Localization-Delocalization Transition in Non-Hermitian Disordered Systems

A. V. Kolesnikov[1] and K. B. Efetov[1,2]

[1]*Fakultät für Physik und Astronomie, Ruhr-Universität, Bochum, Universitätsstrasse 150, Bochum, Germany*
[2]*L. D. Landau Institute for Theoretical Physics, Moscow, Russia*

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Using the supersymmetry technique, we study the localization-delocalization transition in quasi-one-dimensional non-Hermitian systems with a direction. In contrast to chains, our model captures the diffusive character of carriers' motion at short distances. We calculate the joint probability of complex eigenvalues and some other correlation functions. We find that the transition is abrupt and it is due to an interplay between two saddle points in the free energy functional.

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Non-Hermitian models with disorder have attracted recently considerable attention [1–11]. Non-Hermitian Hamiltonians appear in the context of flux lines in superconductors [1], in transfer phenomena in lossy media [8], in hydrodynamics [9], in QCD [10], and in quantum mechanics of open systems [11].

The most important property of non-Hermitian Hamiltonians is that their eigenenergies can be complex independently of the type of non-Hermiticity. Very interesting physical systems are models with a direction. The simplest Hamiltonian $\mathcal{H}$ of such models is written as

$$\mathcal{H} = \frac{(\mathbf{p} + i\mathbf{h})^2}{2m} + V(\mathbf{r}), \tag{1}$$

where $\mathbf{p}$ is the momentum operator, $V(\mathbf{r})$ is a random potential, and $\mathbf{h}$ is a constant vector. One comes to the Hamiltonian $\mathcal{H}$, Eq. (1), by reducing the $d + 1$ dimensional classical problem of vortices in a superconductor with columnar defects to a d-dimensional quantum problem. Then, the vector $\mathbf{h}$ is proportional to the component of the magnetic field perpendicular to the line defects [1].

Considering the one-dimensional (1D) version of the Hamiltonian $\mathcal{H}$, Eq. (1), the authors of Ref. [1] predicted a localization-delocalization transition that occurs at $h_c = l_c^{-1}$, where l_c is the localization length at $\mathbf{h} = 0$. At $h \leq l_c^{-1}$ the "imaginary vector-potential" $\mathbf{h}$ can be removed by the "gauge transformation"

$$\begin{aligned} \phi_k(\mathbf{r}) &= \phi_{0k}(\mathbf{r}) \exp(\mathbf{hr}), \\ \bar{\phi} k(\mathbf{r}) &= \phi_{0k}(\mathbf{r}) \exp(-\mathbf{hr}). \end{aligned} \tag{2}$$

In Eq. (2), $\phi_k(\mathbf{r})$ and $\bar{\phi} k(\mathbf{r})$ are the right and left eigenfunctions; $\phi_{0k}(\mathbf{r})$ are those at $\mathbf{h} = 0$. Carrying out the transformation, Eq. (2), we see that the eigenvalues ϵ_k do not depend on $\mathbf{h}$. However, the gauge transformation applies only if the functions $\phi_k(\mathbf{r})$ do not grow at infinity. For eigenfunctions of the form $\phi_0 \sim \exp(-|\mathbf{r}|/l_c)$, this is possible if $h < h_c$. For $h > h_c$, one should take extended wave functions, which leads to complex eigenenergies.

Qualitative arguments of Ref. [1] have been confirmed for the disordered chains numerically as well as analytically [4–6]. At the same time, they are quite general and one might expect that such a transition occurs in more complex systems, for example, in disordered quasi-one-dimensional wires and films. Using the mapping of Ref. [1], one expects such models to be relevant for description of the vortices in slabs and 3D bulk superconductors. The wires and films are richer than the chains because at distances larger than the mean free path l but smaller than the localization length L_c the motion is diffusive, whereas in disordered chains only ballistic motion or localization are possible. As a result, properties of the localization-delocalization transition in wires and films can differ. The problem has not been addressed yet and our paper is aimed at providing its basic understanding.

