mirror (and the corresponding modification of the boundary conditions). In fact, we show below that  $\Gamma(t)$  is closely connected to the photon emission effect and the associated radiation reaction force that damps the motion so as to enforce energy conservation. The diffusion coefficients  $D_1(t)$  and  $D_2(t)$  are obtained from the symmetric correlation function  $\sigma(t) = \langle \{\mathcal{P}(t), \mathcal{P}(0)\} \rangle - 2\langle \mathcal{P} \rangle^2$ , which represents the vacuum fluctuations.

Since  $\mathcal{P}$  is quadratic in the field operators, the correlation functions are obtained from the two-photon matrix elements  $\langle 0 | \mathcal{P}(t) | \omega_1, \omega_2 \rangle$ , which are calculated by using the normal mode expansion for the field operator. The spectral density  $\Xi(\omega)$  is defined as the Fourier transform of  $\xi(t)$ . For  $\omega > 0$ ,  $\Xi(\omega)$  results from the contribution of two-photon states with  $\omega_1 + \omega_2 = \omega$ . We find  $\Xi(\omega) = (2/\pi)\hbar^2\Omega\zeta(\omega/\Omega)$  with  $\zeta(u) = \ln(1 + u^2)/(2u) + (\arctan u)/u^2 - 1/u$ , whereas the Fourier transform of  $\sigma(t)$  is  $\epsilon(\omega)\Xi(\omega)$  [ $\epsilon(\omega)$  is the sign function]. The transparency frequency  $\Omega$  sets a frequency scale for the behavior of  $\Xi(\omega)$ . Thus, for  $\omega \ll \Omega$  the spectral density is linear ("Ohmic" environment), whereas for high frequencies it goes to zero as  $\zeta(u) \approx \ln(u)/u$ , due to the mirror's transparency at frequencies  $\omega \gg \Omega$ . We find

$$\Delta M_2(t) = \frac{2\hbar\Omega}{\pi^2} \int_{-\infty}^{\infty} d\omega \zeta(\omega/\Omega) \frac{\sin^2[(\omega - \omega_0)t/2]}{\omega - \omega_0}, \quad (5)$$

$$\Gamma(t) = \frac{\hbar\Omega\omega_0}{2\pi^2 M} \int_{-\infty}^{\infty} d\omega \zeta(\omega/\Omega) \frac{\sin[(\omega_0 - \omega)t]}{\omega_0 - \omega}, \quad (6)$$

$$D_1(t) = \frac{\hbar^2\Omega}{2\pi^2 M^2} \int_{-\infty}^{\infty} d\omega \epsilon(\omega)\zeta(\omega/\Omega) \frac{\sin[(\omega_0 - \omega)t]}{\omega_0 - \omega}, \quad (7)$$

$$D_2(t) = \frac{\hbar^2\omega_0\Omega}{\pi^2 M} \int_{-\infty}^{\infty} d\omega \epsilon(\omega)\zeta(\omega/\Omega) \times \frac{\sin^2[(\omega_0 - \omega)t/2]}{\omega_0 - \omega}. \quad (8)$$

The function  $\sin[(\omega_0 - \omega)t]/(\omega_0 - \omega)$  in Eqs. (6) and (7) has a peak of width  $2\pi/t$  at  $\omega = \omega_0$ . For large times,  $\Omega t \gg 1$ , the spectral density is approximately constant over the width of this peak, and then may be taken out of the integral, yielding

$$\Gamma = \frac{\hbar\Omega\omega_0}{2\pi M} \zeta(\omega_0/\Omega) \approx \frac{\hbar\omega_0^2}{12\pi M}, \quad (9)$$

the last approximation being valid in the perfectly reflecting limit. If we also assume that  $\omega_0 t \gg 1$ , Eq. (7) yields  $D_1 = \hbar\Gamma/(M\omega_0)$ . Accordingly, for large times the damping and diffusion coefficients have constant values that result from the contribution of two-photon states  $|\omega_1, \omega_2\rangle$  such that  $\omega_1 + \omega_2 = \omega = \omega_0$ . This is precisely the condition satisfied by the photon pairs generated in the dynamical Casimir effect [6]. In fact, the damping rate  $\Gamma$  as given by Eq. (9) is directly connected to the dissipative force on the moving mirror  $F = \hbar x'''/(6\pi)$  [11] (for simplicity we consider the perfectly reflecting limit). Indeed, the equation of motion for the average position then reads  $x'' = -\omega_0^2 x + \hbar x'''/(6\pi M)$ , whose solution in the limit  $\hbar\omega_0/M \ll 1$  decays as  $\exp[-\hbar\omega_0^2 t/(12\pi M)]$  in agreement with Eq. (9).

