

## Effective “Penetration Depth” in the Vortex State of a $D$ -Wave Superconductor

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The temperature and the field dependence of the effective magnetic penetration depth ( $\lambda_{\text{eff}}$ ) in the vortex state of a  $d$ -wave superconductor, as measured by muon spin rotation ( $\mu$ SR) experiments, is calculated using a nonlocal London model. We show that at temperatures below  $T^* \propto \sqrt{B}$ , the linear  $T$  dependence of  $\lambda_{\text{eff}}^{-2}$  crosses over to a  $T^3$  dependence. This could provide an explanation for the low temperature flattening of the  $\lambda_{\text{eff}}^{-2}$  curve observed in a recent  $\mu$ SR experiment.

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Recent experiments on quasiparticle response in the vortex state of the cuprate superconductors have indicated quite unexpected behavior. Krishana *et al.* discovered an anomalous plateau in the longitudinal thermal conductivity  $\kappa_{xx}$  of  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$  [1] and  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$  (YBCO) [2] at high magnetic fields which they attributed to the opening of a second gap of  $d_{xy}$  symmetry. Scanning tunneling spectroscopy on YBCO compounds [3] lent support to this scenario by suggesting the existence of localized quasiparticles in the vortex core regions which is possible only for a gapped excitation spectrum [4]. Subsequent measurements of Aubin *et al.* [5] discovered hysteretic behavior in the thermal conductivity which is a signature of the influence of the vortex lattice and impurities on  $\kappa_{xx}$ . Furthermore, at temperatures below 1 K they detected an increase of  $\kappa_{xx}$  with the magnetic field instead of a decrease [6]. This is actually in agreement with the existence of nodes in the superconducting gap and in contradiction with the  $d_{x^2-y^2} + id_{xy}$  symmetry scenario. Theoretically, the possibility of a field induced  $d_{xy}$  gap was discussed by Laughlin and others [7], and attempts have been made to explain the observed strange behavior of  $\kappa_{xx}$  without invoking mixed order parameter symmetry [8].

The question of the existence of the second order parameter was recently raised once again by Sonier *et al.* in their muon spin rotation ( $\mu$ SR) experiment [9]. At high magnetic fields, they observed a flattening of  $\lambda_{\text{eff}}^{-2}$  (defined in these experiments as the width of magnetic field distribution) at low temperatures in contrast to the  $T$ -linear behavior expected in a  $d$ -wave superconductor [10,11]. If one assumes that  $\lambda_{\text{eff}}^{-2}$  is proportional to the superfluid density  $\rho_s$ , then such a flattening could be indicative of the opening of a gap in the quasiparticle excitation spectrum, which would result in exponentially activated behavior of  $\rho_s(T)$ . On the other hand, experiment finds strong dependence of  $\lambda_{\text{eff}}(T \rightarrow 0)$  on the magnetic field. This argues against the gap since within conventional BCS-type models the opening of a gap

should not affect the value of  $\rho_s(T \rightarrow 0)$ . In this Letter we argue that the simple relation  $\lambda_{\text{eff}}^{-2} \propto \rho_s$  is not valid for the penetration depth extracted in these experiments at finite fields and that with proper definition the observed behavior of  $\lambda_{\text{eff}}^{-2}(T)$  can be explained very naturally by a nonlocal London model for a  $d$ -wave superconductor in which the superfluid density  $\rho_s$  remains linear in temperature.

High  $T_c$  materials are extremely type II superconductors in the sense that their coherence length  $\xi$  is much smaller than their penetration depth  $\lambda$ . Thus naively one would expect that the local London model well describes their magnetic behavior. However, in order to study the electromagnetic response of a  $d$ -wave superconductor it is necessary to define a momentum dependent coherence length,  $\xi_{\hat{p}} = v_F/\pi\Delta_{\hat{p}}$ , which diverges along the node directions. This divergence gives rise to nonlocal dependence between the supercurrent and the vector potential. Kosztin and Leggett [12] showed that in the Meissner state this nonlocal relation can produce a  $T^2$  dependence of the penetration depth, instead of linear  $T$  dependence, below some crossover temperature given by  $T_{\text{KL}}^* = \Delta_0(\xi_0/\lambda_0)$ , where  $\Delta_0$  is the maximum gap,  $\xi_0 = v_F/\pi\Delta_0$ , and  $\lambda_0$  is the London penetration depth. Such an effect is field independent and can occur only at very low temperatures. Therefore, it cannot directly explain the  $\mu$ SR observation assuming that  $\lambda_{\text{eff}}$  is the same as the Meissner penetration depth. We will show below that an analogous calculation with a new definition for the penetration depth which is similar to its definition in  $\mu$ SR experiments gives  $\lambda_{\text{eff}} \sim T^3$  for  $T < T^*$ . Here  $T^* = \Delta_0(\xi_0/d)$ , and  $d \propto \sqrt{\Phi_0/B}$  is the distance between vortices, with  $\Phi_0$  being the flux quantum.  $T^*$  is now field dependent and could be much larger than  $T_{\text{KL}}^*$ .

