Thermodynamics of a *n*-*p* Condensate in Asymmetric Nuclear Matter

A. Sedrakian[1] and U. Lombardo[2]
[1]*Kernfysisch Versneller Instituut, NL-9747 AA Groningen, The Netherlands*
[2]*Dipartimento di Fisica, Universita di Catania, Corso Italia 57, Catania, Italy and INFN-LNS, Via Santa Sofia 9500, Catania, Italy*
(Received 2 August 1999)

We study the neutron-proton pairing in nuclear matter as a function of isospin asymmetry at finite temperatures and the empirical saturation density using realistic nuclear forces and Brueckner-renormalized single particle spectra. Our computation of the thermodynamic quantities shows that, while the difference of the entropies of the superconducting and normal phases anomalously changes its sign as a function of temperature for arbitrary asymmetry, the grand canonical potential does not; the superconducting state is found to be stable in the whole temperature-asymmetry plane. The pairing gap completely disappears for density asymmetries exceeding $\alpha_c = (\rho_n - \rho_p)/\rho \simeq 0.11$.

PACS numbers: 21.65.+f, 21.30.Fe, 26.60.+c

Neutron-proton pair correlations are potentially important in a number of contexts, including the study of the nuclear structure of medium mass $N \simeq Z$ nuclei produced at the radioactive nuclear beam facilities [1] and the theory of the deuteron formation in the medium energy heavy-ion collisions [2]. In the astrophysical context n-p pairing correlations are relevant for the astrophysical r-process [3,4] and could play a major role in neutron star models which permit pion or kaon condensation [5].

As the existence of the pair correlations crucially depends upon the overlap between the neutron and proton Fermi surfaces, one expects a suppression of the pairing correlations if the system is driven out of the isospin-symmetric state. The mechanism driving the suppression is encountered in many fermionic systems where particles lie on two different Fermi surfaces. The simplest example is the spin zero pairing in a superconducting metal in a magnetic field; here the spin degeneracy is relaxed because of the Pauli paramagnetism [6–8]. Another system is the B state of liquid ^{3}He (Balian-Werthamer state) in a magnetic field [9]. A closely related topic is the transition from a semiconductor to a superconductor, where the role of the separation between the two Fermi energies is played by the semiconductor gap [10]. Apart from condensed matter, a similar situation arises in the finite temperature/density QCD, where color symmetry is spontaneously broken and the system is unstable against the formation of the $\langle q\bar{q}\rangle$ color superconducting condensate [11–14].

The first purpose of this Letter is to show, using the example of the n-p pairing in the nuclear matter, that for systems with broken time-reversal symmetry the difference of the entropies of the superconducting and normal phases anomalously changes its sign as a function of temperature for arbitrarily small perturbation from the symmetric state. Nevertheless, the grand canonical potential still corresponds to a thermodynamically stable superconducting state in the whole temperature-asymmetry plane. The second purpose is to provide a first computation of the key thermodynamic quantities of the asymmetric n-p condensate from the strong coupling BCS theory. In doing so we carry out a fully microscopic calculation of the n-p pairing with realistic nuclear interactions, including ladder-renormalized single particle energies [15].

In recent years much theoretical effort has been devoted to the understanding of the n-p pair correlations. They have been studied both in the infinite nuclear matter within the Thouless [16,17] or the Bardeen-Cooper-Schrieffer [18] theories of superconductivity [19–26] and in finite nuclei within mean-field effective interaction theories of pairing [1,27–33]. In particular, microscopic calculations, based on the BCS theory for the bulk nuclear matter, show that the isospin-asymmetric matter supports Cooper-type pair correlations in the $^3S_1$-$^3D_1$ partial-wave channel due to the tensor component of the nuclear force. The energy gap is of the order of 10 MeV at the empirical saturation density [19–21,26] when the effects of the medium polarization on the pairing force [34–38] are neglected. The studies based on the Thouless criterion [16,17] for the thermodynamic T matrix (the divergence of the ladder resummation scheme at the critical temperature) deduced the suppression of the critical temperature with the isospin asymmetry [22,23]; however, they do not permit one to draw conclusions about properties of the superconducting state with a finite gap. While the studies based on the BCS theory, without self-energy corrections to the single particle spectrum [24,25], give a correct qualitative picture, they overestimate the magnitude of the pairing gap and the critical asymmetries at which the pairing disappears.

