Observation of Breathers in Josephson Ladders

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We report on the observation of spatially localized excitations in a ladder of small Josephson junctions. The excitations are whirling states which persist under a spatially homogeneous force due to the bias current. These states of the ladder are visualized using a low temperature scanning laser microscopy. We also compute breather solutions with high accuracy in corresponding model equations. The stability analysis of these solutions is used to interpret the measured patterns in the I-V characteristics.

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The present decade has been marked by an intense theoretical research on dynamical localization phenomena in spatially discrete systems, namely on discrete breathers (DB). These exact solutions of the underlying equations of motion are characterized by periodicity in time and localization in space. Away from the DB center the system approaches a stable (typically static) equilibrium. (For reviews, see [1,2]). These solutions are robust to changes of the equations of motion, and exist in translationally invariant systems and any lattice dimension. DBs have been discussed in connection with a variety of physical systems such as large molecules, molecular crystals [3], and spin lattices [4,5].

For a localized excitation such as a DB, the excitation of plane waves which might carry the energy away from the DB does not occur due to the spatial discreteness of the system. The discreteness provides a cutoff for the wavelength of plane waves and thus allows one to avoid resonances of all temporal DB harmonics with the plane waves. The nonlinearity of the equations of motion is needed to allow for the tuning of the DB frequency [1].

Though the DB concept was initially developed for conservative systems, it can be easily extended to dissipative systems [6]. There, discrete DBs become time-periodic spatially localized attractors, competing with other (perhaps nonlocal) attractors in phase space. The characteristic property of DBs in dissipative systems is that these *localized excitations* are predicted to persist under the influence of a *spatially homogeneous* driving force. This is due to the fact that the driving force compensates the dissipative losses of the DB.

So far, research in this field was predominantly theoretical. Identifying and analyzing of experimental systems for the direct observation of DBs thus becomes a very actual and challenging problem. Experiments on localization of light propagating in weakly coupled optical waveguides [7], low-dimensional crystals [8], and anti-ferromagnetic materials [9] have been recently reported.

In this work we realize the theoretical proposal [10] to observe DB-like localized excitations in arrays of coupled Josephson junctions. A Josephson junction is formed between two superconducting islands. Each island is characterized by a macroscopic wave function $\Psi \sim e^{i\theta}$ of the superconducting state. The dynamics of the junction is described by the time evolution of the gauge-invariant phase difference $\varphi = \theta_2 - \theta_1 - \frac{2\pi}{\Phi_0}\int \mathbf{A}\cdot d\mathbf{s}$ between adjacent islands. Here Φ_0 is the magnetic flux quantum and $\mathbf{A}$ is the vector potential of the external magnetic field (integration goes from one island to the other one). In the following, we consider zero magnetic fields $\mathbf{A} = \mathbf{0}$. The mechanical analog of a biased Josephson junction is a damped pendulum driven by a constant torque. There are two general states in this system: the first state corresponds to a stable equilibrium, and the second one corresponds to a whirling pendulum state. When treated for a chain of coupled pendula, the DB corresponds to the whirling state of a few adjacent pendula with all other pendula performing oscillations around their stable equilibria. In an array of Josephson junctions the nature of the coupling between the junctions is inductive. A localized excitation in such a system corresponds to a state where one (or several) junctions are in the whirling (resistive) state, with all other junctions performing small forced oscillations around their stable equilibria. According to theoretical predictions [11], the amplitude of these oscillations should decrease exponentially with increasing distance from the center of the excitation.

