Using Slow Light to Enhance Acousto-optical Effects: Application to Squeezed Light

Andrey B. Matsko and Yuri V. Rostovtsev
Department of Physics and Institute for Quantum Studies, Texas A&M University, College Station, Texas 77843

Herman Z. Cummins
Physik Department, Technische Universitat Munchen, D-85747 Garching, Germany

Marlan O. Scully
Max-Planck-Institut für Quantenoptik, D-85748 Garching, Germany
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We propose a technique for achieving phase matching in Brillouin scattering in a dielectric fiber doped by three-level Λ-type ions. This can lead to a dramatic increase of efficiency of ponderomotive nonlinear interaction between the electromagnetic waves and holds promise for applications in quantum optics such as squeezing and quantum nondemolition measurements.

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Mutual phase modulation of two electromagnetic (EM) waves plays an important role in nonlinear and quantum optics with applications ranging from optical Kerr shutters [1] to quantum nondemolition (QND) measurements [2]. To achieve the desired coupling between EM waves a large nonlinear susceptibility with small losses is necessary. Several examples of atomic media with such properties have been proposed and demonstrated [3,4].

There has recently been considerable interest in the mechanical effects of light on macroscopic objects. In such considerations the radiation pressure coupling of the light fields to a freely suspended mirror introduces a nonlinearity which gives rise to a bistability [5] and squeezing of light [6] (Fig. 1). The magnitude of the ponderomotive nonlinearity increases inversely with the mass of the suspended mirror. However, it is difficult to operate experimentally with small mass mirrors [7]. From this point of view, an optical fiber is an appropriate candidate for studying ponderomotive nonlinear effects because of low optical and mechanical losses accompanied by a small fiber mass.

In fibers the scattering of light by an acoustic wave, usually referred to as Brillouin scattering, arises because the refractive index of a medium depends on its density (strain). The acoustic wave modulates the refractive index and may scatter the light wave. This is similar to the ponderomotive nonlinearity mentioned above.

Stimulated backward Brillouin scattering (SBBS) and guided acoustic wave Brillouin scattering (GAWBS) have been demonstrated in optical fibers. In SBBS, light generates sound through electrostriction and is scattered by the acoustic wave [8]. Although SBBS has a relatively low threshold and narrow linewidth, it is not used much in quantum optics, because the high frequency acoustic oscillations have a low quality factor.

GAWBS [9–11] is enhanced by the long phonon lifetime $\tau_s = 20\ \mu$s; however, the scattering is essentially spontaneous. Light is scattered forward by thermal acoustic vibrations of the fiber. Phase mismatch between the guided acoustic and electromagnetic waves is the main reason why the high-Q acoustic oscillations cannot be used for stimulated scattering in usual fibers.

We here show that phase-matched conditions can be established for a wide range of fiber parameters by taking advantage of the large linear dispersion associated with electromagnetically induced transparency (EIT). This makes it possible to slow the group velocity of a laser pulse down to the speed of sound in solids [12,13], and, therefore, to utilize the ponderomotive nonlinearity of the fiber for new phase modulators, frequency shifters, and sensors, on the one hand, and for effective quantum wave mixing, generation of nonclassical states of light, and QND measurements, on the other.

Let us consider two EM waves copropagating in the fiber interacting with an acoustic mode of the fiber. The phonon-photon interaction Hamiltonian is given by

$$H_{\text{int}}(t) = \hbar g \frac{\sin \Delta kL}{\Delta kL} \hat{a}_1(t)\hat{a}_2^+(t)\hat{b}^+(t) + \text{adjoint} \quad (1)$$

where g is a coupling constant, L is the length of the fiber, $\hat{a}_{1,2}$ and $\hat{b}$ are the annihilation operators of the EM fields

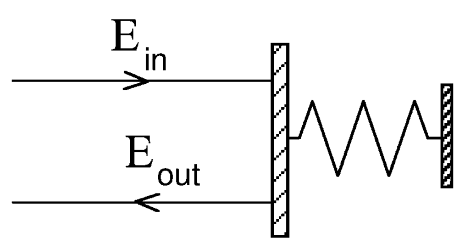

FIG. 1. Amplitude fluctuations of the EM wave E_{in} falling on the mirror cause fluctuations of the mirror's position. This, in turn, leads to a change of the phase of the reflected wave E_{out}. Because usually the velocity of the mirror is small, the Doppler frequency shift of E_{out} can be neglected and the EM wave experiences self-phase modulation without changing its energy. Therefore, the effect can be described in terms of effective $\chi^{(3)}$ nonlinearity.