To set the stage, let us start with the solution of the Dyson equations for the normal and anomalous propagators in the Matsubara formalism. In doing so we shall decouple the isospin singlet SD pairing channel from the isospin triple channels, as the SD coupled channels contain the dominant part of the attractive pairing force. In this case the pairing matrix is diagonal in the spin space, i.e., one deals with the unitary triplet state (see Ref. [21]). The proton/neutron propagators follow from the solution

 0031-9007/00/84(4)/602(4)$15.00

of the Gor'kov equations, and can be cast in the form ($\hbar = 1$)

$$G^{(p/n)}_{\sigma,\sigma'}(\vec{k},\omega_m) = -\delta_{\sigma,\sigma'}\frac{i\omega_m + \xi_k \mp \delta\varepsilon_{\vec{k}}}{(i\omega_m + E^+_{\vec{k}})(i\omega_m - E^-_{\vec{k}})}. \quad (1)$$

The neutron-proton anomalous propagator has the form

$$F^{\dagger}_{\sigma,\sigma'}(\vec{k},\omega_m) = -\delta_{\sigma,\sigma'}\frac{\Delta^{\dagger}(\vec{k})}{(i\omega_m + E^+_{\vec{k}})(i\omega_m - E^-_{\vec{k}})}, \quad (2)$$

where ω_m are the Matsubara frequencies, the upper sign in $G^{(p/n)}$ corresponds to protons, and the lower to neutrons. The isospin asymmetry lifts the degeneracy of the quasiparticle spectra, thus leading to two separate branches for protons and neutrons,

$$E^{\pm}_{\vec{k}} = \sqrt{\xi^2_{\vec{k}} + \Delta^2_{\vec{k}}} \pm \delta\varepsilon_{\vec{k}}, \quad (3)$$

where

$$\xi_{\vec{k}} \equiv \frac{1}{2}(\varepsilon^{(n)}_{\vec{k}} + \varepsilon^{(p)}_{\vec{k}}), \qquad \delta\varepsilon_{\vec{k}} \equiv \frac{1}{2}(\varepsilon^{(p)}_{\vec{k}} - \varepsilon^{(n)}_{\vec{k}}),$$

and $\varepsilon^{(n,p)}_{\vec{k}}$ are the single particle energies of neutrons and protons. The strong coupling BCS theory is coupled to the Brueckner renormalizaton scheme via the single particle energies defined as $\varepsilon^{(n,p)}_{\vec{k}} = k^2/2m + U^{(n,p)}(k) - \mu^{(n,p)}$; here $U^{(n,p)}(k)$ are the single particle potentials which are derived from the Brueckner theory of asymmetric nuclear matter [15] and $\mu^{(n,p)}$ are the chemical potentials for neutrons and protons, which are derived from the BCS theory self-consistently. Both schemes are normalized to the same densities. The small effect of the feedback of the pairing correlation in the Brueckner calculations of the mean field [39] is, however, neglected.

Using the angle-averaging procedure, which is an adequate approximation for the present purpose (see Ref. [21]) the BCS gap equation for asymmetric nuclear matter can be derived. We find

$$\Delta_l(k) = -\sum_{k'}\sum_{l'} V_{ll'}(k,k')\frac{\Delta_{l'}(k')}{2\sqrt{\xi^2_{k'} + D(k')^2}} \times [1 - f(E^+_{\vec{k}}) - f(E^-_{\vec{k}})], \quad (4)$$

where $D(k)^2 \equiv \Delta_0(k)^2 + \Delta_2(k)^2$ is the angle-averaged neutron-proton gap function, $f(E) = [1 + \exp(\beta E)]^{-1}$ is the Fermi distribution function, $\beta^{-1} = k_B T$, where T is the temperature and k_B is the Boltzmann constant. The driving term, $V_{ll'}$, is the bare interaction in the SD channel. The density matrices of neutrons and protons follow from Eq. (1) after summation over frequencies,

$$n^{(p/n)}_{\sigma}(k) = \left\{\frac{1}{2}\left(1 + \frac{\xi_{\vec{k}}}{\sqrt{\xi^2_{\vec{k}} + \Delta^2_{\vec{k}}}}\right) f(E^{\pm}_{\vec{k}}) + \frac{1}{2}\left(1 - \frac{\xi_{\vec{k}}}{\sqrt{\xi^2_{\vec{k}} + \Delta^2_{\vec{k}}}}\right)[1 - f(E^{\mp}_{\vec{k}})]\right\}. \quad (5)$$

