Clogging Time of a Filter

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We study the time until a filter becomes clogged due to the trapping of suspended particles as they pass through a porous medium. This trapping progressively impedes and eventually stops the flow of the carrier fluid. We develop a simple description for the pore geometry and the motion of the suspended particles which, together with extreme-value statistics, predicts that the distribution of times until a filter clogs has a power-law long-time tail, with an infinite mean clogging time. These results and its consequences are in accord with simulations on a square lattice porous network.

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In this Letter, we investigate the time for a filter to clog. In a typical filtration process, a dirty fluid is "cleaned" by passing it through a porous medium to remove the suspended particles. The medium enhances filtering efficiency by increasing both the available filter surface area for trapping suspended particles as well as the exposure time of the suspension to the active surfaces. Such a mechanism is the basis of water purification, air filtration, and many other separation processes [1,2]. As suspended particles become trapped, the fluid permeability of the medium gradually decreases, and eventually the filter clogs. Determining the time dependence of this clogging is basic to predicting when a filter is no longer useful, either because of reduced throughput or reduced filtration efficiency, and should be discarded.

While clogging has been extensively investigated by engineers, much of the previous literature on this phenomenon is either empirical or incorporates so many microscopic details that it is difficult to draw general conclusions about clogging kinetics [3]. In contrast, we develop a minimalist model that provides an intuitive understanding for clogging which should apply to simple and easily prepared porous media, such as unconsolidated bead packs [4]. Our model is based on approximating the clogging of a porous medium by the clogging of a single parallel array of pores which are blocked in decreasing size order (Fig. 1). The use of a single-layer system is based on previous studies of filtration which showed that clogging preferentially occurs in the upstream end of the network when particles and pores are of comparable size [5,6]. The single-layer system represents the ultimate limit of this gradient-controlled process.

The approximation of size-ordered blocking is based on the commonly used picture that a particle has a probability proportional to the relative flux to enter a given open pore from among the outgoing pores at a junction [2,5,7]. For Poiseuille flow, this flux is proportional to r^4, where r is the pore radius. Size-ordered blocking arises by replacing the exponent 4 in this flow-induced entrance probability by ∞. This approximation is relatively accurate for a broad distribution of pore radii. The size ordering also provides a direct correspondence between the radius of the currently blocked pore and the time until it is blocked. By using extreme value statistics [8] to compute the radius distribution of the last few open pores, together with the connection between the radius of the currently blocked pore and the network permeability, we determine the distribution of clogging times. The predictions of this idealized modeling agree well with detailed simulations of the flow field evolution and the attendant clogging kinetics on a square lattice porous network.

In our network simulations, each unit length bond corresponds to a pore and sites represent pore junctions. We assume Poiseuille flow, in which the fluid flux through a bond of radius r_i is proportional to $-r_i^4\nabla p$, where ∇p is the local pressure gradient, when a fixed overall pressure drop is imposed. Our simulations are based on (i) solving the flow field of the network, (ii) constructing the probabilistic motion of dynamically neutral suspended particles in accordance with the local flow and the flow-induced entrance probability at each junction, (iii) implementing bond blocking when mandated, and repeating (i)–(iii) each time a bond gets blocked. Particles are injected at a *finite* rate

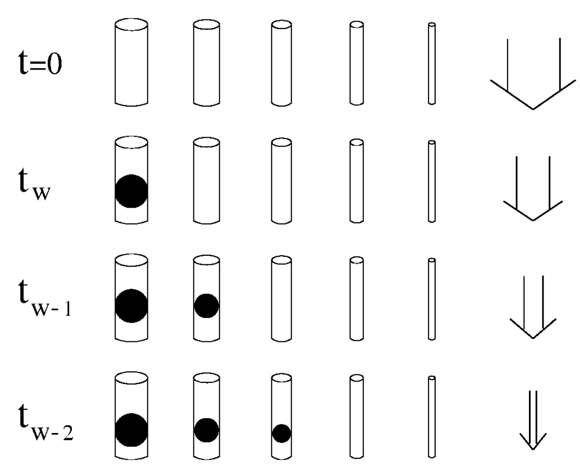

FIG. 1. Schematic clogging evolution in a parallel array of w bonds of distributed radii. Bonds are blocked in decreasing size order. The overall flow decreases more slowly in the latter stages (vertical arrow). The total flow is inversely related to the time until each blockage event, t_k, $k = w, w - 1, \ldots$.

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which is proportional to the overall fluid flux; this corresponds to a fixed nonzero density of suspended particles in the fluid. This new feature is in contrast to many previous simulations, where particles were injected singly and tracked until trapping occurred [2,5,6]. The latter corresponds to an arbitrarily dilute suspension, a limit which is useful for understanding the percolation process induced by clogging [5–7,9]. However, to determine the time evolution of a filter it is necessary to consider a finite-density suspension.