We have conducted experiments with ladders consisting of Nb/Al-AlO_x/Nb underdamped Josephson tunnel junctions [12]. We investigated annular ladders (closed in a ring) as well as straight ladders with open boundaries. The sketch of an annular ladder is given in Fig. 1. Each cell contains 4 small Josephson junctions. The size of the hole between the superconducting electrodes which form the cell is about $3 \times 3\ \mu\mathrm{m}^2$. Here we define *vertical* junctions (JJ_V) as those in the direction of the external bias current, and *horizontal* junctions (JJ_H) as those transverse to the bias. Because of fabrication reasons we made the superconducting electrodes quite broad so that the distance between two neighboring vertical junctions is 30 μm, as can be seen in Fig. 3. The ladder voltage is read across the vertical junctions. According to the Josephson relation, a junction in a whirling state generates a dc voltage

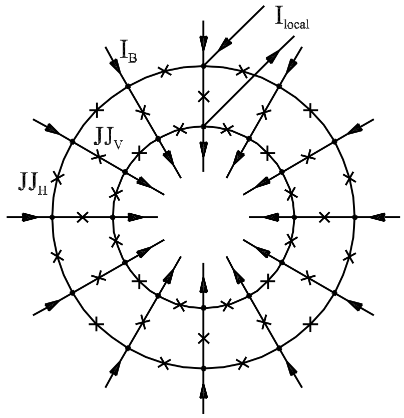

FIG. 1. Schematic view of an annular ladder. Josephson junctions are indicated by crosses ($\times$).

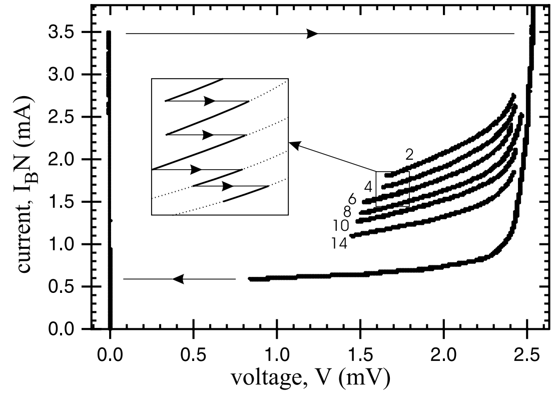

FIG. 2. Current-voltage I_B-V plane for an annular ladder with the parameters $N = 30$, $\eta = 0.44$, and $\beta_L = 2.7$. Digits indicate the number of rotating vertical junctions.

$V = \frac{1}{2\pi}\Phi_0\langle\frac{d\varphi}{dt}\rangle$, where $\langle\cdots\rangle$ means the time average. In order to force junctions into the whirling state we used two different types of bias. The current I_B was uniformly injected at every node via thin-film resistors. Another current I_{local} was applied locally across just one vertical junction.

We studied ladders with different strengths of horizontal and vertical Josephson coupling determined by the junction areas. The ratio of the junction areas is called the anisotropy factor and is expressed in terms of the junction critical currents $\eta = I_{cH}/I_{cV}$. If this factor is equal to zero, vertical junctions will be decoupled and can operate independently one from another. Measurements have been performed at 4.2 K. The number of cells N in different ladders varied from 10 to 30. The discreteness of the ladder is expressed in terms of the parameter $\beta_L = 2\pi L I_{cV}/\Phi_0$, where L is the self-inductance of the elementary cell of the ladder. The damping coefficient $\alpha = \sqrt{\Phi_0/(2\pi I_c C R_N^2)}$ is the same for all junctions as their capacitance C and resistance R_N scale with the area and $C_H/C_V = R_{NV}/R_{NH} = \eta$. The damping α in the experiment can be controlled by temperature and its typical values are between 0.1 and 0.02.

We have measured the dc voltage across various vertical junctions as a function of the currents I_{local} and I_B. In order to generate a localized rotating state in a ladder we started with applying the local current $I_{local} > 2I_{cH} + I_{cV}$. This switches one vertical and the nearest horizontal junctions into the resistive state. After that I_{local} was reduced and, simultaneously, the homogeneous bias I_B was tuned up. In the final state we kept the bias I_B constant and reduced I_{local} to zero. Under these conditions, with a *spatially homogeneous* bias injection, we observed a *spatially localized* rotating state with nonzero dc voltage drops on just one or a few vertical junctions.