We have studied [13–15] the effect of nonlocality and nonlinearity due to the field induced excitations at the gap nodes [16], in the vortex state of a  $d$ -wave superconductor using a generalized London model. In Ref. [15] we defined an effective penetration depth  $\lambda_{\text{eff}}$  so as to closely

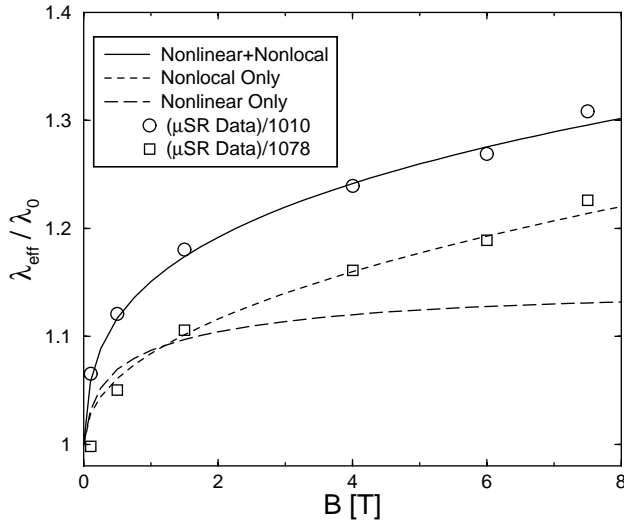


FIG. 1. Comparison between our zero  $T$  nonlinear-nonlocal London theory (with  $\lambda_0 = 1078 \text{ \AA}$ ) and the recent  $\mu$ SR data [9] extrapolated to  $T = 0$ .

correspond to the quantity measured in  $\mu$ SR experiments. Figure 1 presents the result of nonlinear-nonlocal calculation of  $\lambda_{\text{eff}}$  based on the theory developed in Refs. [14,15]. The effect of the nonlinear corrections on  $\lambda_{\text{eff}}$ , as shown in Fig. 1, saturates at high fields and stays effectively field independent for  $B > 1 \text{ T}$ . Most of the field dependence of  $\lambda_{\text{eff}}$ , especially at high fields, therefore comes from the nonlocal contributions. The effect of the nonlinear corrections is just to shift the value of  $\lambda_{\text{eff}}$  by a constant, which can be compensated by rescaling  $\lambda_0$ . In Fig. 1, we compare our theory with the  $\mu$ SR data reported in Ref. [9]. We find that it is possible to fit the experimental data to both nonlinear-nonlocal and nonlocal-only curves, with fairly good agreement, by just changing the scale. At high fields both curves provide good fits. Even below 1 T the agreement between the experimental data and the nonlocal-only curve is fairly good, although including nonlinear corrections enhances the agreement especially at 0.1 T. The excellent agreement between our theory and the experimental data suggests that the same effects might be important for the temperature dependence of  $\lambda_{\text{eff}}$  and might be in fact responsible for the  $\mu$ SR observation.

The effective penetration depth in the vortex state has also been studied numerically by Wang and MacDonald [17] within a lattice model of a  $d$ -wave superconductor. This approach is capable of correctly accounting for the vortex core physics, which is beyond the scope of our London model. On the flip side, because of the system size limitations, the approach of Ref. [17] is limited to relatively high fields  $B \gtrsim 10 \text{ T}$ . The present approach is well suited to address the low-field regime ( $0.1 \text{ T} \lesssim B \lesssim 10 \text{ T}$ ) which is of greatest experimental interest in cuprates.

