Spin-Fermion Model near the Quantum Critical Point: One-Loop Renormalization Group Results

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We consider spin and electronic properties of itinerant electron systems, described by the spin-fermion model, near the antiferromagnetic critical point. We expand in the inverse number of hot spots in the Brillouin zone, N, and present the results beyond the previously studied $N = \infty$ limit. We found two new effects: (i) Fermi surface becomes nested at hot spots, and (ii) vertex corrections give rise to anomalous spin dynamics and change the dynamical critical exponent from $z = 2$ to $z > 2$. To first order in $1/N$ we found $z = 2N/(N - 2)$ which for a physical $N = 8$ yields $z \approx 2.67$.

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The problem of fermions interacting with critical antiferromagnetic spin fluctuations attracts a lot of attention at this time due to its relevance to both high temperature superconductors and heavy-fermion materials [1]. The key interest of the current studies is to understand the system behavior near the quantum critical point (QCP), where the magnetic correlation length diverges at $T = 0$ [2]. Although in reality the QCP is almost always masked by either superconductivity or precursor effects to superconductivity, the vicinity of the QCP can be reached by varying external parameters such as pressure in heavy-fermion compounds or doping concentration in cuprates.

In this paper, we study the properties of the QCP without taking pairing fluctuations into account. We assume that the singularities associated with the closeness to the QCP extend up to energies which exceed typical energies associated with the pairing. This assumption is consistent with the recent calculations of the pairing instability temperature in cuprates [3]. From this perspective, the understanding of the properties of the QCP without pairing correlations is a necessary preliminary step for subsequent studies of the pairing problem.

A detailed study of the antiferromagnetic QCP was performed by Hertz [4] and later by Millis [5] who chiefly focused on finite T properties near the QCP. They both argued that, if the Fermi surface contains hot spots (points separated by antiferromagnetic momentum Q; see Fig. 1), then spin excitations possess purely relaxational dynamics with $z = 2$. They further argued that, in $d = 2$, $d + z = 4$; i.e., the critical theory is at marginal dimension, in which case one should expect that spin-spin interaction yields, at maximum, logarithmic corrections to the relaxational dynamics. Millis argued [5] that this is true provided that the effective Ginzburg-Landau functional for spins (obtained by integrating out the fermions) is an analytic function of the spin ordering field. The analyticity is not guaranteed *a priori* as the expansion coefficients in the Ginzburg-Landau functional are made of particle-hole bubbles and are generally sensitive to the closeness to quantum criticality due to the feedback effect from near critical spin fluctuations on the electronic subsystem. Millis, however, demonstrated that the quartic term in the Ginszurg-Landau functional is governed by high-energy fermions and is free from singularities.

In this Letter, we argue that the regular Ginzburg-Landau expansion is not possible in 2D by reasons different from those displayed in [4,5]. Specifically, we argue that the damping term in the spin propagator (assumed to be linear in ω in [4,5]) is by itself made out of a particle-hole bubble, and, contrary to ϕ^4 coefficient, is governed by low-energy fermions. We demonstrate that due to singular vertex corrections the frequency dependence of the spin damping term at the QCP is actually $\omega^{1-\alpha}$. In the one-loop approximation, we find $\alpha \approx 0.25$.

Another issue which we study is the form of the renormalized quasiparticle Fermi surface near the magnetic instability. In a mean-field spin-density wave (SDW) theory, the Fermi surface in a paramagnetic phase is not affected by the closeness to the QCP. Below the instability, the doubling of the unit cell induces a shadow Fermi surface at $k_F + Q$, with the residue proportional to the deviation from criticality. This gives rise to the opening of the SDW gap near hot spots and eventually (for a perfect antiferromagnetic long range order) yields a Fermi surface in the