The asymptotic values of the dispersive terms  $\Delta M_2(t)$  and  $D_2(t)$  do not originate, on the other hand, from the neighborhood of  $\omega = \omega_0$ . In the perfectly reflecting limit, we neglect  $\omega_0$  in the denominator in Eq. (5), and, when  $\Omega t \gg 1$ , replace the sine squared by one-half. Integration of the resulting expression over the whole frequency interval yields  $\Delta M_2 \approx \hbar\Omega/(2\pi)$ . Accordingly, for large times we find the same mass correction obtained in [10] from stationary perturbation theory.

From these results, we may address two fundamental issues: (i) find out the pointer states; (ii) estimate the decoherence time scale. In the context considered here, pointer states are the most robust elements of the Hilbert space with respect to the motional interaction with the vacuum field. A simple test was proposed in Ref. [12], based on the idea that for pointer states the rate of information loss is minimum. Such a rate is measured with the help of the linear entropy  $s \equiv 1 - \text{Tr}\rho^2$  ( $s = 0$  for a pure state and greater than zero for a mixture). We calculate the rate of

entropy increase from the master equation (4), assuming that the initial state is pure:

$$\dot{s}(t) = 2\Gamma(t)[s(t) - 1] + \frac{4D_1(t)}{\hbar^2}(\Delta p)^2 + \frac{2D_2(t)}{\hbar^2}\sigma_{q,p}, \quad (10)$$

where  $(\Delta p)^2$  is the momentum dispersion and  $\sigma_{q,p} \equiv \langle\langle q, p \rangle\rangle - 2\langle p \rangle \langle q \rangle$  (with all operators evaluated at the same time  $t$ ). The first term in Eq. (10) leads to a decrease of entropy which does not depend on the initial state. Thus, it is not relevant for the determination of the pointer states, and will be left out of our discussion.

We first consider the effect of the last two terms in Eq. (10) in the perfectly reflecting limit. In Fig. 1 we plot the diffusion and damping coefficients as functions of  $\omega_0 t$  for  $\omega_0/\Omega = 10^{-4}$ .  $D_1(t)$  develops an initial jolt for times of the order of  $\Omega^{-1}$  and then decreases to the asymptotic value  $(D_1)_{\text{perf}} = \hbar^2 \omega_0 / (12\pi M^2)$  for  $t \sim 1/\omega_0$ . If we integrate Eq. (10) over many periods of oscillation, from  $t = 0$  to  $t = T = n2\pi/\omega_0$ , the contribution to the entropy of the initial jolt is negligible, allowing us to replace the diffusion coefficients by their constant asymptotic values. When computing  $\sigma_{q,p}(t)$  and  $(\Delta p)^2(t)$  in Eq. (10), we take the free evolution (corresponding to the harmonic oscillator Hamiltonian  $H_M$ ) of the mirror's operators  $q$  and  $p$  (weak coupling approximation). We get

$$s(T) = 2T \frac{D_1}{\hbar^2} [(\Delta p)_0^2 + (M\omega_0)^2(\Delta q)_0^2], \quad (11)$$

where  $(\Delta p)_0^2$  and  $(\Delta q)_0^2$  represent the dispersions for the initial state. Note that  $D_2(t)$  does not contribute

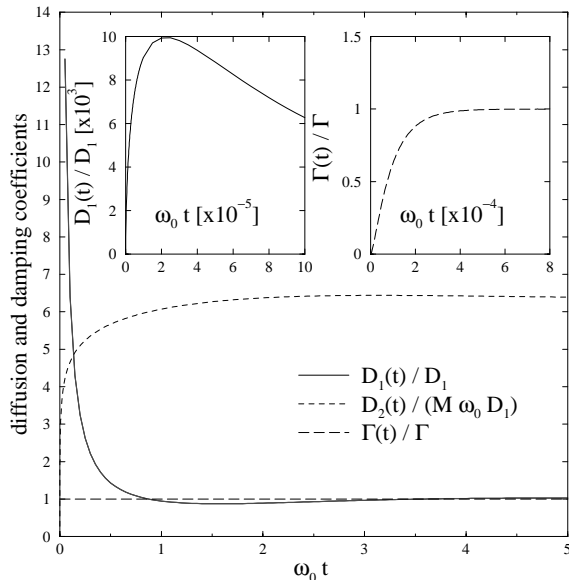


FIG. 1. Diffusion and damping coefficients as a function of time in the perfect-reflector limit  $\omega_0/\Omega = 10^{-4} \ll 1$ . Here  $D_1 = \hbar^2 \omega_0 / (12\pi M^2)$  and  $\Gamma = \hbar \omega_0^2 / (12\pi M)$  are the asymptotic limits of  $D_1(t)$  and  $\Gamma(t)$ . The insets show the behavior of these two time-dependent coefficients for short times.