We study the distribution function of complex eigenenergies $P(\varepsilon, y)$, where ε and y are the real and imaginary parts of the energy, and correlations of eigenfunctions of the Hamiltonian $\mathcal{H}$, Eq. (1), at different points for wires using the supersymmetry technique [12]. A proper nonlinear supermatrix σ model has been derived in Ref. [3]. Its free energy functional $F_h[Q]$ reads

$$F_h[Q] = \frac{\pi\nu}{8} \mathrm{Str} \int [D(\partial Q)^2 - 4(\gamma\Lambda + y\Lambda_1\tau_3)Q]\, d\mathbf{r}, \tag{3}$$

where D is the classical diffusion coefficient, ν is the density of states, and the standard notations for the supertrace Str and supermatrices Q, τ_3, Λ, and Λ_1 are used [3,12]. The free energy functional $F_h[Q]$, Eq. (3), differs from the conventional one $F_0[Q]$ (describing the Hermitian problem) by the presence of the "gauge-invariant" combination $\partial Q = \nabla Q + \mathbf{h}[Q, \Lambda]$ instead of ∇Q.

The zero-dimensional (0D) version of the σ model is equivalent to models of random non-Hermitian matrices [3], where no transition occurs. But the σ model, Eq. (3), holds in any dimension and one might expect that already the 1D version would manifest the transition. So, one could describe the transition using the transfer-matrix technique developed for the σ model [13,14].

Surprisingly, the 1D σ model with $F_h[Q]$, Eq. (3), *does not* have any transition and physical quantities of interest are smooth functions of h giving always a finite probability of complex eigenenergies. At first glance, this result is in an evident contradiction with the arguments of Ref. [1].

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The resolution of this paradox is quite interesting. It turns out that the σ model with $F_h[Q]$, Eq. (3), describes properly only the region of the delocalized states. The functional $F_0[Q]$ should be used in the localized regime, for any finite $h < h_c$, where h_c is a critical vector potential. This replacement of the free energy, resembling a first order transition, results in an *abrupt* change of the eigenvalue distribution. At $h < h_c$, all eigenenergies are real, whereas at $h > h_c$ one gets a broad distribution in the complex plane. Our results are universal for any dimension when $h \gg h_c$. All this is in a contrast with the results obtained for chains, where the spectrum changes smoothly, showing an appearance of "arcs" when increasing h [1,4], or "wings" when starting from the regime of the strong non-Hermiticity [6]. A broad distribution is natural for ensembles of non-Hermitian matrices [2,15]. For a single chain, eigenvalues form a spectral line. Coupling of N chains produces N spectral lines in the complex plane [16]; our broad distribution can be thought of as a continuous limit of this.

Having formulated the rough picture of what happens when changing the parameter h, let us present some details. We first introduce the quantities which are to be studied. Since eigenvalues of the Hamiltonian (1) can be complex, it is convenient to double the size of relevant matrices [3], thus "Hermitizing" the problem. The role of the Green function is played by the function

$$B(\mathbf{r},\mathbf{r}') = \sum_k \frac{\gamma \phi_k(\mathbf{r})\overline{\phi}_k(\mathbf{r}')}{(\epsilon - \epsilon_k')^2 + (y - \epsilon_k'')^2 + \gamma^2} \quad (4)$$

with an eigenenergy $\epsilon_k = \epsilon_k' + i\epsilon_k''$. The "density-density correlator" in the present context is given by

$$Y(\mathbf{r},\mathbf{r}';\epsilon,y) = \frac{1}{\pi}\lim_{\gamma\to 0}\langle B(\mathbf{r},\mathbf{r}')B(\mathbf{r}',\mathbf{r})\rangle\,, \quad (5)$$

where brackets imply impurity averaging. The limit $\gamma \to 0$, understood in all correlators, becomes important as soon as $h \to 0$. The function $Y(\mathbf{r},\mathbf{r}';\epsilon,y)$, Eq. (5), establishes a link between the localization properties and the joint probability density of complex eigenenergies,