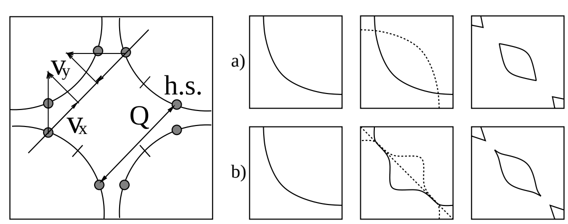

FIG. 1. The Fermi surface with hot spots and the directions of Fermi velocities at hot spots separated by Q, and the evolution of the Fermi surface evolution for (a) mean-field ($N = \infty$) SDW theory and (b) finite N. In both cases, the doubling of the unit cell due to antiferromagnetic SDW ordering introduces a shadow Fermi surface and yields a gap opening near hot spots. At finite N, however, the Fermi surface at the quantum critical point becomes nested at hot spots due to the vanishing of renormalized v_y.

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form of small pockets around $(\pi/2, \pi/2)$ and symmetry related points (see Fig. 1a).

Several groups argued [6,7] that this mean-field scenario is modified by fluctuations, and the Fermi surface evolution towards hole pockets already begins within the paramagnetic phase. We show that the Fermi surface near hot spots does evolve as $\xi \to \infty$, but due to strong fermionic damping (not considered in [6]) this evolution is a minor effect which at $\xi = \infty$ only gives rise to a nesting at the hot spots (see Fig. 1b).

The point of departure for our analysis is the spin-fermion model which describes low-energy fermions interacting with their own collective spin degrees of freedom. The model is described by

$$\mathcal{H} = \sum_{\mathbf{k},\alpha} \mathbf{v}_F(\mathbf{k} - \mathbf{k}_F) c^{\dagger}_{\mathbf{k},\alpha} c_{\mathbf{k},\alpha} + \sum_q \chi_0^{-1}(\mathbf{q}) \mathbf{S}_\mathbf{q} \mathbf{S}_{-\mathbf{q}} + g \sum_{\mathbf{q},\mathbf{k},\alpha,\beta} c^{\dagger}_{\mathbf{k}+\mathbf{q},\alpha} \boldsymbol{\sigma}_{\alpha,\beta} c_{\mathbf{k},\beta} \cdot \mathbf{S}_{-\mathbf{q}} . \quad (1)$$

Here $c^{\dagger}_{\mathbf{k},\alpha}$ is the fermionic creation operator for an electron with momentum $\mathbf{k}$ and spin projection α, σ_i are the Pauli matrices, and g measures the strength of the interaction between fermions and their collective bosonic spin degrees of freedom. The latter are described by $\mathbf{S}_\mathbf{q}$ and are characterized by a bare spin susceptibility which is obtained by integrating out high-energy fermions.

The form of the bare susceptibility $\chi_0(q)$ is an input for the low-energy theory. We assume that $\chi_0(q)$ is nonsingular and peaks at $\mathbf{Q}$, i.e., $\chi_0(\mathbf{q}) = \chi_0/[\xi^{-2} + (\mathbf{q} - \mathbf{Q})^2]$, where ξ is the magnetic correlation length. In principle, χ_0 can also contain a nonuniversal frequency dependent term in the form $(\omega/W)^2$, where W is of the order of a fermionic bandwidth. We, however, will see that, for a Fermi surface with hot spots which we consider here, this term will be overshadowed by a universal $\omega^{1-\alpha}$ term produced by low-energy fermions.