For the trapping mechanism we adopt size exclusion, in which a particle of radius r_p is trapped within the first bond encountered with $r_b < r_p$ [7]. We assume that a trapped particle blocks a pore completely and permanently. The difference between partial and total blockage in a single trapping event appears to be immaterial for long-time properties [6]. The overall trapping rate is then controlled by the relative sizes of particles and bonds. For simplicity and because it often occurs in porous media [10], we consider the Hertz distribution of pore and particle radii, respectively,

$$b(r) = 2\alpha r e^{-\alpha r^2}, \qquad p(r) = 2re^{-r^2}, \tag{1}$$

with the ratio between the average bond and particle radii $s = 1/\sqrt{\alpha}$ a basic parameter which determines the nature of the clogging process.

First, consider the case of pores larger than particles ($s \gtrsim 1$). Many particles must therefore be injected before a sufficiently large particle arises which can block the largest pore. Since the permeability of the system remains nearly constant when only a few pores are blocked, the time between successive particle injection events during this initial stage is nearly constant. Once the largest pores are blocked, it takes many fewer particles to block the remaining smaller pores and clogging proceeds more quickly. Thus the clogging time is dominated by the initial blockage events.

To estimate the clogging time in this large-pore limit, let us find the number of injection events before encountering a particle large enough to block the largest bond. The radius of this bond is given by the criterion [8]

$$\int_{r_{\max}}^{\infty} 2\alpha r e^{-\alpha r^2}\, dr = \frac{1}{w}, \tag{2}$$

that one bond out of w has radius $r_{\max}$ or larger. This gives $r_{\max}^{(b)} = s\sqrt{\ln w}$. Similarly, the largest particle out of N has radius $r_{\max}^{(p)} = \sqrt{\ln N}$. Thus $N_1 \approx w^{s^2}$ particles typically need to be injected before one occurs which is large enough to block the largest bond. Continuing this picture sequentially, the radius of the kth-largest bond is given by Eq. (2), but with $1/w$ replaced by k/w; this gives $r_k = s\sqrt{\ln(w/k)}$. Therefore $N_k \approx (w/k)^{s^2}$ particles need to be injected before the kth-largest bond is blocked. Since N_k decreases rapidly with k, the initial blockage events control the clogging time T, whose lower bound is given by $T \approx N_1/w > w^{s^2-1}$. The factor of $1/w$ arises because the particle injection rate is proportional to w for a fixed-density suspension and a fixed pressure drop. In fact, N_1/w represents a relatively crude lower bound for T in the large-pore limit, since clogging will occur far beyond the first layer of pores.

The latter case where pore sizes are comparable or smaller than particles ($s \lesssim 1$) is simpler, as each particle injection event typically leads to the clogging of the first pore entered. Now initial pores are blocked quickly, while later pores are blocked more slowly because the overall flow rate decreases significantly near the end of the clogging process (Fig. 1). We argue that clogging is dominated by the times for these later blockage events. First, let us determine how the flow rate varies as the last few bonds get blocked. Since only the smallest bonds remain open near clogging, the permeability is determined by these smallest radii. We estimate the radius of the kth smallest bond from $\int_0^{r_k} 2\alpha r e^{-\alpha r^2}\, dr = k/w$, which gives $r_k = s\sqrt{k/w}$. The permeability of a parallel bundle of these k smallest pores is then

$$\kappa(k) = \sum_{j=1}^{k} r_j^4 \approx s^4 \sum_{j=1}^{k} (j/w)^2 \sim s^4 k^3/w^2, \tag{3}$$

while the initial permeability (obtained by setting $k = w$ above) is simply $\kappa(w) = s^4 w$.

Since the total flow is proportional to the permeability for a fixed pressure drop, the time increment t_k between blocking the $(k - 1)$st-smallest and the kth-smallest pore scales as

$$t_k = \frac{\kappa(w)}{w\kappa(k)} \sim \frac{w^2}{k^3}. \tag{4}$$

The factor w in the denominator again accounts for a particle injection rate proportional to w. The clogging time T is now dominated by the time to block the smallest bond, so that $T > t_1 \propto w^2$. Thus the relative pore and particle radii drives a transition in the ultimate clogging behavior. For $s^2 > 3$, clogging is dominated by the initial blockage events and $T > w^{s^2-1}$, while for $s^2 < 3$ the last events dominate, leading to $T > w^2$.

