Parallel Magnetic Field Induced Giant Magnetoresistance in Low Density Quasi-Two-Dimensional Layers

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We provide a possible theoretical explanation for the recently observed giant positive magnetoresistance in high mobility low density *quasi*-two-dimensional electron and hole systems. Our explanation is based on the strong coupling of the parallel field to the *orbital* motion arising from the *finite* layer thickness and the large Fermi wavelength of the *quasi*-two-dimensional system at low carrier densities.

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Recently an intriguing set of low temperature transport properties of low density high mobility two-dimensional (2D) electron (Si inversion layers) and hole (p-type GaAs heterostructures) systems, being collectively referred to as the 2D metal-insulator-transition (MIT) phenomenon, has attracted a great deal of experimental and theoretical attention. Among the puzzling and interesting aspects of the 2D MIT observations is the strong parallel (to the 2D layer) magnetic field (B) dependence of the measured low temperature (T) resistivity ρ of the apparent metallic phase at electron (we will refer to both electrons and holes by the generic name "electron" in this paper) densities (n) above the nominal MIT transition density (n_c). The unexpectedly strong parallel field (B in this paper refers exclusively to a 2D in-plane magnetic field with zero perpendicular field) dependence of the 2D resistivity is suppressed deep into the metallic phase ($n \gg n_c$), and the system behaves more like a "conventional metal." The effect of B is dramatic [1–6]: Modest B fields (5–10 T) spectacularly increase the measured resistance by up to 2 orders of magnitude at low n, independent of whether the system is in the nominal metallic ($n \geq n_c$) or insulating ($n \leq n_c$) phase—in Si inversion layer $\rho(B)$ eventually seems to "saturate" at very high resistances of the order of $\rho \sim 10^6\ \Omega$ (in the nominal "metallic" phase) for $B \sim 10$ T, whereas in p-GaAs systems no such resistivity saturation has yet been reported. In addition to this giant magnetoresistance, the temperature dependence of $\rho(T)$ is also affected by B—in particular, increasing B modifies the sign of $d\rho/dT$ which is positive (metallic behavior) at low B and negative ("insulating" behavior) at high B.

In this Letter we develop a possible theoretical explanation for the observed parallel magnetic field dependence of the 2D resistivity at low T and n. Our theoretical explanation for the observed [1–6] giant magnetoresistance and the associated "suppression" of the metallic phase is compellingly simple conceptually, and is robust in the sense that it is independent of whether the conducting state is a true $T = 0$ novel 2D metallic phase or just an effective "metal" (at finite T) by virtue of the localization length being exceedingly large. Our calculated B-dependent resistivity is in good qualitative agreement with the experimental observations [1–6], particularly in the GaAs-based 2D systems, and can be systematically improved in the future. The agreement between our theory and experimental results in Si inversion layers is not particularly good, and we mostly concentrate on GaAs systems in this paper. In this Letter we use a minimal model implementing the basic theoretical idea to obtain the generic trends in $\rho(B)$ in order to demonstrate that we have found a possible explanation, at least in the GaAs hole systems [3,6], for the giant magnetoresistance reported in recent experiments [1–6]. We also emphasize that the theory presented in this Letter is totally independent of our $B = 0$ theory for $\rho(T, n)$ developed in Ref. [7], and the parallel field results presented herein are *not* an extension and/or application of our $B = 0$ theory.