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and acoustic phonons (we assume that all operators are slowly varying functions of time and space), $k_{1,2}, \omega_{1,2}$ and k_b, ω_b are the wave vectors and frequencies, and $\Delta k = k_1 - k_2 - k_b$. We note that conservation of energy requires $\omega_1 - \omega_2 = \omega_b$. However, in general, we do not have $k_1(\omega_1) - k_2(\omega_2) = k_b(\omega_b)$ because of dispersion effects. Since $\omega_b \ll \omega_{1,2}$, we may write

$$k_1 - k_2 = \frac{\omega_1 n(\omega_1)}{c} - \frac{\omega_2 n(\omega_2)}{c} \simeq \frac{\omega_1 - \omega_2}{c} \frac{\partial[\omega n(\omega)]}{\partial \omega} = \frac{\omega_1 - \omega_2}{V_g},$$

and $k_b = \omega_b / V_s$, where $V_g = c/\{\partial[\omega n(\omega)]/\partial\omega\}$ and V_s are the EM group and the sound velocities, respectively. In the following, we represent the EM field as a sum of large classical expectation and small fluctuation parts $\hat{a}_{1,2} = \langle a_{1,2}\rangle + \delta\hat{a}_{1,2}$ ($\langle a_{1,2}\rangle \gg \delta\hat{a}_{1,2}$, and $\langle a_{1,2}\rangle$ are real values).

It is easy to see from Eq. (1) that, even in the most interesting resonant case $\Delta = \omega_1 - \omega_2 - \omega_b = 0$, we still have phase mismatch $\Delta k L = \omega_b(1/V_g - 1/V_s)L \gg 1$, and the interaction vanishes with increasing L if the light group velocity V_g is different from the phase sound velocity V_s. As noted earlier, the group velocity is relevant here because it characterizes the difference in phase velocities for light waves of different frequencies. It is this phase velocity difference that enters the phase matching condition. If the condition $\omega_b = \delta\omega_\alpha$ is satisfied and the condition $V_g = V_s$ is met, then phase matching is achieved.

To slow the group velocity of light to the speed of sound we propose to use a medium, consisting of a host doped by Λ atoms or ions (Fig. 2) (for example, Eu^{3+}, Er^{3+}, etc.). This could be a doped silica glass or crystal fiber (see, for example, Fig. 3 [14]). The fields are nearly resonant with the corresponding atomic transitions and include a strong driving field with frequency ω_d and Rabi-frequency Ω, and two sufficiently weak fields with carrier frequencies ω_1 and ω_2 and Rabi-frequencies α_1 and α_2 ($\alpha_{1,2} = \wp_{ab}\langle a_{1,2}\rangle\sqrt{4\pi\omega_{ab}/\hbar\mathcal{V}}$, where $\mathcal{V}$ is the total volume of the mode in the fiber, and $\wp_{ab}$ is the dipole momentum of the probe transition). The fields interact via the long-lived coherence of the dipole-forbidden transition between the ground state sublevels $|b\rangle$ and $|c\rangle$. Because in doped materials decay of this ground state coherence (decay rate is about $\sim 10^3$ s^{-1} or more) usually dominates over decay of spin exchange (decay rate is about ~ 1 s^{-1}), we here consider the first type of decay only.