For the small-pore limit, we carry this analysis further and obtain the distribution of clogging times from which the mean clogging time is *divergent.* Nevertheless, suitably defined moments of the clogging time distribution scale as w^2. The clogging time distribution is directly related to the radius distribution of the smallest bond, since this bond ultimately controls the clogging of a single parallel layer of bonds. For the Hertz distribution, the probability that a given bond has a radius greater than or equal to r, $B_>(r)$, is

$$B_>(r) = \int_r^{\infty} 2\alpha r e^{-\alpha r^2}\, dr = e^{-\alpha r^2}. \tag{5}$$

Then the radius distribution of the smallest bond from among w, $S_w(r)$, is given by [8]

$$S_w(r) = wb(r)B_>(r)^{w-1} = 2\alpha wre^{-\alpha wr^2}. \qquad (6)$$

The first equality expresses the fact that one of the w bonds has (smallest) radius r, with probability $b(r)$, and the other $w - 1$ must have radii larger than r.

From the basic connections between permeability, pore radius, and time scale [Eqs. (3) and (4)], and the fact that the clogging time is dominated by t_1, we deduce

$$T \sim t_1 \approx \frac{1}{w}\frac{\kappa(w)}{\kappa(1)} = \frac{s^4}{r_1^4}, \qquad (7)$$

while the clogging time distribution, $P_w(T)$, is directly related to the smallest bond radius distribution through $P_w(T)dT = S_w(r)dr$. Using Eqs. (6) and (7), we obtain the basic result (independent of system length):

$$P_w(T) \sim \frac{w}{T^{3/2}}e^{-w/T^{1/2}}. \qquad (8)$$

The power law applies in the time range $w^2 < t < w^2N^2$. The former limit corresponds to the size of the typical smallest pore in a single realization, $s/\sqrt{w}$, while the latter corresponds to the smallest pore from among N realizations of the system, $s/\sqrt{Nw}$. The short-time cutoff in Eq. (8) arises from those realizations where the smallest bond happens to be anomalously large. Other coincident particle and bond radius distributions lead to similar forms for $P_w(T)$, but with different quantitative details. For example, if $S_w(r) \sim r^\mu$ as $r \to 0$, then the long-time exponent in $P_w(T)$ is $-(\mu + 5)/4$.

We test our predictions by simulations of the motion and trapping of suspended particles in the lattice network. Our algorithm is event driven to bypass the slowing down of the flow as clogging is approached. At an intermediate stage, there are a finite number of particles in the network, consistent with a unit pressure gradient and a constant-density suspension. Particles are defined to be always at lattice sites and each (including newly injected particles) evolves by (i) moving to the next downstream site along an outgoing bond i (with entrance probability equal to the fractional flux into this bond), (ii) blocking the downstream bond it has entered, or (iii) remaining stationary. The probability of each of these possibilities is defined so that the process corresponds to particles moving, on average, at the local velocity.

To achieve this, each particle attempts to move to the next junction, via the preselected bond, with probability proportional to $v_i/v_{\max}$, where v_i is the local bond velocity and $v_{\max}$ is the instantaneous largest particle velocity in the system. If the attempt occurs, the particle either moves to the next downstream site, or, if the particle is too large, blocks the bond. After a single update of all particles, the time is incremented by $\Delta t = 1/v_{\max}$ to ensure that, on average, a particle moving along bond i has speed v_i.

After all particles undergo move attempts, $\mathcal{N} \propto \phi\Delta t$ new particles are injected into the network, where ϕ is the overall fluid flux. This ensures the correct absolute velocity for each particle. Since the time increment systematically increases as bonds get blocked, the particle injection rate progressively slows. This is in contrast to earlier multiparticle filtration simulations, where injection rate was decoupled from overall flow [7].

Figure 2(a) shows the clogging time distribution from simulations on the square lattice and the bubble model of the same size. The latter is a series of parallel bond arrays with perfect mixing at each junction; this idealized system has been previously found to account for the geometrical aspects of clogging [6]. The agreement between the two distributions is remarkably good, suggesting that the schematic evolution proposed in Fig. 1 is quantitatively correct. The tails of the distributions both appear to decay as $t^{-3/2}$, and scaled data for the clogging time distribution