We now present a brief discussion of analytical and numerical calculations leading to the temperature and field dependence of  $\lambda_{\text{eff}}$ . In order to avoid unnecessary complications we only explicitly consider the nonlocal effects.

Nonlinear corrections are included only by rescaling the value of  $\lambda_0$ , as discussed above. A more detailed description of the theory is given in Refs. [14,15]. To linear order, the relation between the supercurrent  $\mathbf{j}$  and the vector potential  $\mathbf{A}$  in a superconductor can be written in Fourier space as

$$\mathbf{j}_{\mathbf{k}} = -(c/4\pi)\hat{\mathbf{Q}}(\mathbf{k})\mathbf{A}_{\mathbf{k}}. \quad (1)$$

where  $\hat{\mathbf{Q}}(\mathbf{k})$  is the electromagnetic response tensor. Applying the linear response treatment of Gorkov equations generalized for an anisotropic gap, one can calculate the kernel  $\hat{\mathbf{Q}}(\mathbf{k})$  in the weak coupling limit. One finds [14]

$$Q_{ij}(\mathbf{k}) = \frac{4\pi T}{\lambda_0^2} \sum_{n>0} \left\langle \frac{\Delta_{\hat{\mathbf{p}}}^2 \hat{v}_{Fi} \hat{v}_{Fj}}{\sqrt{\omega_n^2 + \Delta_{\hat{\mathbf{p}}}^2 (\omega_n^2 + \Delta_{\hat{\mathbf{p}}}^2 + \gamma_{\mathbf{k}}^2)}} \right\rangle, \quad (2)$$

where  $\gamma_{\mathbf{k}} = \mathbf{v}_F \cdot \mathbf{k}/2$ ,  $\lambda_0 = \sqrt{c^2/4\pi e^2 v_F^2 N_F}$  is the London penetration depth,  $\omega_n = \pi T(2n - 1)$  are the Matsubara frequencies, and the angular brackets mean Fermi surface averaging. Equation (2) is valid for an arbitrary Fermi surface and gap function.

Besides the temperature dependence contained in the Matsubara frequencies in (2), the gap itself has temperature dependence which becomes important at temperatures approaching  $T_c$ . Assuming an isotropic Fermi surface, we use the simple form  $\Delta_{\hat{\mathbf{p}}}(T) = \Delta(T) \cos 2\theta$ , for the gap where  $\theta$  is the angle between the internal momentum  $\mathbf{p}$  and the  $x$  direction. In the weak coupling limit and in the absence of magnetic field, the function  $\Delta(T)$  can be obtained by solving the ordinary BCS gap equation which in the  $d$ -wave case is

$$\frac{1}{V} = T \sum_{\mathbf{p}, \omega_n} \frac{\cos^2 2\theta}{\omega_n^2 + \epsilon_p^2 + \Delta(T)^2 \cos^2 2\theta}, \quad (3)$$

where  $V$  is the interaction potential in the  $d$ -wave channel and  $\epsilon_p$  is the dispersion relation. At finite magnetic fields this equation will change and a full self-consistent calculation will be necessary to obtain  $\Delta(T)$ . However, since the magnetic fields of our interest are far below the upper critical field  $H_{c2}$ , and since we focus only on the regions outside the vortex cores, we can assume (to a good approximation) that Eq. (3) holds even in the presence of a weak magnetic field.

One can write the London equation purely in terms of magnetic field, by eliminating  $\mathbf{j}$  from (1) and using Ampère's law  $\mathbf{j} = (c/4\pi)\nabla \times \mathbf{B}$ :

$$\mathbf{B}_{\mathbf{k}} - \mathbf{k} \times [\hat{\mathbf{Q}}^{-1}(\mathbf{k})(\mathbf{k} \times \mathbf{B}_{\mathbf{k}})] = 0. \quad (4)$$

In the vortex state it is furthermore necessary to insert a source term  $F(\mathbf{k})$  on the right-hand side of Eq. (4) which accounts for the phase winding around the vortex cores. We use a source function with the usual Gaussian cutoff  $F(\mathbf{k}) = e^{-\xi_0^2 k^2/2}$  [18].