The new physics we introduce in this paper is *the coupling of the parallel B-field to the orbital motion of the 2D carriers*. It has so far been assumed [8] that the observed parallel field induced giant magnetoresistance must *necessarily* be a spin effect because the orbital motion does not couple to a B field parallel to the 2D plane. This would certainly be true if the systems being studied were strictly 2D systems with zero thickness (i.e., perfect δ-function confinement) in the direction (z) normal to 2D (x-y) plane. In reality, however, the systems being studied are quasi-2D with their average thickness in the z direction $\langle z \rangle$, being of the order of 30–300 Å depending on the system, carrier density, and other parameters (e.g., depletion charge density) which are often not accurately known. Therefore, a parallel magnetic field in the x-y plane does couple to the z-orbital degree of freedom of the system, and such an orbital coupling could, in fact, be strong when $l_c < \langle z \rangle$ where $l_c = (\hbar c/eB)^{1/2}$ is the magnetic length associated with the parallel field. Since $\langle z \rangle$ depends on (increases with decreasing n) the electron density n [9,10] through the self-consistent confinement potential, and may be quite large at low electron densities, the condition $l_c < \langle z \rangle$ is fulfilled in the low density experimental regime where the phenomenon of giant magnetoresistance is observed. In addition, the effect of the parallel field is enhanced at low carrier densities by the fact that the 2D Fermi wavelength λ_F ($= 2\pi/k_F$) at $B = 0$ is substantially larger than the magnetic length l_c associated with the typical B indicating

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a massive nonperturbative orbital effect associated with the applied parallel field. For example, for $B \approx 5$ T and $n = 5 \times 10^{10}$ cm^{-2}, $\lambda_F \approx 1600$ Å (Si); 1100 Å (p-GaAs), and $l_c \approx 100$ Å—thus $\lambda_F \approx (10-15) l_c$, a situation which would be achieved in a simple metal only for astronomical fields around $B \approx 10^7$ T. One should therefore expect a strong orbital effect arising from the applied field as reflected in the observed giant magnetoresistance in these systems. Since we are interested in only the orbital motion, we neglect any Zeeman spin splitting in our consideration assuming a spin degeneracy of 2 throughout.

We note that the new physics arising from the coupling of the orbital motion to the parallel field in these quasi-2D inversion layer type systems has no analog in purely 3D or 2D systems. The zero field z motion in these quasi-2D systems is quantized into subbands [9,10] due to the confining potential—the coupling of the B field to the orbital motion thus necessarily involves intersubband dynamics or scattering, and this intersubband dynamics gets coupled with the in-plane 2D dynamics in the presence of the parallel field, leading to the giant magnetoresistance. The effect should, in fact, persist to high densities (i.e., deep in the so-called metallic phase) except the size of the magnetoresistance should be only a few percent at high electron densities ($l_c \gg \langle z \rangle, \lambda_F$), whereas in the nonperturbative strong field limit the effect can be huge. Another way of understanding this strong (and unusual) orbital coupling is to note that the cyclotron energy associated with $B \approx 10$ T is about 6 meV, which is larger than the zero field subband splitting and much larger than the 2D Fermi energy of the system in the low density regime. This interplay of cyclotron and subband dynamics is a novel feature of the quasi-2D system which has no purely 2D or 3D analog. The physics of orbital coupling introduced in this paper could be thought of as a parallel field induced effective 2D to 3D crossover in the low density quasi-2D systems.

We implement the above idea by assuming a simple harmonic confinement of the z wave function at $B = 0$. We adjust the parabolic confinement potential variationally to obtain the best fit to the $B = 0$ wave function and energy of the actual system at the appropriate density n. The harmonic confinement allows us to incorporate the effect of the parallel B field nonperturbatively [10], and in the high field limit ($l_c \ll \langle z \rangle_{B=0}$) our harmonic approximation (the Fock-Darwin levels) becomes almost exact. Any error arising from our somewhat inaccurate choice of the z wave function at $B = 0$ has only a small effect on the rather large magnetoresistance we calculate. We take the B-field direction to be the x axis without any loss of generality, and denote by ρ_{xx} (ρ_{yy}) the 2D resistivity associated with the electric current flowing along (perpendicular) to the direction of B. As we describe below in detail, one of our specific predictions is that $\rho_{yy} \gg \rho_{xx}$ due to the large 2D anisotropy introduced by the applied B field, provided there is no other anisotropy in the system.