Summation over frequencies in Eq. (2) leads to the density matrix of the particles in the condensate,

$$\nu(\vec{k}) = \frac{\Delta(\vec{k})}{2\sqrt{\xi^2_{\vec{k}} + \Delta^2_{\vec{k}}}}[1 - f(E^+_{\vec{k}}) - f(E^-_{\vec{k}})]. \quad (6)$$

The partial densities of nucleons, in terms of the density matrices, are $\rho^{(p/n)} = \sum_{\vec{k},\sigma} n^{(p/n)}_{\sigma}(k)$. It is essential that the coupled system of Eqs. (4) and (5) is solved self-consistently.

Now we are in the position to write down the key thermodynamic quantities. As the occupation of the quasiparticle states is given by the Fermi-Dirac distribution function, the entropy of the system is given by the combinatorical expression:

$$S = -2k_B\sum_{\vec{k}}\{f(E^+_{\vec{k}})\ln f(E^+_{\vec{k}}) + \bar{f}(E^+_{\vec{k}})\ln\bar{f}(E^+_{\vec{k}}) + f(E^-_{\vec{k}})\ln f(E^-_{\vec{k}}) + \bar{f}(E^-_{\vec{k}})\ln\bar{f}(E^-_{\vec{k}})\}, \quad (7)$$

where $\bar{f}(E^{\pm}_{\vec{k}}) = 1 - f(E^{\pm}_{\vec{k}})$. Equation (7) is an obvious extension of the mean-field expression for the entropy in the symmetrical nuclear matter [21] to the asymmetric case. The internal energy, defined as the grand canonical statistical average of the Hamiltonian, $\mathcal{U} = \langle \hat{H} - \mu^{(n)}\hat{\rho}_n - \mu^{(p)}\hat{\rho}_p\rangle$, reads

$$\mathcal{U}(T,\mu) = \sum_{\sigma\vec{k}}[\varepsilon^{(n)}_{\vec{k}} n^{(n)}_{\sigma}(k) + \varepsilon^{(p)}_{\vec{k}} n^{(p)}_{\sigma}(k)] + \sum_{\vec{k}_1\vec{k}_2} V(\vec{k}_1,-\vec{k}_1;\vec{k}_2,-\vec{k}_2)\nu(\vec{k}_1)\nu(\vec{k}_2), \quad (8)$$

where we have carried out the spin summation in the second term. The nonpairing interaction energy among the quasiparticles and the contribution from the chemical potentials are included in the single particle energies (the first term) of Eq. (8). The second term includes the BCS mean-field interaction among the particles in the condensate. Note that the interaction in the second term of Eq. (8) can be eliminated in terms of the gap Eq. (4). Finally, the thermodynamic potential is given as

$$\Omega(T,\mu) = \mathcal{U}(T,\mu) - TS. \quad (9)$$

The expressions for the entropy and the internal energy in the normal state follow from Eqs. (7) and (8) in the limit $\Delta = 0$. Our main interest below is the change in the thermodynamic potential of the superconducting state with respect to the normal state $\delta\Omega = \Omega_s - \Omega_n$.

Numerical calculations of the pairing gap were carried out using the Paris potential. Figure 1 shows the values of the pairing gap in the $^3S_1$-$^3D_1$ partial wave channel as a function of the temperature at the empirical saturation density $\rho = 0.17$ fm^{-3}. The asymmetry parameter is defined as $\alpha \equiv (\rho_n - \rho_p)/\rho$. For the value $\alpha = 0$, one finds the usual BCS solution: the gap is a monotonically decreasing function of the temperature and vanishes at the critical temperature $T_c = \Delta(T = 0)/1.76$. In the asymmetric