to the time-averaged entropy production. The minimum  $s(T)$  given the constraint  $\Delta q \Delta p \geq \hbar/2$  is for  $\Delta q^2 = \hbar / (2M\omega_0)$ ,  $\Delta p^2 = M\hbar\omega_0/2$ . Thus, as in the problem of quantum Brownian motion (QBM) with interaction Hamiltonian linear in the position operator [12], the pointer basis consists of coherent states.

The opposite limit  $\omega_0 \gg \Omega$  corresponds to dominant frequencies of the environment slow with respect to the mirror's own time scale. However, since the spectral density  $\Xi(\omega)$  decays too slowly for  $\omega \gg \Omega$ , field frequencies of the order of  $\omega_0$  provide a significant contribution even in this limit. As a consequence, the vacuum field does not behave as an adiabatic environment in the sense of Ref. [13]. In Fig. 2, we plot the diffusion and damping coefficients as functions of  $\omega_0 t$  for  $\omega_0/\Omega = 10^4$ . They oscillate around their asymptotic values with (angular) frequency  $\omega_0$  and with an amplitude of oscillation that decays in a time  $t \sim 1/\Omega$  [14]. The oscillatory terms do not contribute to the entropy increase when we average over many oscillations. Hence Eq. (11) also holds in this case, although the rate of entropy increase is much smaller than in the perfect-reflecting limit, since the asymptotic limit of  $D_1(t)$  is now  $D_1 = 6(\Omega/\omega_0)^2 \ln(\omega_0/\Omega)(D_1)_{\text{perf}} \ll (D_1)_{\text{perf}}$ .

In order to estimate the decoherence time scale, we take, at  $t = 0$ , the superposition state  $|\psi\rangle = (|\alpha\rangle + |-\alpha\rangle)/\sqrt{2}$ , with  $\alpha = iP_0/\sqrt{2M\hbar\omega_0}$ . The corresponding Wigner function is

$$W = W_m + \frac{1}{\pi\hbar} \exp\left[-\frac{q^2}{2(\Delta q)^2} - \frac{2p^2(\Delta q)^2}{\hbar^2}\right] \times \cos\left(\frac{2P_0 q}{\hbar}\right), \quad (12)$$

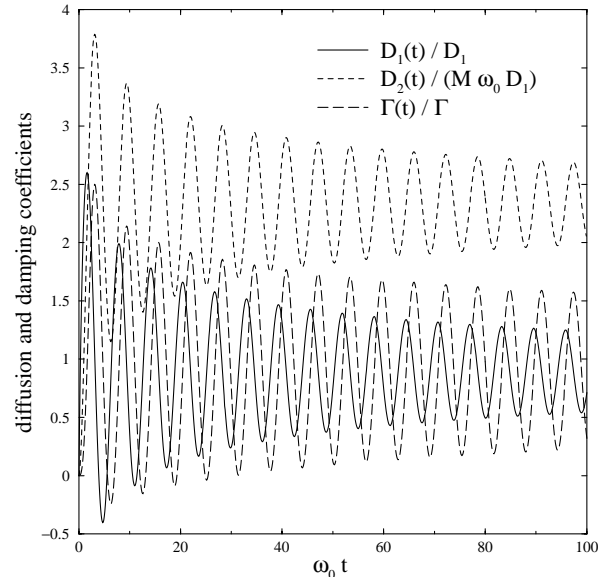


FIG. 2. Diffusion and damping coefficients as a function of time in the high-transmission limit  $\omega_0/\Omega = 10^4 \gg 1$ . Here  $D_1 = \hbar^2 \Omega^2 \ln(\omega_0/\Omega) / (2\pi M^2 \omega_0)$  and  $\Gamma = \hbar \Omega^2 \ln(\omega_0/\Omega) / (2\pi M)$  are the asymptotic limits of  $D_1(t)$  and  $\Gamma(t)$ .

