Quantum Logic Gates based on Coherent Electron Transport in Quantum Wires

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(Received 1 November 1999)

It is shown that the universal set of quantum logic gates can be realized using solid-state quantum bits based on coherent electron transport in quantum wires. The elementary quantum bits are realized with a proper design of two quantum wires coupled through a potential barrier. Numerical simulations show that (a) a proper design of the coupling barrier allows one to realize any one-qbit rotation and (b) Coulomb interaction between two qbits of this kind allows the implementation of the CNOT gate. These systems are based on a mature technology and seem to be integrable with conventional electronics.

PACS numbers: 03.67.Lx, 68.65.+g, 73.23.Ad

Quantum information theory is being rapidly developed on the basis of the new possibilities offered by quantum mechanics. As in classical information theory, two-state ($|0\rangle$ and $|1\rangle$) systems (qbits) are required in order to encode and process information. However, the possibility for a qbit to be in any superposition of the two states has no correspondence in classical information theory.

In order to perform any quantum computation on the qbits one must find how to physically define a number of quantum gates operating on them. It has been proved that any logic gate can be decomposed into a sequence of two elementary quantum gates: an arbitrary single qbit rotation and the two-qbit CNOT gate [1].

Even though the quantum theory of computation and communication has been rapidly developed in recent years, it still seems very difficult to find a quantum system suitable for the realization of the fundamental quantum logic gates, technologically feasible, and easily integrable into a traditional circuitry. Furthermore, since many qbits are necessary to manipulate quantum information, the physical system which represents the qbit must be reliable and easily reproducible with high quality standards even for structures with a large number of units.

In this paper we present a proposal for semiconductor qbits based on the physical properties of coherent electron propagation in quantum wires, and we show how to design the qbits in order to implement some fundamental logic gates. The properties of the qbits have been studied by means of a numerical solution of the time-dependent 2D Schrödinger equation for a number of physical cases of interest. The discretized time-dependent Schrödinger equation has been solved by means of a simple finite-difference relaxation method. It has been applied at each step of the time evolution performed in a Crank-Nicholson scheme with $\Delta t = 0.2$ fs. The electron-electron interaction is accounted for, at each time step, by means of an explicit calculation of the Coulomb potential generated by each electron, added to the structure potential.

Let us consider a system of two identical semiconductor quantum wires separated by a potential barrier of finite height (see Fig. 1). We will assume in the following coherent electron propagation along the wires. This condition can nowadays be achieved at low temperatures in very pure materials (see, e.g., [2]). To our purposes the quantized space dimension orthogonal to the plane of the figure is not involved in the physical processes under discussion and, accordingly, is not included in our calculations. The wave functions of the two lowest-energy bound states along the quantized direction x orthogonal to the wires have even and odd parities and their corresponding energies are ε_e and ε_o, with $\varepsilon_e < \varepsilon_o$. As for the case

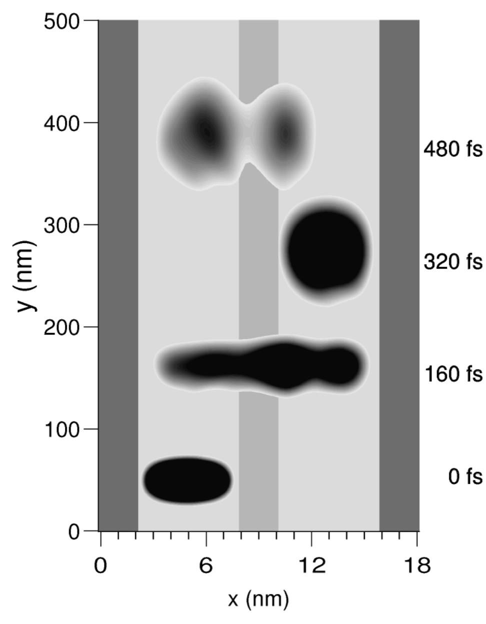

FIG. 1. Oscillation of the electron density between two coupled quantum wires at different times. The lateral extension of the single wire is $L = 6$ nm. The potential in the outer zones is assumed to be infinite. The barrier height between the two wires is 0.1 eV. It is assumed that at $t = 0$ the electron is confined in the left wire with a wave function given by Eq. (1) with $x_0 = 5$ nm, $y_0 = 50$ nm, and $\sigma = 10$ nm. k_0 corresponds to an energy of 0.1 eV. Note the different scales of the x and y axes.