As in Ref. [15], we define  $\lambda_{\text{eff}}$  by

$$\lambda_{\text{eff}}^{-4} = \lambda_0^{-4} \left( \frac{\delta B^2}{\delta B_0^2} \right), \quad (5)$$

where  $\overline{\delta B^2}$  is the second moment of the field distribution in the vortex lattice and  $\delta B_0^2$  is the same quantity for the magnetic field  $B_0(\vec{r})$  obtained by solving the ordinary London model on a triangular lattice with the same  $\bar{B}$  and  $\lambda_0$ . We emphasize that this way of defining  $\lambda_{\text{eff}}$  is roughly equivalent to the way it is computed from the  $\mu\text{SR}$  data. Using (4) and (5) we get

$$\lambda_{\text{eff}}^{-4} = C \sum_{\mathbf{k} \neq 0} \frac{e^{-\xi_0^2 k^2}}{(1 + \mathcal{L}_{ij} k_i k_j)^2} \approx C \sum_{\mathbf{k} \neq 0} \frac{e^{-\xi_0^2 k^2}}{(\mathcal{L}_{ij} k_i k_j)^2}, \quad (6)$$

where  $\mathbf{k}$  are the reciprocal lattice wave vectors and

$$\mathcal{L}_{ij}(\mathbf{k}) = \frac{Q_{ij}(\mathbf{k})}{\det \hat{\mathbf{Q}}(\mathbf{k})}, \quad C^{-1} = \sum_{\mathbf{k} \neq 0} \frac{e^{-\xi_0^2 k^2}}{k^4}. \quad (7)$$

Notice that only wave vectors of  $O(d^{-1})$  and larger contribute to the vortex lattice field distribution and the response to a magnetic field at these wave vectors can be much different than at zero wave vector in a superconductor with gap nodes. At low  $T$ , only the regions near the gap nodes are important in the calculation of  $Q_{ij}(\mathbf{k})$ . We can therefore linearize the gap as  $\Delta_{\hat{p}} \approx \Delta_{\theta} \equiv 2\Delta(T)\theta$ , where  $\theta$  is the angle measured from the node, and let  $\mathbf{v}_F$  in Eq. (2) take only the node directions. As a result,  $Q_{ij}$  will be diagonal in the  $45^\circ$  rotated frame. Defining  $q_{1,2} = (k_x \pm k_y)/\sqrt{2}$  we can write  $Q_{ij} = \lambda_0^{-2}[1 + K(q_i, T)]\delta_{ij}$ . We then have

$$\begin{aligned} \mathcal{L}_{11}(\mathbf{q}) &= \lambda_0^2[1 - K(q_2, T)], \\ \mathcal{L}_{22}(\mathbf{q}) &= \lambda_0^2[1 - K(q_1, T)], \\ \mathcal{L}_{12}(\mathbf{q}) &= \mathcal{L}_{21}(\mathbf{q}) = 0, \end{aligned} \quad (8)$$

This is actually a Taylor expansion in  $K$  which is small at low  $B$  and  $T$ . Substituting into (6) we get

$$\frac{\lambda_{\text{eff}}}{\lambda_0} = 1 - \frac{\tilde{C}}{2} \sum_{\tilde{\mathbf{q}} \neq 0} \frac{\tilde{K}(\tilde{q}_1, T)\tilde{q}_2^2 + \tilde{K}(\tilde{q}_2, T)\tilde{q}_1^2}{\tilde{q}^6}, \quad (9)$$

where  $\tilde{\mathbf{q}} = \mathbf{q}d$ ,  $\tilde{C} = Cd^4$ , and  $\tilde{K}(x, T) = K(x/d, T)$ . We have also taken the upper cutoff  $\zeta_d^{-1} = d/\xi_0$  to infinity since it does not affect our calculations (we keep  $\zeta_d$  finite in our numerical calculation but our results are insensitive to its exact value).

At  $T = 0$  for  $\tilde{q}\zeta_d \ll 1$  one can write [12]  $\tilde{K}(\tilde{q}, 0) \approx -(\pi^2/8)\zeta_d\tilde{q}$ . Substituting back into (9) we get

$$\delta\lambda_{\text{eff}}(T=0) \propto \zeta_d \sim \sqrt{\bar{B}}. \quad (10)$$

This is in complete agreement with our numerical calculation and  $\mu\text{SR}$  data [9] (cf. Fig. 1), and also with other calculations [19].