$$P(\epsilon,y) = \frac{1}{V}\int d\mathbf{r}\, d\mathbf{r}'\, Y(\mathbf{r},\mathbf{r}';\epsilon,y)\,, \quad (6)$$

where V is the volume. We introduce also another important correlator, $X(\mathbf{r},\mathbf{r}';\epsilon,y) = C(\mathbf{r},\mathbf{r}';\epsilon,y) + C(\mathbf{r}',\mathbf{r};\epsilon,y)$,

$$C(\mathbf{r},\mathbf{r}';\epsilon,y) = \frac{1}{2\pi}\lim_{\gamma\to 0}\langle B(\mathbf{r},\mathbf{r}')B^*(\mathbf{r},\mathbf{r}')\rangle\,. \quad (7)$$

The correlator $Y(\mathbf{r},\mathbf{r}';\epsilon,y)$, Eq. (5), is invariant under the transformation, Eq. (2), but $X(\mathbf{r},\mathbf{r}';\epsilon,y)$ is not.

We further express as usual [3,12] the correlation functions in terms of integrals over eight-component superfield $\psi(\mathbf{r})$, average over the white-noise disorder potential $V(\mathbf{r})$, decouple the ψ^4 term by 8×8 matrix $Q(\mathbf{r})$, and integrate over $\psi(\mathbf{r})$. As a result, we obtain a functional integral over Q with a free energy functional

$$F[Q] = \frac{1}{2}\int \mathrm{Str}\left(\frac{\pi\nu Q^2(\mathbf{r})}{4\tau} - \ln\left[-i\mathcal{H} + \frac{Q(\mathbf{r})}{2\tau}\right]\right) d\mathbf{r}\,,$$

$$\mathcal{H} = (\mathbf{p}^2 - \mathbf{h}^2)/2m - \epsilon + i\gamma\Lambda + i\Lambda_1\mathcal{H}', \quad (8)$$

where $\mathcal{H}' = -i\mathbf{h}\nabla/m + y\tau_3$ and τ is the mean free time.

The next standard step is to find the minimum of $F[Q]$ neglecting $\mathcal{H}'$. The minimum is reached at

$$Q = V\Lambda\overline{V}\,, \quad (9)$$

$V\overline{V} = 1$, $V^+ = K\overline{V}K$. (The notations are the same as in Ref. [3,12].) Expanding the functional $F[Q]$ near the minimum in the gradients of Q and in $\mathcal{H}'$ one comes to the functional $F_h[Q]$, Eq. (3). This is exactly the way that the σ model, Eq. (3), was derived in Ref. [3]. However, it has been mentioned that this σ model is not valid in the localized regime. What is wrong in the derivation?

It is not difficult to find that the functional $F[Q]$ has also another minimum. Performing the transformation

$$\widetilde{Q} = \exp(\Lambda_1\mathbf{rh})Q\exp(-\Lambda_1\mathbf{rh})\,, \quad (10)$$

we rewrite the functional $F[Q]$, Eq. (8) in terms of $\widetilde{Q}$. As a result, the imaginary vector potential $\mathbf{h}$ is removed from F and the minimum is achieved at

$$\widetilde{Q} = V\Lambda\overline{V}\,, \quad (11)$$

which corresponds to Q varying in space.

Which of these two saddle points should be chosen? The answer depends on the value of h. To clarify this question we consider the correlation functions $Y(\mathbf{r},\mathbf{r}';\epsilon,y)$ and $X(\mathbf{r},\mathbf{r}';\epsilon,y)$, Eqs. (5) and (7). They can be written as functional integrals over the supermatrices Q,

$$Y(X) = \frac{\pi\nu^2}{4}\langle Q_{42}^{1\pm}Q_{24}'^{1\pm} - Q_{42}^{2\pm}Q_{24}'^{2\pm}\rangle_Q\,, \quad (12)$$