EM wave propagation in the fiber occurs with the absorption coefficients $\beta_{1,2} = (k/2)\chi''(\omega_{1,2})$, and the group velocity $V_g = c/n_g$, $n_g = 1 + \omega/2(\partial\chi'/\partial\omega)$. The pump power should be large enough to sustain EIT in the system ($\Omega^2 \gg \gamma_{bc}\gamma$ and $\Omega \gg \alpha_{1,2}$) and large enough bandwidth to provide EIT for both fields $\alpha_{1,2}$ ($\Omega^2 \gg \omega_b\gamma$). In the above limits we obtain expressions for the group velocity index and the resonant losses in the form [13]

$$\beta_{1,2} \simeq \frac{3}{8\pi} N\lambda^2 \frac{\gamma_r \gamma_{bc}}{|\Omega|^2}, \tag{2}$$

$$n_{g1,2} \simeq \frac{3}{8\pi} N\lambda^2 \frac{c\gamma_r}{|\Omega|^2}, \tag{3}$$

where N is the density of dopants, λ is the EM wavelength, γ_r is the natural linewidth of optical transitions, γ_{bc} is the decay rate of the ground state coherence, and γ is the total linewidth of the transitions.

To meet the phase-matching condition, we use Eq. (3) to write

$$V_s = V_g = \frac{8\pi|\Omega|^2}{3N\lambda^2\gamma_r}. \tag{4}$$

Noting that Rabi frequency is related to the laser power by

$$|\Omega|^2 = \frac{3\lambda^3\gamma_r P_\Omega}{8\pi^2\hbar c\mathcal{A}}, \tag{5}$$

where $P_{\alpha 1}$, $P_{\alpha 2}$, and P_Ω are the powers of the probe and drive fields, respectively, $\mathcal{A}$ is the cross-sectional area of the fiber, from Eqs. (4) and (5) we derive

$$\frac{P_\Omega}{\mathcal{A}} = NV_s\frac{\hbar\omega_1}{2}. \tag{6}$$

Once the phase-matching condition has been established (i.e., $V_s = V_g$), interaction between the fields [see Eq. (1)] is sustained through the whole fiber length L.

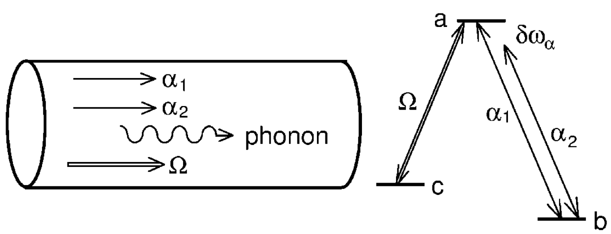

FIG. 2. Probe EM waves $\alpha_1(z,t)$ and $\alpha_2(z,t)$ and drive EM wave $\Omega(z,t)$ propagate into a fiber doped by three-level ions. Appropriate choice of the drive power allows phase matching between probe waves and a guided acoustic wave, propagating in the same direction as the EM waves. Effective resonant interaction between the waves ($\delta\omega_\alpha = \omega_b$) leads to large nearly noiseless cross phase modulation between the probe waves.

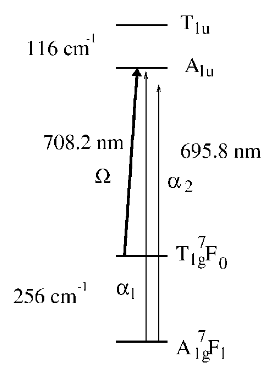

FIG. 3. Energy-level diagram for CaF_2:Sm^{2+} showing an appropriate Λ configuration for reducing group velocity of the probe waves $\alpha_{1,2}$.

Solving the Hamiltonian equation for the acoustic oscillations (neglecting memory effects) and assuming that the laser powers are not changing significantly during propagation, we arrive at

$$\hat{a}_{1,2}(t) = \hat{a}_{1,2}(0) \exp\left(\frac{g^2 \hat{a}^{+}_{2,1} \hat{a}_{2,1}}{i\Delta + \gamma_{\rm ph}} - \frac{\beta_{1,2} c}{n} - \frac{\beta_f c}{n}\right) t, \tag{7}$$

and where β_f and $\gamma_{\rm ph}$ describe the losses of EM and phonon fields, respectively, n is the refractive index of the host material (for example, silica). The interaction time t can be replaced by Ln/c when we consider propagation of the cw EM fields through the fiber with length L.