For our transport calculation we apply the Boltzmann theory (in the presence of the parallel field which is treated nonperturbatively by including it explicitly in our one electron wave function and energy via the harmonic confinement model [10]) assuming scattering by short-range random impurities distributed uniformly throughout the quasi-2D layer. Our neglect of screening effects [7] and the associated assumption of δ-function impurity scattering potential arising from random impurity centers is done purely for the purpose of simplicity and in order to keep the number of free parameters at a minimum. Since the effect we are considering is a rather gross effect (involving a very large increase in magnetoresistance), any errors associated with our simplified scattering model are not of any qualitative significance in understanding the basic phenomenon. One will have to improve the model (both for confinement and for impurity scattering) if one is interested in precise quantitative agreement with the experimental data in a specific system—we believe that such improvements may be computationally extremely demanding. Details of our calculation will be given in a future long publication.

We show our calculated results in Figs. 1 and 2, concentrating on the p-GaAs samples of Ref. [6]. In Fig. 1(a) we show our calculated ρ_{xx} at $T = 50$ mK for various carrier densities as a function of the applied B field. The harmonic confinement at each density has been variationally adjusted to give the best wave function for the holes in the GaAs heterostructure appropriate for that density at $B = 0$. The overall qualitative trends of our calculated results agree well with the experimental data (cf. Fig. 3 of Ref. [6]). In fact, the specific experimental results of Ref. [6] agree very well with our calculations as can be seen in our Fig. 1(a) where we have put some representative experimental data. At low B ($\omega_c < \omega_0$), $\ln(\rho_{xx})$ shows a B^2 dependence, changing to a linear B dependence at high B ($\omega_c > \omega_0$) in agreement with experiment [6], where $\omega_c = eB/mc$ is the cyclotron frequency of the B field and ω_0 is the curvature (or the subband splitting) of the confinement potential. We note that the overall resistivity scale in our results is set by the density N_b of the δ-function impurity scatterers which uniquely defines the $B = 0$ value of ρ_{xx} in a particular sample. In Fig. 1(b) we show our calculated qualitative behavior for the Si inversion layer situation where the impurity scattering centers, instead of being randomly distributed throughout the layer, are located at the Si-SiO_2 interface or mostly on the insulating side. In this situation (when the dominant scattering mechanism is planar, as due to the charged impurities at the interface and the surface roughness scattering at the Si-SiO_2 interface) elementary considerations show that the scattering effect must saturate eventually at high enough B fields as can be seen in Fig. 1(b) with the approximate saturation field increasing with increasing electron density consistent with experimental observations [1,2,4,5]. The qualitative difference between the nonsaturation behavior [Fig. 1(a)] and the saturation behavior [Fig. 1(b)] arises from the δ-function random scattering centers being distributed three dimensionally in Fig. 1(a) and in a 2D plane at the interface in Fig. 1(b). The results in Fig. 1(a) correspond qualitatively