At  $T > 0$ , Eq. (9) can be written as

$$\frac{\delta\lambda_{\text{eff}}}{\lambda_0} = -\frac{\tilde{C}}{2} \sum_{\tilde{\mathbf{q}} \neq 0} \frac{\delta\tilde{K}(\tilde{q}_1, T)\tilde{q}_2^2 + \delta\tilde{K}(\tilde{q}_2, T)\tilde{q}_1^2}{\tilde{q}^6}$$

where  $\delta\lambda_{\text{eff}} = \lambda(B, T) - \lambda(B, 0)$  and  $\delta\tilde{K}(\tilde{q}, T) = \tilde{K}(\tilde{q}, T) - \tilde{K}(\tilde{q}, 0)$ . We follow Ref. [12] by writing

$$\tilde{K}(\tilde{q}, T) = \tilde{K}(0, T)F\left(\frac{\tilde{q}}{t}\right), \quad (11)$$

where  $t = T/T^*$ ,  $\tilde{K}(0, T) \approx -2(\ln 2)T/\Delta_0$  is the kernel obtained in local approximation, and  $F(z)$  is a universal function which can be approximated by

$$\begin{aligned} F(z) &\approx 1 - c_1 z \quad \text{for } z < 2, \\ F(z) &\approx c_0/z^2 \quad \text{for } z > 2. \end{aligned} \quad (12)$$

Here,  $c_0$  and  $c_1$  are constants. Substituting (12) and (11) into (9) we find that, for  $t \ll 1$ ,  $F(z)$  falls into the  $z > 2$  regime for all reciprocal lattice vectors  $\tilde{\mathbf{q}}$ . This immediately gives  $\delta\lambda_{\text{eff}} \propto T^3$ . At higher temperatures on the other hand, there will exist a significant number of points with  $z < 2$  which would give linear  $T$  behavior. Thus in general

$$\begin{aligned} \delta\lambda_{\text{eff}} &\propto T^3 \quad \text{for } T \ll T^*, \\ \delta\lambda_{\text{eff}} &\propto T \quad \text{for } T^* \ll T \ll T_c. \end{aligned} \quad (13)$$

Figure 2 shows the results of our numerical calculation of  $\lambda_{\text{eff}}$  as a function of temperature. We take  $\lambda_0 = 1078 \text{ \AA}$ , which is the value that produces the best fit to the experimental data (cf. Fig. 3), and  $\kappa = \lambda_0/\xi_0 = 68$ . We also assume a triangular vortex lattice aligned with  $x$  and  $y$  ( $a$  and  $b$ ) directions. We have tried changing the core size as well as the shape of the vortex lattice and our results remained unchanged. The penetration depth is no longer linear at low temperatures, unlike the superfluid density (the upper curve). The deviation from linearity is stronger at higher fields, in complete agreement with the  $\mu\text{SR}$  observation [9] and previous theoretical work [17]. At high enough fields, the dependence is in complete agreement with the  $T^3$  form obtained analytically. The curves for different fields join to a single curve at higher temperatures. This is essentially because the nonlocal corrections are most pronounced for the quasiparticles close to the gap nodes. At higher temperatures the response becomes dominated by the quasiparticles far from the nodes which feel much weaker nonlocal effects.

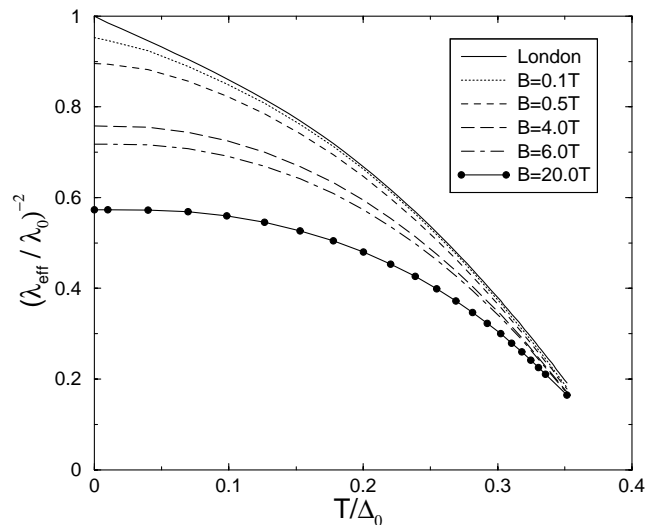


FIG. 2. Temperature dependence of  $\lambda_{\text{eff}}^{-2}/\lambda_0^{-2}$  at different magnetic fields. Here  $\Delta_0 = \Delta(T=0)$ .