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of two coupled potential wells [3], if the electron is put into one of the two wires, i.e., in a state which is a linear combination of the even and odd states, it oscillates between the two wires with frequency $\omega = \Delta\varepsilon/\hbar$, where $\Delta\varepsilon = \varepsilon_o - \varepsilon_e$. This result is shown in Fig. 1 as obtained by a numerical solution of the time-dependent Schrödinger equation. The material parameters of GaAs have been used. The initial condition along the x axis has been taken as the ground state of an infinite square well having the same lateral extension L of the single wire; along the "free" y direction a minimum uncertainty wave packet has been assumed for simplicity (a different choice for the longitudinal part of the initial state does not compromise the operation performed by the gate):

$$\psi(x,y) = \sqrt{\frac{2}{L}}\cos[\pi(x - x_0)]\frac{1}{\sqrt{\sigma\sqrt{2\pi}}} \times e^{-(y-y_0/2\sigma)^2}e^{ik_0y}. \quad (1)$$

The values of the parameters used in the calculations are specified in the caption of Fig. 1.

Let us now consider two wires separated by a potential barrier of height V_0 enough to prevent tunneling of the electron. A "window" is introduced between the two wires, defined by a region of length L_W where the central barrier height is lowered to a value V_W. By adjusting the parameters of this coupling-potential window and the velocity of the electron, the system can be designed in such a way as to produce an assigned transfer process of the wave function between the two wires while the electron crosses the region of the window.

This effect is illustrated in Fig. 2. In Fig. 2a the window is such that the electron undergoes half a period of oscillation. If the electron is injected into the left wire, it will exit from the right wire and vice versa. In Fig. 2b a shorter window equally splits the wave function between the two wires.

The physical properties discussed above suggest that the considered couple of wires can be used as a quantum bit. The two states of the bit are represented by the electron in the transverse ground state of each of the two wires. The dynamical evolution occurring in the coupling-window region implements a quantum logic gate. In the case of Fig. 2a, the NOT operation is realized by transferring the electron from one wire to the other.

In order to implement the CNOT gate two qbits must be considered. To this purpose the following structure has been studied. A NOT gate is first realized similar to the one shown in Fig. 2a, where, however, the coupling window corresponds to five half periods of the electron wave function between the two wires (referred to in the following as wires 1 and 2 or "data qbit"). The functionality of the NOT gate is preserved also for this case, as confirmed by numerical simulations, shown in Fig. 3a. Another couple of wires (the control qbit) is added to this quantum bit. One of them (wire 3) is symmetrically located above