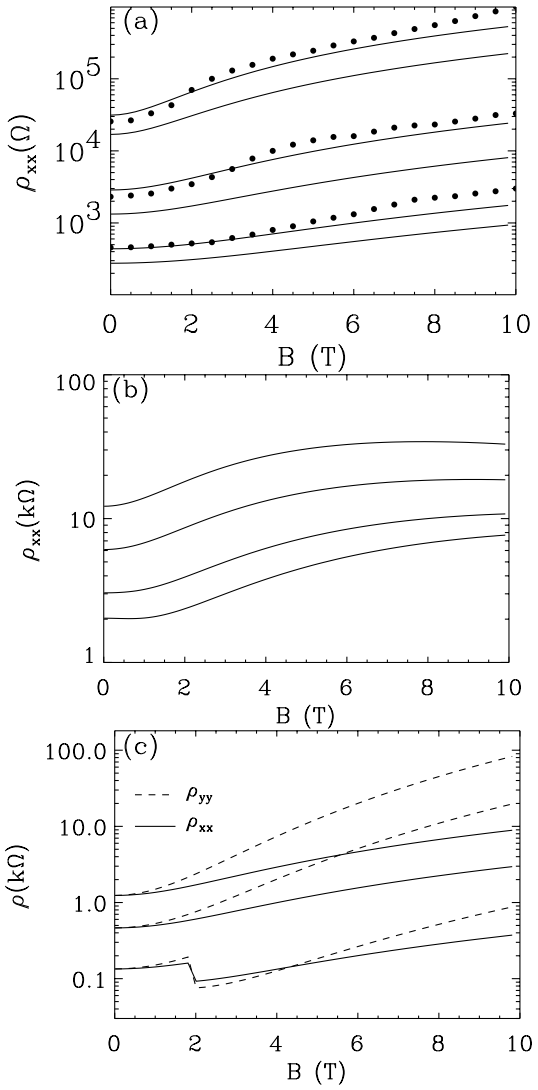

FIG. 1. (a) The calculated 2D resistivity ρ_{xx} along the field direction as a function of the parallel field B at $T = 50$ mK in GaAs hole systems (appropriate for the samples in Ref. [6]) for $n = 0.89, 1.1, 1.63, 2.12, 3.23, 4.11 \times 10^{10}$ cm^{-2} (top to bottom) with the confinement curvature $\omega_0 = 0.85, 1.0, 1.35, 1.7, 2.3, 2.6$ meV (top to bottom). Experimental points from Ref. [6] are shown as dots for comparison. Random δ impurities are distributed uniformly through the layer. (b) The calculated ρ_{xx}-B plot when the random δ impurities are at the interface plane 85 Å from the center of the Gaussian ground state wave function (at $B = 0$). The top to bottom curves are for $n = 1$, 2, 4 , 6×10^{10} cm^{-2} and $\omega_0 = 1$ meV. The saturation shifts to higher B values for higher densities. (c) The calculated ρ_{xx}- and ρ_{yy}-B plots showing the strong 2D anisotropy with $\omega_0 = 1.5$ meV and $n = 1.5, 4.0, 10.0 \times 10^{10}$ cm^{-2}.

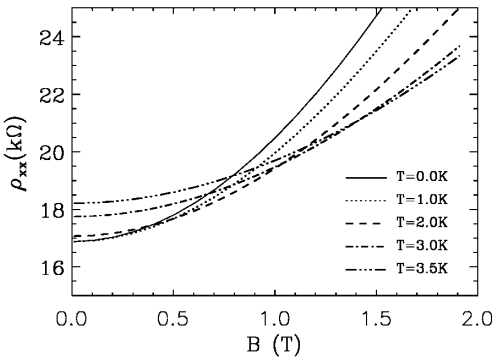

FIG. 2. The calculated ρ_{xx}-B plots for various T values as shown with $n = 1.1 \times 10^{10}$ cm^{-2}, $\omega_0 = 1.0$ meV.

to GaAs where the main resistive scattering centers are the random impurities in GaAs, whereas the results in Fig. 1(b) correspond more to the Si inversion layer where the main scattering centers (charged impurities and surface roughness) are located in a plane near the interface. We have not adjusted the parameters used for Fig. 1(b) to get agreement with Si inversion layer data—our only point here is to demonstrate the qualitative physics underlying the saturation behavior. In principle, we can get semiquantitative agreement with any given set of data by adjusting the confinement parameter ω_0, but we do not believe such an exercise to be meaningful particularly for the highly simplified model used in our theoretical calculations.

