Meyer Replies: van Enk's observation that there are classical models for Q's strategy in PQ penny flipover [1] is, of course, correct; but this does not mean that "there is nothing quantum mechanical about that strategy" [2]. One might equally well say that there is nothing classical about Picard's pure strategies since there are quantum models for flipping (or not) a two-state system [3]. Clearly, the expansion of Q's pure strategy set [4] which enables him to win every game can be realized in either a quantum or a classical system, but to argue that "A single qubit is not a truly quantum system" because it can be "mocked up by a classical hidden-variable model" [2] is, as Heisenberg put it, to "attempt to put new wine into old bottles. Such attempts are always distressing, for they mislead us into continually occupying ourselves with the inevitable cracks in the old bottles, instead of rejoicing over the new wine" [5].

Nevertheless, since van Enk suggests that we should put PQ penny flipover into an old bottle, let us identify the cracks. Our present interest is less in ruling out classical hidden-variable models for quantum mechanics and more in demonstrating computational advantages for quantum over classical systems. From this perspective models should be dynamical and scale up as the number of Hilbert space factors (e.g., qubits) increases. The Deutsch-Jozsa [6] and Simon [7] algorithms, each of which is structured as a PQ game [1], describe quantum computations for which *any* classical model—including ones like those suggested by van Enk [8]—must scale exponentially badly. The old bottles can hold only a few drops of new wine—any more leaks out through the cracks.

Although van Enk alludes to entanglement when he mentions the Bell inequalities [9], in fact, entanglement of intermediate states is not even necessary for quantum algorithms to outperform classical ones: Imagine Picard and Q playing a two qubit game initialized at $|00\rangle$, where Picard is constrained to make one of four moves—corresponding to the possible maps $f: \mathbb{Z}_2 \to \mathbb{Z}_2$ via $|x, y\rangle \to |x, y \oplus f(x)\rangle$ for $x, y \in \mathbb{Z}_2$, a basis for $\mathbb{C}^2$, and where $\oplus$ denotes addition mod 2. If Q's objective is to identify Picard's choice of function as surjective or not at the end of the game, no classical strategy can ensure he wins more than half the time. But the simple improvement [10] on the one bit Deutsch-Jozsa [6] and Simon [7] algorithms consisting of Q first acting by $H \otimes H\sigma_z$ and last by $H \otimes \mathbf{1}_2$ [where $H = \left(\begin{smallmatrix} 1 & 1 \\ 1 & -1 \end{smallmatrix}\right)/\sqrt{2}$ is the Hadamard transform] guarantees a win with probability 1 since the first qubit is $|1\rangle$ when Picard's choice is surjective and $|0\rangle$ otherwise. *At no turn in the game are the two qubits entangled; Q's strategy works by a clever interference of amplitudes just as it does in PQ* penny flipover. There the amplitudes for the computational paths terminating at tails (T) cancel independently of Picard's move; see Fig. 1 in [1].

The relevance of classical models for quantum systems depends upon the use to which they are put. In the modern context of quantum information processing, models must scale with the number of qubits and be dynamical. Thus, despite the fact that there is a classical model for a single qubit, it is most useful to consider the simple quantum strategy illustrated in PQ penny flipover [1] as quantum mechanical. Even a single drop holds the taste of a new wine.

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[1] D. A. Meyer, Phys. Rev. Lett. **82**, 1052 (1999).
[2] S. J. van Enk, preceding Comment, Phys. Rev. Lett. **84**, 789 (1999).
[3] Let me take this opportunity to correct a minor substantive error in [1]. The single qubit error correcting code discussed at the end of the paper must be used only to encode a classical bit.
[4] Notice that it is only the pure strategy set which is extended: Q's pure quantum strategies do *not* include the mixed strategy which would be optimal were he to play classically. It is certainly possible—and upon reflection "unsurprising"—that extending a player's pure strategy set may confer no advantage [when the additional strategies are *strongly dominated;* see, e.g., R. B. Myerson, *Game Theory: Analysis of Conflict* (Harvard University Press, Cambridge, 1991), Chap. 2] and further, that disallowing mixed strategies may be detrimental in repeated games. That this is *not* the case for quantum strategies is the content of Theorem 1 in [1].
[5] W. Heisenberg, in *Niels Bohr and the Development of Physics,* edited by W. Pauli with the assistance of L. Rosenfeld and V. Weisskopf (Pergamon, New York, 1955), p. 12.
[6] D. Deutsch and R. Jozsa, Proc. R. Soc. London A **439**, 553 (1992).
[7] D. R. Simon, in *Proceedings of the 35th Symposium on Foundations of Computer Science, Santa Fe, 1994,* edited by S. Goldwasser (IEEE, Los Alamitos, CA, 1994), p. 116.
[8] Despite van Enk's implication [2] that classical simulation may be impossible for larger games, there is *always* a classical model, be it only multiplication of vectors in $\mathbb{C}^N$ by $N \times N$ unitary matrices on a classical computer. The quantum player wins the game no matter the size of the board—just more easily as it gets larger.
[9] J. S. Bell, Physics **1**, 195 (1964).
[10] R. Cleve, A. Ekert, C. Macchiavello, and M. Mosca, Proc. R. Soc. London A **454**, 339 (1998).

 0031-9007/00/84(4)/790(1)$15.00