To estimate the coupling parameter g we note that a single-mode optical fiber is essentially a long, narrow cylinder of fused silica, and the light interacts with vibrational eigenmodes of the cylinder. As an example, we consider the radial modes, which are responsible for the forward scattering of light and derive, using the results and parameters from [9],

$$g \simeq \xi \frac{n^3 \nu}{2} \sqrt{\frac{\hbar \omega_b}{2m V_s^2}}, \tag{8}$$

where the coefficient ξ, depending on strain-optic properties of fused silica, is about unity, m is the total mass of the fiber, and $\nu \simeq \omega_{1,2}/2\pi$. We assume here that the total length of the fiber exceeds the wavelength of the acoustic wave.

According to Eqs. (5)–(8) the expressions for the cross-phase modulation $\Delta\phi_1$ of EM wave α_1 due to the ponderomotive interaction is

$$\Delta\phi_1 = \eta_{xpm} P_{\alpha 2} L, \tag{9}$$

$$\eta_{xpm} = \frac{n^6}{8\pi\lambda c} \frac{Q}{\rho \mathcal{A} V_s^2} \frac{\gamma_{\rm ph}\Delta}{\Delta^2 + \gamma_{\rm ph}^2}, \tag{10}$$

where ρ is the density of the material, η_{xpm} is the cross-phase modulation coefficient, $Q^{-1} = 2\gamma_{\rm ph}/\omega_b$; the corresponding equation for $\Delta\phi_2$ is obtained by $2 \leftrightarrow 1$.

To show that the ponderomotive nonlinearity can be used for demonstration of quantum effects (e.g., squeezing of light), we compare the relaxation rate in the system with the nonlinear response (the second should be much larger than the first). Assuming that the losses of the host material are much less than the resonant losses due to the EM wave coupling with the dopants and phonons, and considering $P_{\alpha_1} \simeq P_{\alpha 2} = P_\alpha$, we write the ratio between losses and nonlinearity in the fiber:

$$\frac{\beta_{1,2} L + \beta_f L}{\Delta\phi_{1,2}} = \zeta \frac{\Delta^2 + \gamma_{\rm ph}^2}{\Delta \gamma_{\rm ph}}, \tag{11}$$

$$\zeta = \frac{8\rho\lambda^2 \gamma_{bc}}{n^5 \hbar N Q} \frac{P_\Omega}{P_\alpha}. \tag{12}$$

This ratio reaches minimum at the optimal detuning $\gamma_{\rm ph} = \Delta_{\rm opt}$. For the parameters, typical for a fiber doped by rare earth ions (e.g., see Fig. 3) ($\rho = 1.2$ g/cm^3, $\lambda = 1$ μm, $n = 1.5$, $N = 10^{19}$ cm^{-3}, $\gamma_{bc} = 10^3$ s^{-1}, $\gamma_r = 2 \times 10^4$ s^{-1}, $\gamma = 10^9$ s^{-1}) the nonlinearity exceeds losses if $Q > 4 \times 10^3 (P_\Omega/P_\alpha)$.

The value of the nonlinearity can be large. For example, for $Q = 5 \times 10^6$, $V_s = 6 \times 10^5$ cm/s, $\mathcal{A} = 10^{-7}$ cm^2, we find $\eta_{xpm} \simeq 2(P_\Omega/P_\alpha)^{1/2}$ cm^{-1} W^{-1}, (for silica $\eta_{xpm} \simeq 3 \times 10^{-5}$ cm^{-1} W^{-1}) while total losses are $2.5\ 10^3 \times (P_\alpha/P_\Omega)$ times less.