Various measured states of the annular ladder in the current-voltage I_B-V plane with $I_{local} = 0$ are presented in Fig. 2. The voltage V is recorded locally on the same vertical junction which was initially excited by the local current injection. The vertical line on the left side corresponds to the superconducting (static) state. The rightmost (also the bottom) curve accounts for the spatially homogeneous whirling state (all vertical junctions rotate synchronously). Its nonlinear $I_B(V)$ shape is caused by a strong increase of the normal tunneling current at a voltage of about 2.5 mV corresponding to the superconducting energy gap. The series of branches represent various localized states. These states differ from each other by the number of rotating vertical junctions.

In order to visualize various rotating states in our ladders we used the method of low temperature scanning laser microscopy [13]. It is based on the mapping of a sample voltage response as a function of the position of a focused low-power laser beam on its surface. The laser beam locally heats the sample and, therefore, introduces an additional dissipation in the area of a few micrometers in diameter. Such a dissipative spot is scanned over the sample and the voltage variation at a given bias current is recorded as a function of the beam coordinate. The resistive junctions of the ladder contribute to the voltage response, while the junctions in the superconducting state show no response. The power of the laser beam is modulated at a frequency of several kHz and the sample voltage response is measured using a lock-in technique.

Several examples of the ladder response are shown in Fig. 3 as 2D gray scale maps. The spatially homogeneous whirling state is shown in Fig. 3(A). Here all vertical junctions are rotating, but the horizontal ones are not. Figure 3(B) corresponds to the uppermost branch of Fig. 2. We observe a localized whirling state expected for a DB, namely a rotobreather [10]. In this case 2 vertical junctions and 4 horizontal junctions of the DB are rotating, with all others remaining in the superconducting state. The same state is shown on an enlarged scale in Fig. 3(C). Figure 3(D) illustrates another rotobreather found for the next lower branch of Fig. 2 at which 4 vertical junctions are in the resistive state. The local current at the beginning of each experiment is passing through the vertical junction being the top one on each map. In Fig. 3(E), which

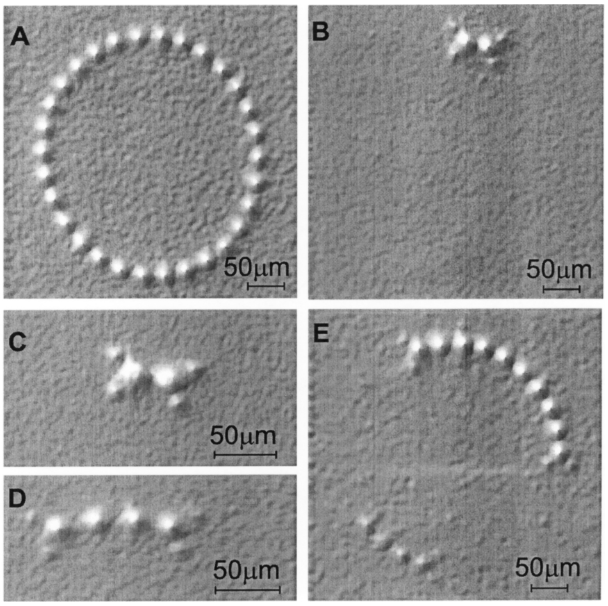

FIG. 3. Whirling states measured in the annular ladder using the low temperature scanning laser microscope: (A) spatially homogeneous whirling state, (B)–(E) various localized states corresponding to discrete breathers.

accounts for one of the lowest branches, we find an even broader localized state. Simultaneously, on the opposite side of the ring we observe another DB excited spontaneously (without any local current). An interesting fact is that in experiments with open boundary ladders (not closed in a ring) we also detected DBs with even or odd numbers of whirling vertical junctions.

Various states shown in Fig. 3 account for different branches in the I_B-V plane in Fig. 2. Each resistive configuration is found to be stable along its particular branch. On a given branch the damping of the junctions in the rotating state is compensated by the driving force of the bias current I_B. The transitions between the branches are discontinuous in voltage. In Fig. 2, we see that all branches of localized states lose their stability at a voltage of about 1.4 mV. Furthermore, as indicated in the inset on Fig. 2, a peculiar switching occurs: upon *lowering* the bias current I_B the system switches to a *larger* voltage. According to our laser microscope observations, the lower the branch in Fig. 2, the larger the number M of resistive vertical junctions. The slope of these branches is $dV/dI_B \approx MR_{NV}/(M + \eta)$, thus the branches become very close to each other for large M. The fact that the voltage at the onset of instability is independent of the size of the DB, indicates that the instability is essentially local in space and occurs at the border between the resistive and nonresistive junctions.