where the sign $+$ $(-)$ corresponds to the correlator Y (X), $Q^{1\pm} = Q^{11}(\mathbf{r}) \pm Q^{22}(\mathbf{r})$, $Q^{2\pm} = Q^{12}(\mathbf{r}) \pm Q^{21}(\mathbf{r})$, and the matrices Q' are taken at $\mathbf{r}'$. The symbol $\langle\cdots\rangle_Q$ stands for averaging with the functional $F[Q]$, Eq. (8). To calculate the integral in Eq. (12) let us use the transformation, Eq. (10), and take the saddle point, Eq. (11). Since the combination of the supermatrices Q entering Eq. (12) for the function Y is invariant under the transformation, Eq. (10), $\mathbf{h}$ is gauged out in this function. Therefore, one can use the standard results of the transfer-matrix approach [12] developed for the Hermitian case. The final result for the correlator $Y(\mathbf{r},\mathbf{r}';\epsilon,y)$,

$$Y(\mathbf{r},\mathbf{r}';\epsilon,y) = p_\infty(r)\delta(y)\,, \quad (13)$$

where $r = |\mathbf{r} - \mathbf{r}'|$, contains the function $p_\infty(r)$ characterizing localization properties

$$p_\infty(r) = \sum_k |\phi_{0k}(\mathbf{r})|^2|\phi_{0k}(\mathbf{r}')|^2\delta(\epsilon - \epsilon_k)\,. \quad (14)$$

In disordered wires, the function $p_\infty(r)$ describes the decay of the wave functions at $r \gg L_c$,

$$p_\infty(r) \approx \frac{1}{4\sqrt{\pi}L_c}\left(\frac{\pi^2}{8}\right)^2\left(\frac{4L_c}{r}\right)^{3/2}\exp\left(-\frac{r}{4L_c}\right), \quad (15)$$

where L_c is the localization length. For the unitary ensemble it equals $L_c = 2\pi\nu SD$; S is the cross section.

In contrast, the vector potential $\mathbf{h}$ enters explicitly the preexponential of the function X after making the transformation, Eq. (10). However, the dependence on $\mathbf{h}$ is simple and calculations for $X(\mathbf{r},\mathbf{r}';\epsilon,y)$ can be performed in the same way as for $Y(\mathbf{r},\mathbf{r}';\epsilon,y)$, yielding

$$X(\mathbf{r},\mathbf{r}';\epsilon,y) = \cosh(2hr)p_\infty(r)\delta(y). \quad (16)$$

Equations (13) and (16) demonstrate that, provided one may perform the transformation, Eq. (10), all eigenenergies remain real. However, the validity of this procedure depends on the value of h. The function $X(\mathbf{r},\mathbf{r}';\epsilon,y)$ does not grow at infinity only if $h < h_c$ where

$$h_c = (8L_c)^{-1}. \quad (17)$$

At $h > h_c$ Eqs. (13) and (16) cannot be used because this would correspond to growing wave functions, which are forbidden for a closed geometry. This agrees with the arguments of Ref. [1]. According to Ref. [1] one should use at $h > h_c$ extended states having complex eigenenergies.

In the present formalism, the other saddle point, Eq. (9), of the free energy $F[Q]$ should be taken in the regime $h > h_c$. This leads to the σ model in the form of Eq. (3). Expecting that the eigenenergies become complex, we determine their distribution function $P(\epsilon,y)$, Eq. (6). Following the transfer-matrix technique [12,13], we write the function $P(\epsilon,y)$ in the form

$$P(\epsilon,y) = \frac{\pi\nu^2 S}{4}\int \Psi(Q)\,(Q_{42}^{1+}P_{24}^{1+} - Q_{42}^{2+}P_{24}^{2+})\,dQ\,. \quad (18)$$

In Eq. (18), $\Psi(Q)$ is the partition function of the semi-infinite wire; the matrix function P is the partition function between the points r and r' multiplied by $Q'\Psi(Q')$ and integrated over r'. As usual, proper differential equations for Ψ and P are derived from $F[Q]$, Eq. (3).