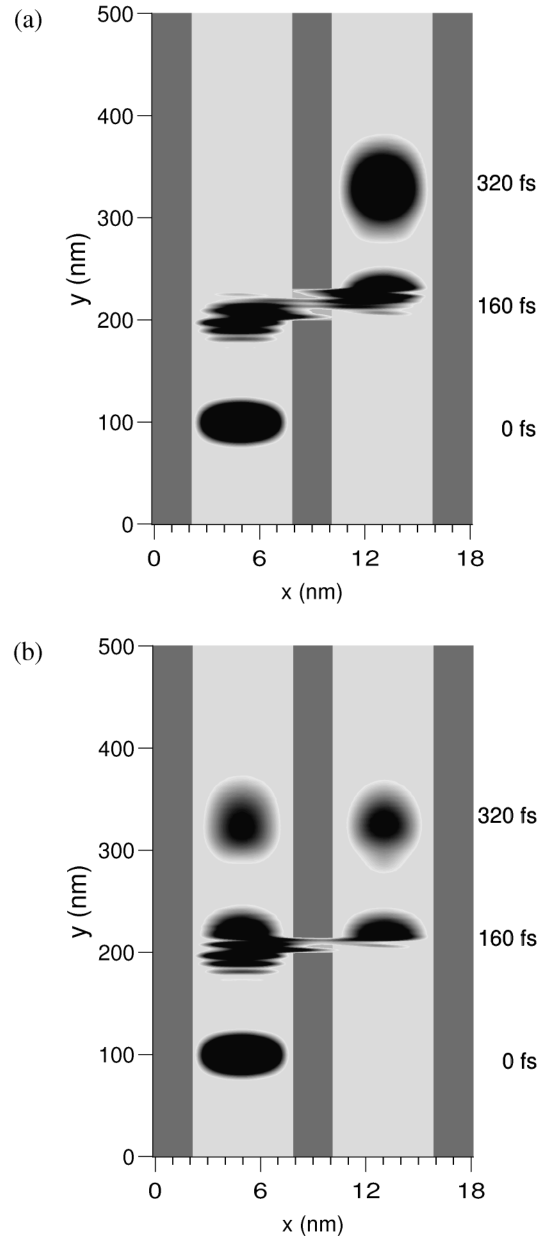

FIG. 2. Electron density in the two wires of Fig. 1 at different times, where the coupling region is reduced to a suitable window in order to realize (a) a complete transfer of the electron from the left to the right wire (NOT gate) and (b) an equal splitting of the wave function between the two wires (see text). Again it is assumed that at $t = 0$ the electron is confined in the left wire with the same longitudinal central wave vector as in Fig. 1.

the two wires of the NOT gate at a distance such that Coulomb interaction between electrons in different wires is effective. The second wire of the control qbit (wire 4) is located far enough from the other three in order to prevent any Coulomb interaction or electron transfer. The two states of the control qbit are again defined as the two transverse ground states of wires 3 and 4. The structure realizes a CNOT gate as follows. Suppose that an electron coherently propagates along one of the two wires of the