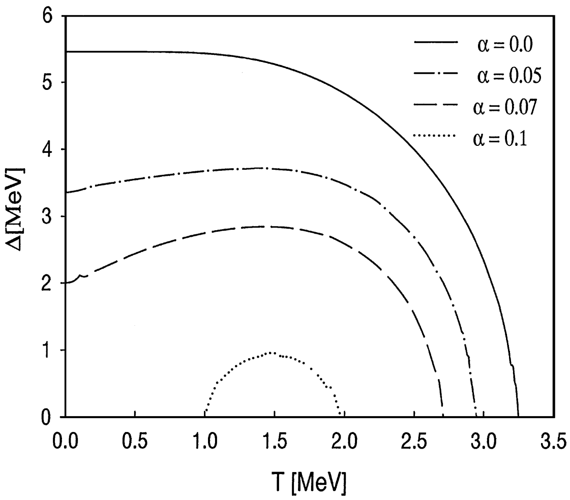

FIG. 1. The pairing gap at the empirical saturation density as a function of the temperature for asymmetries $\alpha = 0$, 0.05, 0.07, and 0.1.

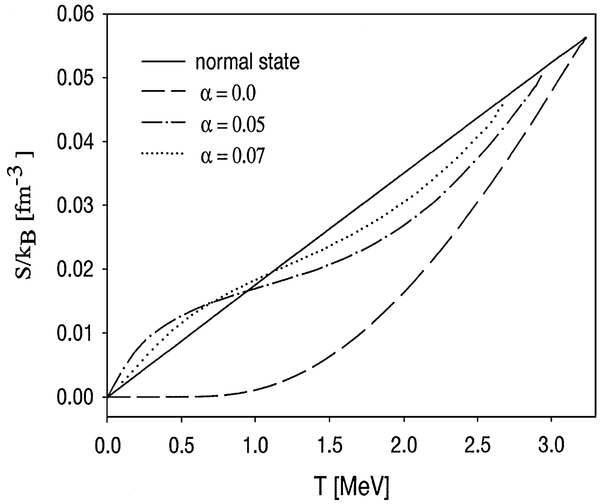

FIG. 2. The entropy of normal (solid line) and superconducting state for $\alpha = 0$, 0.05, and 0.07; the $\alpha = 0.1$ curve is indistinguishable from the normal state entropy curve on the figure's scale.

state the gap develops a maximum at a certain intermediate temperature in the range $0 < T < T_c$; the value of the critical temperature is reduced and the BCS relation between the critical temperature and the value of the gap at $T = 0$ does not hold any longer. For large asymmetries $\alpha \sim 0.1$ the superconducting state exists only in a finite temperature range; it completely vanishes for the critical asymmetry $\alpha_c = 0.11$. This behavior is the result of the interplay between the increasing shift between the radii of the Fermi spheres of neutrons and protons with increasing asymmetry and the smearing of the Fermi surfaces due to *both* interaction driven correlations and the temperature. While the first factor tends to suppress the pairing, the role of the second factor is twofold. On the one hand, the temperature and correlations smear the Fermi surfaces thus increasing the overlap between the two Fermi spheres which promotes the pairing. If, however, the temperatures are high enough the pair correlated states are quenched by the thermal excitation. The pairing gap, hence, has a maximum at some intermediate temperature. The two critical temperatures are controlled by two different mechanisms. The superconductivity vanishes with decreasing temperature at a lower critical temperature when the smearing becomes insufficient to support the pairing. The superconductivity vanishes at the upper critical temperature because of the thermal excitation of the system, as in the standard BCS theory.

Figure 2 displays the entropies of the normal and superconducting states as a function of the temperature at the empirical saturation density. The $\alpha = 0$ behavior of the normal and superconducting entropies is the one expected from the theory of the normal Fermi liquids and the BCS theory, respectively: the entropy of the normal state is a linear function of the temperature; the entropy of the superconducting state is linear close to the critical temperature and decreases exponentially in the low temperature limit. For finite asymmetries the entropy of the normal state is unchanged. The curve of the entropy of the superconducting state, however, shows an anomalous behavior by crossing the entropy curve of the normal state. For temperatures below the crossing point *the entropy of the superconducting state is larger than that of the normal state*. The crossover to the anomalous behavior occurs at the temperature at which the gap reaches its maximum. This anomalous dependence of the entropy of the superconducting state on the temperature implies that this state could be unstable for temperatures below the temperature at which the gap attains its maximum. The internal energy, the first term in Eq. (9), has a definite sign as it vanishes in the normal state and is a quadratic form of the pairing gap. Thus the sufficient condition for the onset of the instability is that the contribution of the second term in Eq. (9) dominates that of the first in the range of the temperatures below the crossover point.