In order to carry out these calculations it is necessary to choose a parametrization of the supermatrices Q. For the non-Hermitian problem involved the parametrization of Ref. [3] is most natural. For simplicity we consider the unitary ensemble, where the supermatrices Q can be parametrized in the form

$$Q = T(\hat\theta)Q_0\overline{T}(\hat\theta), \qquad Q_0 = \begin{bmatrix} \cos\hat\varphi & -\tau_3\sin\hat\varphi \\ -\tau_3\sin\hat\varphi & -\cos\hat\varphi \end{bmatrix}. \quad (19)$$

In Eqs. (19), the supermatrices $T(\hat\theta)$ contain not only real variables $\hat\theta$ but also Grassmann variables. The diagonal matrices $\hat\varphi$ and $\hat\chi$ contain compact variables, θ and φ, and noncompact ones, χ and θ_1.

The variables $\hat\theta$ and $\hat\varphi$ are not equivalent. For example, neglecting the gradient terms in Eq. (3) one comes to the 0D free energy $F^{(0)}$, containing the variables $\hat\varphi$ only:

$$F^{(0)}[\hat\varphi] = \tilde h^2(\lambda_1 - i\tilde y/2\tilde h^2)^2 + \tilde h^2(\lambda + \tilde y/2\tilde h^2)^2, \quad (20)$$

where $\lambda_1 = \sinh\chi$, $\lambda = \sin\varphi$, $\tilde h^2 = 2h^2L_c^2$, and $\tilde y = 2yL_c^2/D$. In contrast to a real magnetic field H, which gradually suppresses and finally freezes out some degrees of freedom with increasing H, the imaginary magnetic field h shifts the saddle point as a whole. Noticeable changes in behavior occur only at $\pm\tilde y \gtrsim 2\tilde h^2$, where $\sin\varphi \approx 1$.

Calculations performed in Ref. [3] for the 0D case show that the variables $\hat\theta$ play a minor part in 0D. They do not enter $F^{(0)}[\hat\varphi]$ but their role is even less pronounced due to the singularity of the Jacobian at θ, $\theta_1 \to 0$. A detailed discussion of Ref. [3] leads to the conclusion that one should replace the $\hat\theta$-dependent part of the Jacobian by a constant and put everywhere else $\hat\theta = 0$. It is also interesting to note that the free energy $F_h[Q]$, Eq. (3), is not invariant against the replacement, Eq. (10). Using the parametrization, Eqs. (19), one obtains that this replacement leads to the shift $\tilde\theta = \theta - 2i\mathbf{rh}$, $\tilde\theta_1 = \theta_1 - 2\mathbf{rh}$. This shift changes the contour of the integration over $-\pi < \theta < \pi$ in a complicated way, thus demonstrating the violation of the "gauge symmetry."

As in the 0D case, one should put everywhere θ, $\theta_1 = 0$ when deriving the transfer matrix equations. As a result, differential equations for Ψ and P contain λ and λ_1 only:

$$\hat{\mathcal{H}}\Psi = F^{(0)}\Psi, \qquad \hat{\mathcal{H}}P^+ = L_c(i\lambda_1 - i\lambda)\Psi\,,$$
$$\hat{\mathcal{H}} = \frac{1}{J_0}\partial_\lambda(1-\lambda^2)J_0\partial_\lambda + \frac{1}{J_0}\partial_{\lambda_1}(1+\lambda_1^2)J_0\partial_{\lambda_1}\,, \quad (21)$$

where $J_0 = 1/(\lambda_1 + i\lambda)^2$.