The earlier studies of the spin-fermion model demonstrated that the perturbative expansion for both fermionic and bosonic self-energies holds in powers of $\lambda = 3\bar{g}/(4\pi v_F \xi^{-1})$, where v_F is the Fermi velocity at a hot spot, and $\bar{g} = g^2\chi_0$. This perturbation theory obviously does not converge when $\xi \to \infty$. As an alternative to a conventional perturbation theory, we suggested [8] the expansion in the inverse number of hot spots in the Brillouin zone, N. Whether Q is commensurate or not is irrelevant for this expansion. Indeed, for $Q = (\pi, \pi)$, $N = 8$. For incommensurate Q, $N = 16$ as Q and $2\pi - Q$ are no longer identical. However, each of the two incommensurate peaks in $\chi(q, \omega)$ has a residue which is half of that for $Q = (\pi, \pi)$, which cancels the increase in N. Physically, large N implies that a spin fluctuation has $N/2$ independent channels to decay into a particle-hole pair. This gives rise to a strong ($\sim N$) spin damping rate. At the same time, a fermion near a hot spot can only scatter into a single hot spot separated by $\mathbf{Q}$, i.e., fermionic self-energy does not contain N as the overall factor. Power counting arguments then show that a large damping rate appears in the denominators of the expressions for the fermionic self-energy and vertex corrections, and makes them small to the extent of $1/N$. The only exception to this rule is the fermionic self-energy due to a single spin fluctuation exchange. This self-energy is singular and has a piece which comes from static spin fluctuations and does not contain $1/N$ [9].

The set of coupled equations for fermionic and bosonic self-energies at $N = \infty$ has been solved in [9], and we merely quote the result. Near hot spots, we have

$$G_k^{-1}(\omega) = \omega - \epsilon_k + \Sigma(\omega), \qquad \chi(q, \Omega_m) = \chi_0 \xi^2/[1 + (\mathbf{q} - \mathbf{Q})^2 \xi^2 - i\Pi_\Omega]. \quad (2)$$

Here $\epsilon_k = v_x \tilde{k}_x + v_y \tilde{k}_y$, where $\tilde{\mathbf{k}} = \mathbf{k} - \mathbf{k}_{\rm hs}$, and v_x and v_y, which we set to be positive, are the components of the Fermi velocity at a hot spot ($v_F^2 = v_x^2 + v_y^2$). The fermionic self-energy $\Sigma_k(\omega)$ and the spin polarization operator Π_Ω are given by

$$\Sigma(\omega) = 2\lambda \frac{\omega}{1 + \sqrt{1 - \frac{i|\omega|}{\omega_{\rm sf}}}}, \qquad \Pi_\Omega = \frac{|\Omega|}{\omega_{\rm sf}}, \quad (3)$$

and $\omega_{\rm sf} = (4\pi/N) v_x v_y \xi^{-2}/\bar{g}$.

We see from Eq. (3) that, for $\omega \le \omega_{\rm sf}$, $G(k_{\rm hs}, \omega) = Z/[\omega + i\omega|\omega|/(4\omega_{\rm sf})]$, i.e., as long as ξ is finite, the system preserves the Fermi-liquid behavior at the lowest frequencies. The quasiparticle residue Z, however, depends on the interaction strength, $Z = (1 + \lambda)^{-1}$, and progressively goes down when the spin-fermion coupling increases. At larger frequencies, $\omega \ge \omega_{\rm sf}$, the system crosses over to a region, which is in the basin of attraction of the quantum critical point, $\xi = \infty$. In this region, $G^{-1}(k_F, \omega) \approx (i|\omega|\bar{\omega})^{1/2}\,{\rm sgn}(\omega)$ [9,10], where $\bar{\omega} = 9\bar{g} v_x v_y \pi N v_F^2$ is the upper frequency cutoff for the quantum critical behavior. At the same time, the spin propagator has a simple $z = 2$ relaxational dynamics unperturbed by the strong frequency dependence of the fermionic self-energy [11].

Our present goal is to go beyond the $N = \infty$ limit and analyze the role of $1/N$ corrections. The $1/N$ terms give rise to two new features: vertex corrections which renormalize both fermionic and bosonic self-energies, and static fermionic self-energy Σ_k. The corresponding diagrams are presented in Fig. 2.