The occurrence of DBs is inherent to our system. We have also found various localized states to arise without any local current. Namely, when biasing the ladder by the homogeneous current I_B slightly below NI_{cV}, we sometimes observed the system switching to a spatially inhomogeneous state, similar to that shown in Fig. 3(E).

To interpret the experimental observations, we analyze the equations of motion for our ladders (see [11] for details). Denote by $\varphi_l^v, \varphi_l^h, \tilde{\varphi}_l^h$ the phase differences across the lth vertical junction and its right upper and lower horizontal neighbors. Using $\nabla\varphi_l = \varphi_{l+1} - \varphi_l$ and $\Delta\varphi_l = \varphi_{l+1} + \varphi_{l-1} - 2\varphi_l$, the Josephson equations yield

$$\ddot{\varphi}_l^v + \alpha\dot{\varphi}_l^v + \sin\varphi_l^v = \gamma - \frac{1}{\beta_L}(-\Delta\varphi_l^v + \nabla\tilde{\varphi}_{l-1}^h - \nabla\varphi_{l-1}^h), \tag{1}$$

$$\ddot{\varphi}_l^h + \alpha\dot{\varphi}_l^h + \sin\varphi_l^h = -\frac{1}{\eta\beta_L}(\varphi_l^h - \tilde{\varphi}_l^h + \nabla\varphi_l^v), \tag{2}$$

$$\ddot{\tilde{\varphi}}_l^h + \alpha\dot{\tilde{\varphi}}_l^h + \sin\tilde{\varphi}_l^h = \frac{1}{\eta\beta_L}(\varphi_l^h - \tilde{\varphi}_l^h + \nabla\varphi_l^v). \tag{3}$$

Here $\gamma = I_B/(NI_{cV})$. First, we compute the dispersion relation for Josephson plasmons $\varphi \propto e^{i(ql-\omega t)}$ at $\alpha = 0$ in the ground state (no resistive junctions). We obtain three branches: one degenerated with $\omega = 1$ (horizontal junctions excited in phase), the second one below $\omega = 1$ with weak dispersion (mainly vertical junctions excited), and finally the third branch with the strongest dispersion above the first two branches (mainly horizontal junctions excited out of phase), cf. the inset in Fig. 4. The region of experimentally observed DB stability is also shown. Note, that for DBs with symmetry between the upper and lower horizontal junctions the voltage drop on the horizontal junctions is half the drop across the vertical ones. This causes the characteristic frequency of the DB to be 2 times smaller than the value expected from the measured voltage drop on the vertical junctions [11].

In order to compare experimental results of Fig. 2 to the model given by Eqs. (1)–(3), we have integrated the latter equations numerically. We also find localized DB solutions, in particular solutions similar to the ones reported in previous numerical studies [11]. These solutions are generated with using initial conditions when M vertical junctions of the resistive cluster (cf. Fig. 3) have $\varphi = 0$ and $\dot{\varphi} = 2V_0$ and the horizontal junctions adjacent to the vertical resistive cluster have $\varphi = 0$ and $\dot{\varphi} = V_0$, while all other phase space variables are set to zero. The obtained I_B-V curves are shown in Fig. 4. The superconducting gap structure and the nonlinearity of slopes are not reproduced in the simulations, as we use a voltage independent dissipation constant α in (1)–(3). We find several instability windows of DB solutions, separating stable parts of the I_B-V characteristics.