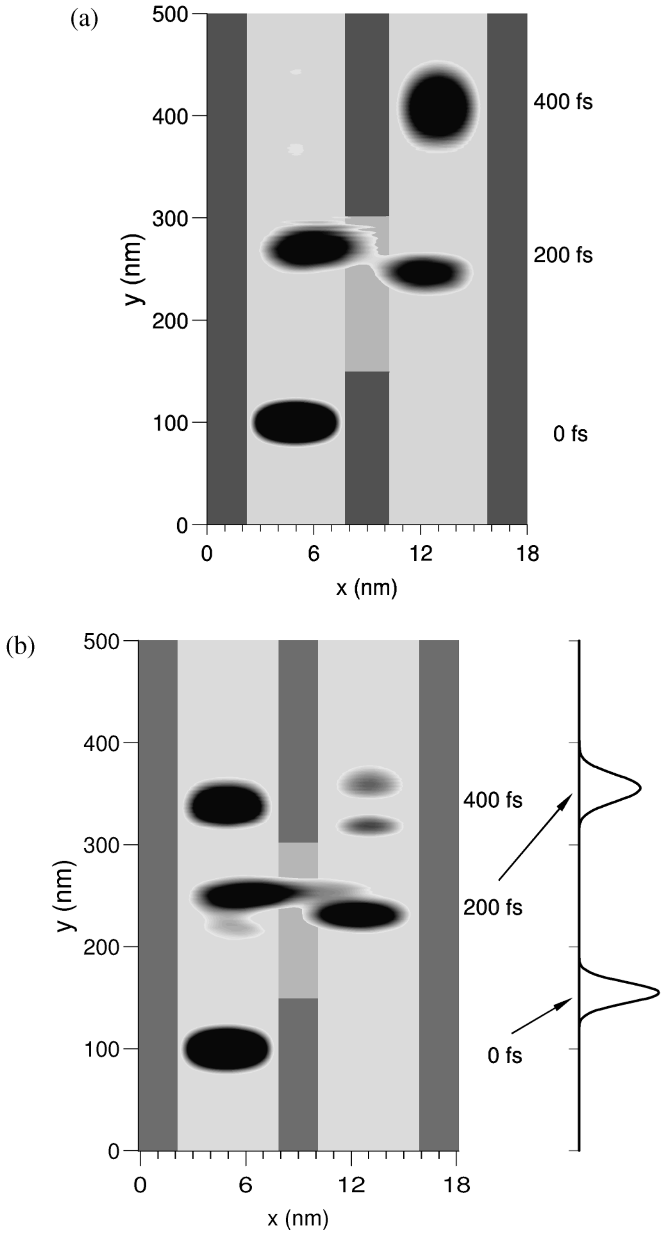

FIG. 3. Electron density in the two wires of the data qbit at different times for the CNOT gate. The control qbit, not shown in the picture, consists of two not-coupled quantum wires parallel to the data-qbit wires and lying in an xy plane 4 nm distant from the plane of the figure. The 0 control-qbit wire; is over the barrier between the two data-qbit wires; the 1 control-qbit wire is far away from data-qbit wires. In case (a) the logic state of the control qbit is $|1\rangle$ (see text) so the data qbit changes from $|0\rangle$ to $|1\rangle$; in case (b) the control qbit (whose evolution is shown in the one dimensional graph on the side) is in state $|0\rangle$ and prevents the data qbit from changing his state.

data qbit. If another electron is launched in wire 4, then, due to the large distance between the two electrons in the two qbits, the Coulomb interaction between the two particles is very weak and does not affect at all the evolution of the electronic motion in the first qbit, which changes its logical state, thus leading to the same result reported in Fig. 3a. If instead the electron of the control qbit is launched in wire 3 slightly anticipated with respect to the first electron, due to the Coulomb interaction between the two electrons, the motion of the wave packet in the data qbit is slowed down. The physical parameters of the system and the anticipation of the control electron have been properly designed, so that the crossing time of the coupling-potential window corresponds to six half periods of oscillation of the electron wave function between the two wires. As a result the electron eventually ends up in the original wire and the data qbit does not change its logical state as can be seen from the simulation results reported in Fig. 3b, where the electron density in wire 3 (modeled in a one-dimensional scheme) at different times is shown in the plot on the right side of the figure. As can be seen in the figure the electron in the data qbit terminates its evolution almost entirely in the original wire. Residues of the electron wave function in the second wire of the data qbit do not jeopardize the functionality of the gate and should be reduced through a more accurate search for optimal parameters.

The preparation of the synchronized states of the qbits involved in the CNOT gate is probably the highest experimental difficulty for the realization of our proposal. It could be managed using two single electron pumps [2,4] driven by the same periodic potential pulse.

The measurement of the electrical state of the wire could be performed using tunnel double junctions that are sensitive detectors for charge and can be used to count single electrons [5]. Concerning possible interferences on the gate functionality of the detection devices, they can be avoided by connecting such devices to the system through a long enough quantum wire.

For a formal description of the qbit proposed above, let us consider the qbit states $|0\rangle$ and $|1\rangle$ as linear combinations of the even and odd states which constitute the transverse (x direction) component of the wave function at the initial time:

$$|0\rangle = \frac{1}{\sqrt{2}}\,(|\psi_e\rangle + |\psi_o\rangle), \quad |1\rangle = \frac{1}{\sqrt{2}}\,(|\psi_e\rangle - |\psi_o\rangle)\,. \tag{2}$$

The phase rotation due to the longitudinal (y direction) dynamics does not influence the quantum computation performed by the system, since it enters as a multiplying factor in Eqs. (2). The same is true for the time-evolution phase factor due to the transverse dynamics outside the coupling window.