Analysis of Eqs. (21) for $hL_c \gg 1$ is to some extent similar to the one in the limit of high frequencies for the conventional problem of localization [12]. We obtain

$$\Psi \simeq \exp[-\tilde h(\lambda_1 + i\tilde y/2\tilde h^2)^2 - \tilde h(\lambda - \tilde y/2\tilde h^2)^2],$$
$$P^+ \simeq \Psi/2\tilde h\,,$$

which gives after substitution into Eq. (18)

$$P(\epsilon,y) \simeq \frac{\nu}{4\tilde h^2}\begin{cases} 1, & |\tilde y| < \tilde h^2, \\ 0, & |\tilde y| > \tilde h^2. \end{cases} \quad (22)$$

The form of the function $P(\epsilon,y)$, Eq. (22), is the same as the 0D result of Refs. [2,3]. This result does not depend on the dimensionality and corresponds to the elliptic law for strongly non-Hermitian random matrices [15].

Analogous calculations for Y, Eqs. (4) and (5) yield

$$Y(r) \simeq \nu\beta\exp(-2\beta|r|)\,, \qquad \beta = \sqrt{(h^2 - y^2/4Dh^2)}\,. \quad (23)$$

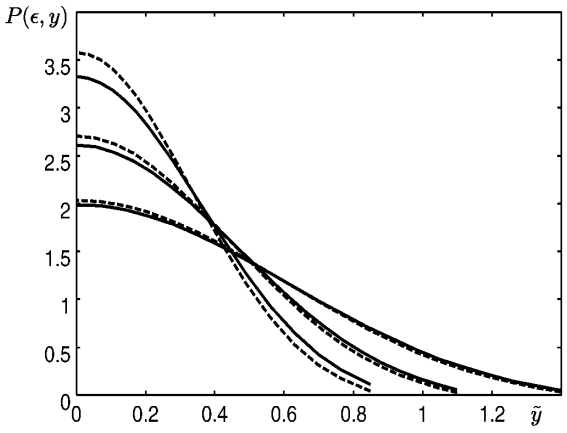

FIG. 1. The joint distribution function of complex eigenvalues $P(\epsilon)$, Eqs. (6), for the quasi-1D infinite wire (solid line) and the 0D case (dashed line), for $\tilde{h} = \tilde{h}_c, 0.2, 0.25$.

Analytical study of Eqs. (21) is hardly feasible at $hL_c \sim 1$. To solve them numerically, we use the standard overrelaxation method with Chebyshev acceleration. The results for the distribution $P(\epsilon, y)$ are presented in Fig. 1 for several values of $\tilde{h}$. The lowest value of $\tilde{h} = 1/4\sqrt{2}$ corresponds to the critical h_c, Eq. (17). For comparison we present the 0D result of Refs. [2,3].

The Q matrix does not vary in the space in 0D and the coordinate-independent saddle point, Eq. (9), governs the behavior. The same saddle point is relevant in 1D for $h > h_c$ so that the curves for the 1D and 0D cases are rather close to each other. The evolution of eigenvalue distribution with further decrease of h is drastically different for the 1D and 0D cases. The 0D curve tends smoothly to the δ function $\nu\delta(y)$ when $h \to 0$, whereas the 1D curve changes abruptly to this expression at $h = h_c$. This is because the space-varying saddle point, Eq. (11), becomes essential at $h < h_c$ which follows from the behavior of the correlator X, Eq. (16). In the region $h > h_c$ the states are extended and the exponential decay of the correlation function $Y(r)$, Eq. (23), is obtained after summation over many states with different phase differences.

In conclusion, we studied the localization-delocalization transition for non-Hermitian disordered systems with a direction, Eq. (1), and broken time-reversal symmetry. We found that the transition occurs due to an interplay between two saddle points in the free energy $F[Q]$, which results in the *abrupt* transition at the critical field $h = h_c$. Below this field, all eigenvalues are real and wave functions are localized. Above the critical field the eigenvalues form a broad distribution in the complex plane. We believe that the abrupt transition found is quite natural for the original problem of the vortex lines in the presence of columnar defects [1]. Provided the geometry of the sample is closed and it is infinitely long, vortex lines should abruptly change their behavior, from being pinned by columns to becoming oriented along the magnetic field.

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