The lowest-order $1/N$ corrections have been calculated previously [9,12]. Both vertex correction and the static self-energy are logarithmic in ξ:

$$\frac{\Delta g}{g} = \frac{Q(v)}{N} \log\xi , \quad (4)$$

$$\Delta\epsilon_k = -\epsilon_{k+Q} \frac{12}{\pi N} \frac{v_x v_y}{v_F^2} \log\xi , \quad (5)$$

where $\epsilon_{k+Q} = -v_x \tilde{k}_x + v_y \tilde{k}_y$, and $Q(v) = (4/\pi) \times \arctan(v_x/v_y)$ interpolates between $Q = 1$ for $v_x = v_y$, and $Q = 2$ for $v_y \to 0$.

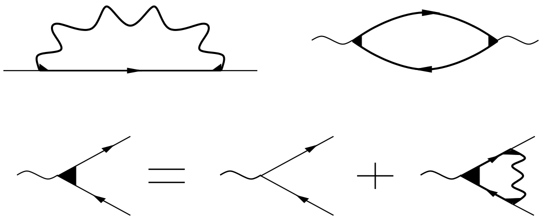

FIG. 2. The one-loop RG diagrams for the fermionic self-energy and vertex renormalization. Solid lines are full fermionic propagators, wavy lines are full spin susceptibilities, and black triangles are full vertices. The lowest-order diagrams are obtained by replacing full internal lines and vertices by their $N = \infty$ forms.

Besides, the $1/N$ corrections also contribute $(1/N)\omega \log\xi$ to $G_k^{-1}(\omega)$, but this term is negligible compared to $\Sigma(\omega)$ and we neglect it.

We see from Eqs. (4) and (5) that the $1/N$ corrections to the vertex and to the velocity of the excitations are almost decoupled from each other: the velocity renormalization does not depend on the coupling strength at all, while the renormalization of the vertex depends on the ratio of velocities through only a nonsingular $Q(v)$. This is a direct consequence of the fact that the dynamical part of the spin propagator is obtained self-consistently within the model. Indeed, the overall factors in $\Delta\epsilon_k$ and $\Delta g/g$ are $\bar{g}(\omega_{\rm sf}\xi^2)$, where $\omega_{\rm sf}\xi^2$ comes from the dynamical part of the spin susceptibility. Since the fermionic damping is produced by the same spin-fermion interaction as the fermionic self-energy, $\omega_{\rm sf}$ scales as $1/\bar{g}$, and the coupling constant disappears from the right-hand side of Eqs. (4) and (5).

The logarithmic dependence on ξ implies that the $1/N$ expansion breaks down near the QCP, and one has to sum up the series of the logarithmic corrections. We do this in a standard one-loop approximation by summing up the series in $(1/N)\log\xi$ but neglecting regular $1/N$ corrections to each term in the series. We verified that in this approximation the cancellation of the coupling constant holds even when g is a running, scale dependent coupling. This in turn implies that one can separate the velocity renormalization from the renormalization of the vertex to all orders in $1/N$.

By separating the corrections to v_x and v_y and performing standard renormalization group (RG) manipulations, we obtain a set of two RG equations for the running v_x^R and v_y^R:

$$\begin{aligned} \frac{dv_x^R}{dL} &= \frac{12}{\pi N}\,\frac{(v_x^R)^2 v_y^R}{(v_x^R)^2 + (v_y^R)^2}\,, \\ \frac{dv_y^R}{dL} &= -\frac{12}{\pi N}\,\frac{(v_y^R)^2 v_x^R}{(v_x^R)^2 + (v_y^R)^2}\,, \end{aligned} \tag{6}$$

where $L = \log\xi$. The solution of these equations is straightforward, and yields

$$\begin{aligned} v_x^R &= v_x Z, \qquad v_y^R = v_y Z^{-1}, \\ Z &= \left(1 + \frac{24L}{\pi N}\,\frac{v_y}{v_x}\right)^{1/2}, \end{aligned} \tag{7}$$

where, we recall that v_x and v_y are the bare values of the velocities (the ones which appear in the Hamiltonian).