It should be mentioned that the self-phase modulation is much less than the cross-phase modulation here due to the resonant feature of the nonlinearity, while in ordinary fibers they are equal. The total power of the driving field needed for establishing of the phase-matching condition is quite reasonable $P_\Omega = 38$ mW, and due to the phase-matched operation, the ponderomotive nonlinearity of the fiber holds promise for application in quantum and nonlinear optics.

Let us next demonstrate how the ponderomotive nonlinearity can be used for the generation of squeezed states of light. In the strong field approximation, Eq. (7) can be linearized to yield

$$\delta\hat{a}_{1,2}(L) = \delta\hat{a}_{1,2}(0) - i \frac{Lg^2 n}{\Delta c} \langle a_1 \rangle \langle a_2 \rangle \times [\delta\hat{a}_{2,1}(0) + \delta\hat{a}^{\dagger}_{2,1}(0)], \tag{13}$$

where $\delta\hat{a}_{1,2}(0)$ describes the initial variant of the coherent fields. It follows from Eq. (13) that the sum of the fields,

$$\delta\hat{a}_\Sigma = \delta\hat{a}_1(L) + \delta\hat{a}_2(L), \tag{14}$$

is squeezed if the fields were initially prepared in a coherent state.

We have neglected the losses and associated noises here because, as it has been discussed above, they can be considered as small. However, the maximum squeezing is determined by the total losses in the system

$$\langle (\delta\hat{a}_\Sigma e^{i\varphi} + \delta\hat{a}^{\dagger}_\Sigma e^{-i\varphi})^2 \rangle_{\rm min} \simeq 4\left(\frac{\gamma_{\rm ph}}{|\Delta|} + \frac{\gamma_{bc} L}{V_s}\right), \tag{15}$$

where φ is the optimal squeezing angle.

The squeezing of the sum of the field operators is governed by the quantum correlation between the fields. Such cross-phase modulation has been used in QND measurements [2]. The detection of the quadrature amplitude of one of the fields allows us to obtain information about the corresponding quadrature amplitude of the other field without destroying it.

There is another interesting feature of the above. Equation (7) shows that field α_2 can be amplified through the propagation along the fiber in the case of resonant tuning $\Delta = 0$ and when $\alpha_2 < \alpha_1$. The condition for such an amplification is $\zeta < 2$, which is not unreasonable. For parameters as these the threshold power is $P_{\alpha 1} \simeq 4$ μW and detection of stimulated GAWBS in the fiber is feasible.

Furthermore, it is possible to relax the threshold condition and to enhance the interaction between EM waves by decreasing $\gamma_{\rm ph}$ and γ_{bc}. Another way to increase effective gain is using a ring resonator, which allows one to decrease the effective phonon "mass" while keeping the interaction length the same. The threshold condition does not depend on relaxation rates, γ_r or γ. The relaxation rates need only meet the condition required to maintain the atomic coherence ($\Omega^2 \gg \gamma_{bc}\gamma$). This allows us to use a variety of dopants, the best having the smallest γ_{bc}. It is interesting to note, that the acoustic oscillation of the fiber can lead to modulation of frequency of the transitions of the dopants, which can result in additional parametric effects.

It should be also mentioned that the intensity and linewidth of the acoustic resonances are affected by the fiber diameter variations, bulk attenuation of the acoustic wave, and damping due to the fiber surface. Increasing the fiber quality yields strong nonlinear interaction between EM waves resulting from the large resonant ponderomotive $\chi^{(3)}$ nonlinearity. Thus, the (usually) undesirable photon-phonon interaction may be used to advantage establishing a phase-matching condition between the acoustic and EM waves.

In conclusion, we have demonstrated how ultraslow light can yield phase matching in optical fibers which allows us to achieve strong coupling between high quality acoustic waves of the fiber and multifrequency EM fields. The method is based on doping the fiber by three-level Λ atoms or ions which possess steep dispersion with low absorption close to the point of two-photon resonance. We predict that the fiber holds promise for an effective wave mixer and/or amplifier at low temperatures due to the large ponderomotive nonlinearity associated with acoustic oscillations of the fiber.

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