Figure 3 shows the difference in the thermodynamic potentials of the normal and superconducting state $\delta\Omega$. This quantity is negative for all temperatures and asymmetries. The anomalous change of sign of $S_n - S_s$, therefore, does not change the net balance between the energies of the normal and superconducting states. The temperature dependence of $\delta\Omega$ is reminiscent of the temperature dependence of the gap function by virtue of the (approximately) quadratic dependence of the internal energy on the pairing gap. We thus conclude that the superconducting state is stable, at least in the present model, in the whole temperature-asymmetry plane, whenever a nontrivial solution to the gap equation exists. We have verified that the deduced behavior of the $\delta\Omega$ is the consequence of the fact that the contribution of the internal energy to the thermodynamic potential dominates the contribution from the entropy.

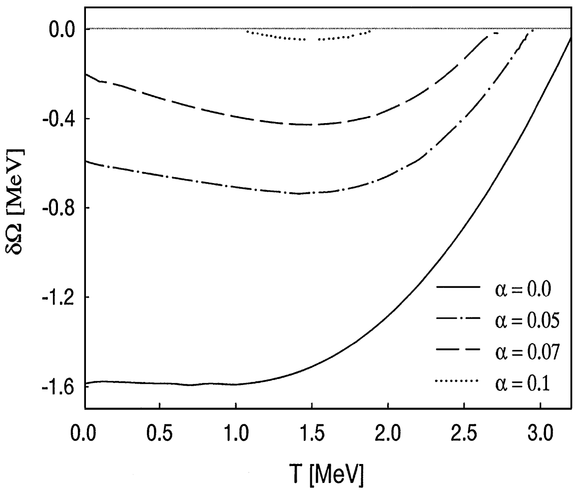

FIG. 3. The difference in the thermodynamic potentials of the superconducting and normal states at the empirical saturation density as a function of the temperature for asymmetries $\alpha = 0$, 0.05, 0.07, and 0.1.

In conclusion, we studied the pairing in the isospin asymmetric nuclear matter using realistic nuclear interactions combined with ladder-renormalized single particle energies. The pairing at the empirical saturation density vanishes for the critical density asymmetry $\alpha_c = 0.11$ at the finite temperature $T \simeq 1.5$ MeV. Our evaluation of the thermodynamic quantities of the isospin asymmetric nuclear matter shows that, while the entropy of the superconducting state becomes larger than that of the normal state below a certain temperature, the thermodynamic potential at the empirical saturation density corresponds to a stable superconducting state in the whole asymmetry-temperature plane.

We acknowledge discussions and correspondence with Professor P. Nozières, Professor G. Röpke, and Professor P. Schuck. A. S. thanks the LNS, INFN for the warm hospitality during his stay in Catania. The work at KVI, Groningen, has been supported through the Stichting voor Fundamenteel Onderzoek der Materie with financial support from the Nederlandse Organisatie voor Wetenschappelijk Onderzoek.