We see that v_y^R vanishes logarithmically at $\xi \to \infty$. This implies that right at the QCP the renormalized velocities at $k_{\rm hs}$ and $k_{\rm hs} + Q$ are antiparallel to each other, i.e., the Fermi surface becomes nested at hot spots (see Fig. 1b). This nesting creates a "bottleneck effect" immediately below the criticality as the original and the shadow Fermi surfaces approach hot spots with equal derivatives (see Fig. 1b). This obviously helps in developing a SDW gap at $k_{\rm hs}$ below the magnetic instability. However, above the transition, no SDW precursors appear at $T = 0$. Earlier works which found SDW precursors considered either a toy model without spin damping [6] or the case of high temperatures when classical, thermal spin fluctuations dominate [7].

Another feature of the RG equations (6) is that they leave the product $v_x v_y$ unchanged. This is a combination in which velocities appear in $\omega_{\rm sf}$. The fact that $v_x v_y$ is not renormalized implies that, without vertex renormalization, $\omega_{\rm sf}\xi^2$ remains finite at $\xi = \infty$, i.e., spin fluctuations preserve a simple $z = 2$ relaxational dynamics.

We next consider vertex renormalization. Using again the fact that $\bar{g}\omega_{\rm sf}$ does not depend on the running coupling constant, one can straightforwardly extend the second-order result for the vertex renormalization [Eq. (4)] to the one-loop RG equation,

$$\frac{dg^R}{dL} = \frac{Q(v)}{N}\, g^R, \tag{8}$$

where g^R is a running coupling constant, and $Q(v)$ is the same as in (4) but contains renormalized velocities v_x^R and v_y^R. At the QCP, the dependence on ξ obviously transforms into the dependence on frequency [$L = \log\xi \to (1/2)\log|\bar{\omega}/\omega|$, where, we recall, $\bar{\omega}$ is the upper cutoff for the quantum critical behavior]. Using the fact that, for $\xi \to \infty$, $v_y^R/v_x^R \approx N\pi/24L$ and expanding $Q(v)$ near $v_y^R = 0$, we find $Q(v) \approx 2[1 - (2/\pi)v_y^R/v_x^R] = 2 - N/3L$. Substituting this result into (8) and solving the differential equation, we obtain ($\tilde{\omega} \equiv \omega/\bar{\omega}$),

$$g^R = g|\tilde{\omega}|^{-1/N}|\log\tilde{\omega}|^{-1/6}. \tag{9}$$

We see that, at the QCP, the running coupling constant diverges as $\omega \to 0$ roughly as $|\tilde{\omega}|^{-1/N}$. Substituting this result into the spin polarization operator and using the fact that $\omega_{\rm sf} \propto (g^R)^{-2}$, we find that, at the QCP,

$$\Pi_\Omega \propto |\tilde{\omega}|^{(N-2)/N}|\log\tilde{\omega}|^{-1/3}. \tag{10}$$

This result implies that vertex corrections change the dynamical exponent z from its mean-field value $z = 2$ to $z = 2N/(N-2)$. For $N = 8$, this yields $z \approx 2.67$ and $\chi^{-1}(Q,\omega) \propto |\omega|^{1-\alpha}$, where $\alpha = 0.25$.

Singular vertex corrections also renormalize the fermionic self-energy $\Sigma(\omega) \propto (g^R/v_F^R)\sqrt{|\omega|}$. Using the results for g^R and $v_F^R \approx v_x^R$ we obtain at criticality

$$\Sigma(\omega) \propto |\tilde{\omega}|^{(N-2)/2N}|\log\tilde{\omega}|^{-2/3}. \tag{11}$$

Equations (7), (10), and (11) are the central results of this paper. We see that the singular corrections to the Fermi velocity cause nesting, but do not affect the spin dynamics. The corrections to the vertex, on the other hand, do not affect velocities, but change the dynamical critical exponent for spin fluctuations.