While the electron is crossing the window, the $|0\rangle$ and $|1\rangle$ states develop into

$$\frac{e^{-i\omega_e T}}{\sqrt{2}}\,(|\psi_e\rangle + e^{-i\theta}|\psi_o\rangle) \tag{3}$$

and

$$\frac{e^{-i\omega_e T}}{\sqrt{2}}\left(|\psi_e\rangle - e^{-i\theta}|\psi_o\rangle\right), \tag{4}$$

respectively, where $\theta = (\omega_o - \omega_e)T$; ω_o and ω_e are the frequencies of the odd and even states, respectively, and T is the propagation time along the coupling window. The matrix representing the time evolution of the system results in being

$$\mathbf{S}(\theta) = \frac{1}{2}\, e^{-(i/2)(\omega_e+\omega_o)T}\begin{pmatrix} \cos(\frac{\theta}{2}) & i\sin(\frac{\theta}{2}) \\ i\sin(\frac{\theta}{2}) & \cos(\frac{\theta}{2}) \end{pmatrix}. \tag{5}$$

The length L_W of the coupling-potential window is chosen in such a way as to obtain the desired value for θ, i.e., $\theta = \theta(L_W)$. For the case in Fig. 2 L_W has been chosen such that (a) $\theta = \pi$ and (b) $\theta = \pi/2$. Moreover, by introducing the rotation matrix $\mathbf{R}(\phi)$, expressed by

$$\mathbf{R}(\phi) = \begin{pmatrix} e^{i(\phi/2)} & 0 \\ 0 & e^{-i(\phi/2)} \end{pmatrix} = e^{-i(\phi/2)}\begin{pmatrix} e^{i\phi} & 0 \\ 0 & 1 \end{pmatrix}, \tag{6}$$

the universal transformation $\mathbf{U}$ [6] can be constructed as

$$\mathbf{U}(\alpha,\beta,\theta) = \mathbf{R}\left(\alpha - \frac{\pi}{2}\right)\mathbf{S}(\theta)\mathbf{R}\left(\beta + \frac{\pi}{2}\right). \tag{7}$$

The rotation $\mathbf{R}$ of the qbit state by an arbitrary phase can be realized by introducing a suitable potential barrier in one of the wires that slightly delays the propagation of the wave packet along that wire [7]. This is in strict analogy with the introduction of a slab of a medium with suitable refraction index along the optical path of a photon in an interference experiment.

In conclusion, coherent electron transport in systems of couples of quantum wires can be used to design qbits and to implement fundamental quantum logic gates. In particular, numerical solutions of the Schrödinger equation show that, with suitable design, the quantum NOT and CNOT gates can be realized. Furthermore, other devices for universal gates have been proposed. The structures of wires studied in this paper can be experimentally realized in a vertical configuration where the two wires of the data qbit are grown one above the other and the "close" wire of the control qbit is located on a side.

The physical systems proposed in this paper should be already experimentally realizable and tested using frontier mesoscopic semiconductor technology [8]. Furthermore, in contrast with other proposals present in the literature, they are based on a mature technology and should be better integrable with conventional electronics.

As a last relevant consideration, the features of coherent electron propagation discussed above offer interesting and innovative new possibilities for single-electron architectures in classical computation. In this last case coherence of the electron wave function is required only during the time required to cross the potential window.

This work has been partially supported by MURST under the 40% Project on Quantum Computation and by the U.S. Office of Naval Research (Contract No. N00014-98-1-0777).

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[7] It should be noticed that typical values for the height of the barrier used to this purpose are low enough to keep the reflected part of the wave function negligible. For example, in order to obtain a rotation $\mathbf{R}(\phi = \pi)$ the potential barrier introduced is 0.01 eV high and 80 nm long. This barrier, for an electron with a kinetic energy of 0.1 eV (as in Fig. 2), has a transmission coefficient of about $T = 0.998$.
[8] Even if the functionality of the device turns out to be quite sensitive to variations of its geometrical parameters (larger than a few nm), our simulations have shown that an external potential can be used to retune the potential profile experimented by the electron, thus recovering the gate functionality.