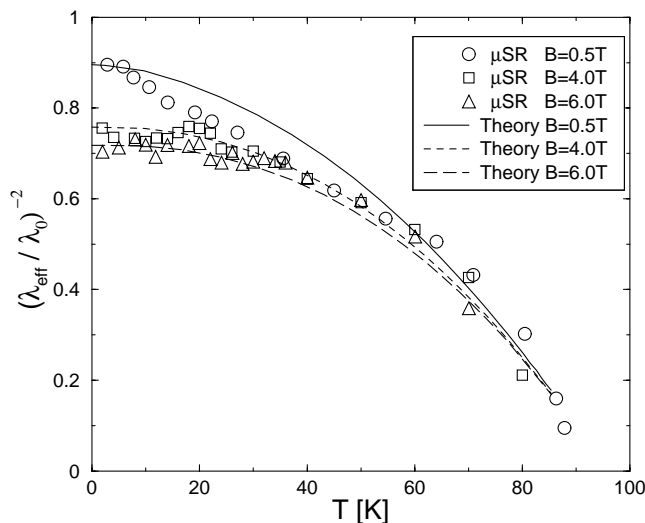


FIG. 3. A comparison between our nonlocal calculation and  $\mu$ SR data.

Figure 3 presents a comparison between our results and the experimental data. In order to get a good fit, we set  $\Delta_0 \equiv \Delta(0) = 2.65T_c$  which is also what one obtains from the gap equation (3). We have normalized both experimental and theoretical data with  $\lambda_0 = 1078 \text{ \AA}$ . The agreement between our theory and the data is good at  $B = 4 \text{ T}$  and  $6 \text{ T}$ , but at  $B = 0.5 \text{ T}$  the theory shows less linearity at low temperatures compared to the experimental data, although they agree fairly well at  $T = 0$  and high  $T$ .

As we mentioned earlier, a complete calculation should include nonlinear corrections [15,16] in addition to the nonlocal corrections considered here. However, as apparent from Fig. 1, at  $T = 0$  the main source of the field dependence of  $\lambda_{\text{eff}}$  is the nonlocal effect. Since this field dependence is closely related to the flattening of the curves at low  $T$ , it is reasonable to assume that nonlocal effects also dominate the  $T$  dependence. The main discrepancy between the theory and experiment in Fig. 3 is the lack of linearity at  $0.5 \text{ T}$ ; however, it is difficult to envision how nonlinear corrections could cure this. Given the inherent uncertainties in the extraction of  $\lambda$  from the  $\mu$ SR data we consider the overall agreement to be reasonable even without inclusion of the nonlinear effects.

In summary, we have calculated the field and temperature dependence of the effective penetration depth  $\lambda_{\text{eff}}$  from a nonlocal London model of a  $d$ -wave superconductor. We used a definition of  $\lambda_{\text{eff}}$  which permits a direct comparison to the  $\mu$ SR experimental data. Our results exhibit a  $T^3$  dependence in the  $\lambda_{\text{eff}}^{-2}$  curve below  $T^* = \Delta_0(\xi_0/d) \sim \sqrt{B}$ , quantitatively consistent with the experimental data on YBCO [9]. This flattening has nothing to do with the reduction of the superfluid density or opening a true gap at high magnetic fields. Rather, it is a conse-

quence of the nonlocal response of a  $d$ -wave superconductor which modifies the magnetic field distribution in the vortex lattice as compared to an ordinary London model. Thus, as pointed out previously [15,17], it is essential to make a distinction between the London penetration depth  $\lambda_L$  (as measured, e.g., in the microwave experiment [10]) and the effective penetration depth  $\lambda_{\text{eff}}$  deduced from the magnetic field profile in a  $\mu$ SR experiment. In the present model  $\lambda_{\text{eff}}$  flattens at low temperatures and finite field while  $\lambda_L$  remains linear in  $T$ .

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