We now briefly discuss the form of the susceptibility at finite T. Previous studies have demonstrated [2,5] that the scattering of a given spin fluctuation by classical, thermal spin fluctuations yields, up to logarithmic prefactors, $\xi^{-2} \propto uT$, where u is the coefficient in the ϕ^4 term in the Ginzburg-Landau potential. This implies that, at the QCP, $\chi^{-1}(Q, \omega) \propto T - i|\omega|$, at $N = \infty$.

We, however, argue that the linear in T and the linear in ω terms have completely different origins: the linear in ω term comes from low energies and is universal, while the linear in T term comes from high energies and is model dependent. This can be understood by analyzing the particle-hole bubble at finite T. We found that, for a linearized $\epsilon_k \approx v_F(k - k_F)$, Π_Ω preserves exactly the same form as at $T = 0$. The temperature dependence of Π appears due to a nonzero curvature of the electronic dispersion and is obviously sensitive to the details of the dispersion at energies comparable to the bandwidth. Similarly, the derivation of the Landau-Ginzburg potential from (1) shows [5] that u vanishes for linearized ϵ_k, and is finite due to a nonzero curvature of the fermionic dispersion.

The different origins of T and ω dependences in $\chi(Q, \omega)$ imply that the anomalous $\omega^{1-\alpha}$ frequency dependence of $\chi(Q, \Omega)$ is not accompanied by the anomalous temperature dependence of $\chi(Q, 0)$. In view of this, it is not clear whether our theory explains the anomalous spin dynamics observed in heavy fermion $CeCu_{6-x}Au_x$ [13], or the explanation should involve the local Kondo physics [14]. On one hand, the observed exponent for the frequency dependence of Π_Ω is $1 - \alpha \sim 0.8$ which is very close to our $1 - \alpha = 0.75$. On the other hand, experimental data demonstrate Ω/T scaling, which our model does not have.

Finally, we consider how anomalous vertex corrections affect the superconducting problem. We and Finkel'stein argued recently [3] that at $\xi = \infty$ the kernel $K(\omega, \Omega)$ of the Eliashberg-type gap equation for the d-wave anomalous vertex $F(\Omega) = (\pi T/2) \sum_\omega K(\omega, \Omega) F(\omega)$ behaves as $K(\omega, \Omega) \propto g^2/[v_F^2 \Sigma^2(\omega) \Pi_{\Omega-\omega}]^{1/2}$. At $N = \infty$, this yields (including the prefactor) $K(\omega, \Omega) = |\omega(\Omega - \omega)|^{-1/2}$. Although this kernel is qualitatively different from the one in the BCS theory because it depends on both frequencies, it still scales as an inverse frequency due to an interplay between a non-Fermi liquid form of the fermionic self-energy and the absence of the gap in the spin susceptibility which mediates pairing. We demonstrated in [3] that this inverse frequency dependence gives rise to a finite pairing instability temperature even when $\xi = \infty$.

To check how the kernel is affected by vertex corrections, we substitute the results for g^R, v_F, $\Sigma(\omega)$, and Π_Ω into $K(\omega, \Omega)$. We find after simple manipulations that *despite singular vertex corrections the kernel in the gap equation still scales inversely proportional to frequency.* A simple extension of the analysis in [3] then shows that the system still possesses a pairing instability at $\xi = \infty$ at a temperature which differs from that without vertex renormalization by only $1/N$ corrections.

To summarize, in this paper we considered the properties of the antiferromagnetic quantum critical point for itinerant electrons by expanding in the inverse number of hot spots in the Brillouin zone $N = 8$. We went beyond a self-consistent $N = \infty$ theory and found two new effects: (i) the Fermi surface becomes nested at hot spots which is a weak SDW precursor effect, and (ii) vertex corrections account for anomalous spin dynamics and change the dynamical critical exponent from $z = 2$ to $z > 2$. To first order in $1/N$ we found $z = 2N/(N - 2) \approx 2.67$. We argued that anomalous frequency dependence is not accompanied by anomalous T dependence.

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