In addition to direct numerical calculation of I_B-V curves, we have also computed numerically exact DB solutions of (1)–(3) by using a generalized Newton map [1]. We have studied the linear stability of the obtained DB [1] by solving the associated eigenvalue problem. The spatial profile of the eigenmode which drives the

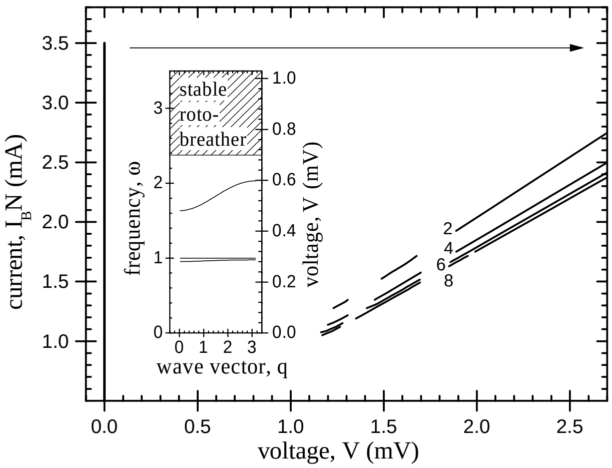

FIG. 4. I_B-V characteristics for numerically obtained breather states with 2, 4, 6, and 8 whirling vertical junctions. The inset shows the dispersion relation of an annular ladder with $\gamma = 0.3$, $\alpha = 0.07$, $N = 30$, $\eta = 0.44$, and $\beta_L = 2.7$.

instability (associated with the edges of the instability windows in Fig. 4) is localized on the DB, more precisely on the edges of the resistive domain. This is in accord with the experimental observation (Fig. 3) where several independent DBs can be excited in the system.

The DB states turn to be either invariant under $\varphi_l^h \leftrightarrow \tilde{\varphi}_l^h$ transformation or not. Both such solutions have been obtained numerically. To understand this, we consider the equations of motion (1)–(3) in the limit $\eta \to 0$ and look for time-periodic localized solutions. In this limit the brackets on the right-hand side of (2) and (3) vanish, and vertical junctions decouple from each other. Let us then choose one vertical junction with $l = 0$ to be in a resistive state and all the others to be in the superconducting state. To satisfy periodicity in the horizontal junction dynamics we arrive at the condition

$$\varphi_0^h = \frac{1}{k}\varphi_0^v, \qquad \tilde{\varphi}_0^h = -\frac{k-1}{k}\varphi_0^v$$

or

$$\varphi_0^h = \frac{k-1}{k}\varphi_0^v, \qquad \tilde{\varphi}_0^h = -\frac{1}{k}\varphi_0^v \tag{4}$$

and a similar set of choices for $\varphi_{-1}^h, \tilde{\varphi}_{-1}^h$, with all other horizontal phase differences set to zero. Here k is an arbitrary positive integer. Continuation to nonzero η values should be possible [6]. The symmetric DBs in Fig. 3 correspond to $k = 2$. The mentioned asymmetric DBs correspond to $k = 1$. The current-voltage characteristics for asymmetric $k = 1$ DBs show a different behavior from that discussed above. These DBs are stable down to very small current values, and simply disappear upon further lowering of the current, so that the system switches from a state with finite voltage drop to a pure superconducting state with zero voltage drop.

The observed DB states are clearly different for the well-known row switching effect in 2D Josephson junction arrays. The DBs demonstrate localization transverse to the bias current (driving force), whereas the switched states of *noninteracting junction rows* are localized along the current. At the same time, DB states inherent to Josephson ladders are closely linked to the recently discovered meandering effect in 2D arrays [14].

In summary, we have experimentally detected various types of rotobreathers in Josephson ladders and visualized them with the help of laser microscopy. Our experiments show that DBs in Josephson ladders may occupy several lattice sites and that the number of occupied sites may increase at specific instability points. The possibility of exciting DBs spontaneously, without using any local force, demonstrates their inherent character. The observed DBs are stable in a wide frequency range. Numerical calculations confirm the reported interpretation and allow for a detailed study of the observed instabilities.

Note added.—After this work was completed we became aware of the experiment [15] which detected 1- and 2-site rotobreathers by using several voltage probes across a ladder. We note that our method allows for direct visual observation of any multisite breathers.

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