[1] A. L. Goodman, Phys. Rev. C **60**, 014311 (1999), and references therein.
[2] M. Baldo, U. Lombardo, and P. Schuck, Phys. Rev. C **52**, 975 (1995).
[3] B. K.-L. Kratz, J.-P. Bitouzet, F.-K. Thielmann, and B. Pfeiffer, Astrophys. J. **403**, 216 (1993).
[4] B. Chen, J. Dobaczewski, K.-L. Kratz, K. Langanke, B. Pfeiffer, F.-K. Thielmann, and P. Vogel, Phys. Lett. B **355**, 37 (1995).
[5] G. E. Brown and H. A. Bethe, Astrophys. J. **423**, 659 (1994).
[6] A. I. Larkin and Yu. N. Ovchinnikov, Sov. Phys. JETP **20**, 762 (1965).
[7] P. Fulde and R. A. Ferrel, Phys. Rev. **135**, 550 (1964).
[8] A. Leggett, Phys. Rev. **140**, A1869 (1965).
[9] A. Leggett, Rev. Mod. Phys. **41**, 331 (1975).
[10] P. Nozières and F. Pistolesi, Eur. Phys. J. B **10**, 649 (1999).
[11] M. Alford, K. Rajagopal, and F. Wilczek, Phys. Lett. B **422**, 247 (1998); M. Alford, J. Berges, and K. Rajagopal, hep-ph/9908235.
[12] R. Rapp, T. Schäfer, E. V. Shuryak, and M. Velkovsky, Phys. Rev. Lett. **81**, 53 (1998).
[13] D. Blaschke and C. D. Roberts, Nucl. Phys. **A642**, 197 (1998).
[14] R. Pisarski and D. Rischke, nucl-th/9903023.
[15] W. Zuo, I. Bombaci, and U. Lombardo, Phys. Rev. C **60**, 024605 (1999).
[16] D. J. Thouless, Ann. Phys. (N.Y.) **10**, 553 (1960).
[17] V. J. Emery and A. M. Sessler, Phys. Rev. **119**, 248 (1960).
[18] A. L. Fetter and J. D. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, New York, 1971).
[19] T. Alm, G. Röpke, and M. Schmidt, Z. Phys. A **337**, 355 (1990).
[20] B. E. Vonderfecht, C. C. Gearhart, W. H. Dickhoff, A. Polls, and A. Ramos, Phys. Lett. B **253**, 1 (1991).
[21] M. Baldo, I. Bombaci, and U. Lombardo, Phys. Lett. B **283**, 8 (1992).
[22] T. Alm, B. L. Friman, G. Röpke, and H. Schulz, Nucl. Phys. **A551**, 45 (1993).
[23] T. Alm, G. Röpke, A. Sedrakian, and F. Weber, Nucl. Phys. **A406**, 491 (1996).
[24] A. Sedrakian, T. Alm, and U. Lombardo, Phys. Rev. C **55**, R582 (1997).
[25] M. Baldo, U. Lombardo, P. Schuck, and A. Sedrakian, *Condensed Matter Theories*, edited by J. W. Clark (Nova Science Publishers, Commack, New York, 1997), Vol. 12, pp. 265–277.
[26] Ø. Elgarøy, L. Engvik, E. Osnes, and M. Hjorth-Jensen, Phys. Rev. C **57**, R1069 (1998).
[27] J. Engel, K. Langanke, and P. Vogel, Phys. Lett. B **389**, 211 (1996).
[28] J. Engel, S. Pittel, M. Stoitsov, P. Vogel, and J. Dukelsky, Phys. Rev. C **55**, 1781 (1997).
[29] J. Dobaczewski, W. Nazarewicz, T. R. Werner, J. F. Berger, C. R. Chinn, and J. Decharge, Phys. Rev. C **53**, 2809 (1996).
[30] W. Satula and R. Wyss, Phys. Lett. B **393**, 1 (1997).
[31] O. Civitarese, M. Reboiro, and P. Vogel, Phys. Rev. C **56**, 1840 (1997).
[32] O. Civitarese and M. Reboiro, Phys. Rev. C **56**, 1179 (1997).
[33] G. Röpke, A. Schnell, P. Schuck, and U. Lombardo, nucl-th/9906092.
[34] J. W. Clark, C. G. Källman, C. H. Yang, and D. A. Chakkalakal, Phys. Lett. **61B**, 331 (1976).
[35] J. M. C. Chen, J. W. Clark, R. D. Davé, and V. V. Khodel, Nucl. Phys. **A555**, 59 (1993).
[36] J. A. Niskanen and J. A. Sauls (unpublished).
[37] J. Wambach, T. L. Ainsworth, and D. Pines, Nucl. Phys. **A555**, 128 (1993).
[38] H.-J. Schulze, J. Cugnon, A. Lejeune, M. Baldo, and U. Lombardo, Phys. Lett. B **375**, 1 (1996).
[39] U. Lombardo, H.-J. Schulze, and W. Zuo, Phys. Rev. C **59**, 2927 (1999).