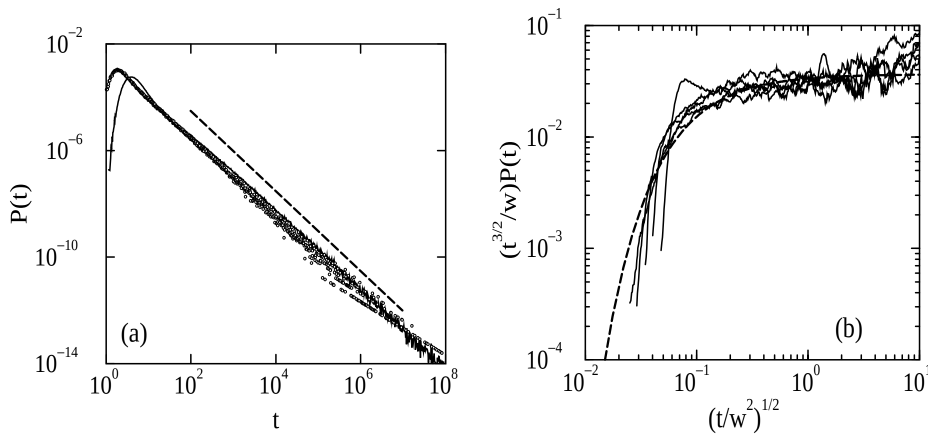

FIG. 2. (a) Clogging time distribution for the bubble model (continuous curve) and the square lattice (data points) for length $L = 10$ and width $w = 20$. Time units correspond to an initial injection rate of 0.05 particles per unit width per unit time. The dashed line has slope $-3/2$. (b) Scaled clogging time distribution for square lattices of $L = 10$ and $w = 20$, 40, 60, 100, and 200 (right to left). The data are based on 10 000 realizations and are smoothed over a 12-point range. The dashed line is the prediction of Eq. (8).

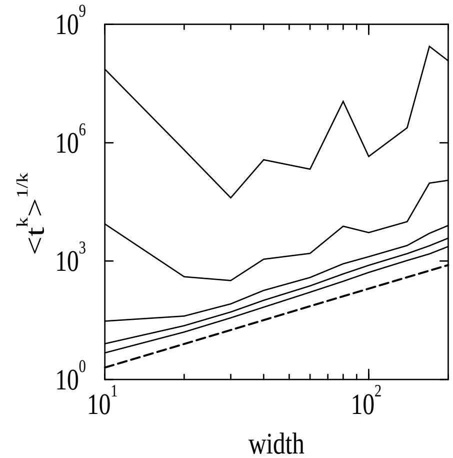

FIG. 3. Width dependence of the moments $\langle t^k \rangle^{1/k}$ for $k = 1, 1/2, 1/3, 1/4$, and $1/6$ (top to bottom). The dashed line corresponds to w^2 growth.

for systems of various widths w also agree with the functional form in Eq. (8) [Fig. 2(b)].

An important consequence of the power-law tail in the clogging time distribution is a transition in the corresponding moments

$$\langle t^k \rangle = \int_0^\infty t P_w(t)\, dt \approx \int_{w^2}^{N^2 w^2} w t^{k-3/2}\, dt\,. \qquad (9)$$

Here the short-time cutoff in Eq. (8) is approximated by the lower limit in the integral. Consequently,

$$M_k(w) \equiv \langle t^k \rangle^{1/k} \sim \begin{cases} w^2 N^{2-1/k} & k > 1/2 \\ w^2 (\ln N)^2 & k = 1/2 \\ w^2 & k < 1/2\,. \end{cases} \qquad (10)$$

Thus the mean clogging will exhibit large sample-to-sample fluctuation and diverge when the number of realizations is infinite. In contrast, the moments M_k with $k < 1/2$ are well behaved. This is illustrated in Fig. 3, where $M_k(w)$ varies erratically with w for $k = 1$ and $1/2$, but then grows smoothly as w^2 for $k \leq 1/3$.

The behavior of M_k raises the question of when a filter is no longer useful. Waiting until complete clogging is impractical because of large fluctuations in the clogging time and late-stage poor filter performance. It would be more useful to operate a filter only until the permeability decays to a (small) fraction of its initial value, such that reasonable flow and trapping efficiency are maintained, while minimizing fluctuations in this threshold time. Simulations indicate that the time to reach permeability fraction f, t_f, is proportional to $w^{2/3} f^{-1/2}$ for $10^{-10} \lesssim f \lesssim 10^{-3}$, with the distribution of t_f progressively broadening for decreasing f (Fig. 4). Thus waiting until the permeability decays to a fixed but not too small fraction provides a reliable criterion for when a filter should be discarded. A consequence of $t_f \propto w^{2/3} f^{-1/2}$ is that the permeability decays as $1/t^2$ over a wide range. Because of this rapid decrease, a practical filter needs to have pores typically larger than particles to have a reasonable lifetime. These issues and understanding the efficiency of a filter throughout its useful life are currently under investigation.

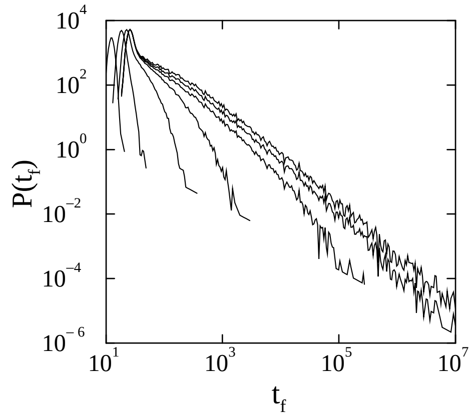

FIG. 4. Probability that a filter with $w = 200$ reaches permeability fraction f at time t_f, for $f = 4^{-n}$, with $n = 2, 4, 6, 8, 12, 16$, together with the time distribution until complete clogging (left to right).

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