with  $\Delta q = \sqrt{\hbar/(2M\omega_0)}$ , and where  $W_m$  corresponds to the statistical mixture  $\rho_m = (1/2)(|\alpha\rangle\langle\alpha| + |-\alpha\rangle\langle-\alpha|)$ . In phase space,  $W_m$  has two peaks along the momentum axis (at  $\pm P_0$ ). The second term in Eq. (12) originates from the interference between the two state components, and hence represents the coherence of the state. Since it oscillates along the  $q$  axis in phase space, diffusion in position will damp the coherence at a maximum rate given by  $-D_1\partial_q^2 W/W$ , with, from Eq. (12),  $\partial_q^2 W \approx -(2P_0/\hbar)^2 W$ . After averaging the decoherence rate over a period of oscillation [15], we find that the decoherence time scale  $t_d$  is

$$t_d = \frac{\hbar^2}{2P_0^2 D_1} = \frac{\Gamma^{-1}}{4|\alpha|^2}. \quad (13)$$

To clarify the connection between decoherence and the dynamical Casimir effect, we present a second derivation of Eq. (13), based on the concept of entanglement between mirror and field on account of the generation of photon pairs. At  $t = 0$ , the quantum state  $|\Psi\rangle$  of the complete mirror-plus-field system is  $|\Psi\rangle_0 = |\psi\rangle \otimes |0\rangle$ . Instead of tracing over the field operators, we follow the evolution of the field state (in the interaction picture) to find  $|\Psi\rangle_t = (|\alpha\rangle \otimes |\phi^+\rangle_t + |-\alpha\rangle \otimes |\phi^-\rangle_t)/\sqrt{2}$ , where  $|\phi^\pm\rangle_t$  is computed from first order perturbation theory assuming a classical prescribed motion:

$$|\phi^\pm\rangle_t = B(t)|0\rangle \pm \frac{1}{2} \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 b(\omega_1, \omega_2; t) \times |\omega_1, \omega_2\rangle, \quad (14)$$

where

$$b(\omega_1, \omega_2; t) = \frac{i}{\hbar} \langle \omega_1, \omega_2 | \mathcal{P} | 0 \rangle \int_0^t dt' e^{i(\omega_1 + \omega_2)t'} \dot{q}(t')$$

is the two-photon amplitude corresponding to the mirror's velocity  $\dot{q}(t) = -i\sqrt{2\hbar\omega_0/M}\alpha \cos(\omega_0 t)$  associated to the state  $|\alpha\rangle$ , whereas  $|B(t)|^2$  is determined by the normalization condition  $\langle \phi^\pm | \phi^\pm \rangle = 1$ . Since the amplitude is proportional to the velocity, it has an *opposite* sign when associated to  $|-\alpha\rangle$ , as shown in Eq. (14). When  $\omega_0 t \gg 1$ , the two-photon probabilities are proportional to the time  $t$ , and related to the relaxation rate  $\Gamma$ . Then, from Eq. (14) we derive  $\rho(t) - \rho_m = (1 - t/t_d)[\rho(0) - \rho_m]$ , with  $t_d$  given by (13).

According to Eq. (13), decoherence is faster than energy dissipation by a factor that represents the degree of "macroscopicity" of the coherent states. In fact,  $|\alpha|^2$  is twice the ratio between the energy of the coherent state and the zero-point energy of the harmonic oscillator. Therefore, Eq. (13) provides an additional illustration of the meaning of the limit  $|\alpha| \gg 1$  as the classical limit of the quantum harmonic oscillator. Moreover, Eq. (13) also shows that the decoherence rate increases with the distance between the two coherent components in phase space. We

have confirmed the role of coherent states in the understanding of the classical limit by showing that they are the pointer states. Remarkably, classical behavior is obtained from the mere inclusion of an unavoidable, intrinsically quantum effect, the radiation pressure coupling with the quantum vacuum field.

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- [14] If a stronger high-frequency cutoff is introduced in our model, so as to render the correlation function  $\sigma(0) = \int_0^\infty d\omega \Xi(\omega)/\pi$  finite, then it would follow from Eq. (7) that  $D_1 \approx [\sigma(0)/2M^2\omega_0] \sin(\omega_0 t)$  when  $\omega_0$  is much larger than the frequency cutoff. In this case, the vacuum field behaves as an adiabatic environment coupled linearly to the harmonic oscillator, and, as discussed in Ref. [13], no decoherence takes place.
- [15] The decoherence rate decreases as the two coherent components of  $|\psi\rangle$  rotate away from their initial positions in phase space, so that the average rate is one-half of the maximum value. When the interaction is linear in the position rather than in the momentum operator, as in the usual QBM models [J. P. Paz, S. Habib, and W. H. Zurek, *Phys. Rev. D* **47**, 488 (1993)], the opposite applies, and then the maximum rate occurs when the coherent components are along the position axis.