In Fig. 1(c) we show our predicted anisotropic magnetoresistivity with $\rho_{yy} \gg \rho_{xx}$—note that the anisotropy can be very large, and decreases with increasing density. (The corresponding 2D Fermi surfaces, not shown here due to lack of space, are strongly anisotropic in shape, being elliptic rather than circular with the eccentricity increasing with increasing field.) The highest density results (the lowest set of curves) in Fig. 1(c) show another predicted feature of our theory: At relatively high density, if the first excited subband of the system is occupied by the carriers at $B = 0$ (a situation which in principle is achievable), the calculated resistivity would, in fact, first exhibit a negative magnetoresistance, as the excited subband depopulates with increasing B, before showing the characteristic giant magnetoresistance phenomenon. This oscillatory feature in the lowest set of curves in Fig. 1(c) is the analog of the usual SdH oscillations in this problem. It is important to mention that the features predicted in Fig. 1(c) have already been *observed experimentally* [11] in parabolic *n-type* GaAs structures at higher densities (where the overall magnetoresistivity is a factor of 6 for ρ_{yy} and only a factor of 2 for ρ_{xx}), and our calculations for this structure [11] agree well with the experimental findings.

It is important to emphasize that our predicted anisotropy [Fig. 1(c)], which has been observed in n-type GaAs systems [11], is *not* seen [1,12] in Si inversion layers where $\rho_{xx} \sim \rho_{yy}$ even in the presence of a strong applied parallel magnetic field. In this context we also point out that the saturation behavior of Fig. 1(b) calculated in our theory is not in particularly good agreement with the observed behavior [1,2,4,5] in Si inversion layers where the saturation sets in more abruptly than in our theory. On the other hand, our calculated magnetoresistance for GaAs systems, as shown in Fig. 1(a), is in excellent qualitative agreement with the corresponding GaAs results reported in Ref. [6]. The reason for this difference between the observed experimental behavior between Si and GaAs

based systems is currently not known. Our current theory applies rather well to the GaAs-based 2D systems but not to Si systems for reasons which are not clear at this stage. One possibility for the difference between the two systems could be spin effects neglected in our theory.

Finally, in Fig. 2 we provide the calculated temperature dependence of $\rho(T, B, n)$ which shows interesting nonmonotonicity ("the suppression of the metallic phase") experimentally [6]. We emphasize that since we neglect all screening effects (due to our δ-function impurity scattering model) our calculated temperature dependence, which arises entirely from Fermi surface and thermal occupancy effects, is necessarily simplistic [compared, for example, with our $B = 0$ theory for $\rho(T, n)$ as given in Ref. [7]]. Nevertheless, we believe that even this drastically simplified model catches the basic physics of the phenomenon, and explains on a qualitative level why $d\rho/dT$ changes sign from being positive (metallic) at low field to negative (insulating) at high fields. This is essentially a "quantum-classical crossover" type phenomenon [7] where the strong modification of the Fermi surface leads to nonmonotonic temperature dependence at various B values. The physics of the negative $d\rho/dT$ at high fields is entirely a Fermi surface effect. In Fig. 2 we plot $\rho_{xx}(B)$ for various fixed values of T, and we note that we obtain rough qualitative agreement with the experimental observations [3,6] that there is a transition [around $B_c \sim 1$ T in Fig. 2] point B_c where $d\rho(B)/dT$ changes its sign from being positive (metallic) for $B < B_c$ to being negative (insulating) for $B > B_c$—in our theoretical results B_c is *not* a sharp transition point, rather a rough transition regime, whereas in the experiment B_c seems to be a sharp point. We see no particular reason for B_c to be a sharp single transition point since this phenomenon is obviously not a phase transition (both $B < B_c$ and $B > B_c$ are effective metallic phases), and the crossover behavior is only a Fermi surface (which is drastically distorted by high B values) effect. We suggest more precise measurements to check whether B_c is really a single transition point or more a rough transition regime.

We conclude by summarizing our theory and by briefly discussing our various approximations and limitations. We have shown that the observed giant positive magnetoresistance phenomenon in quasi-2D systems in the presence of a parallel magnetic field can be qualitatively explained as arising from the coupling of the field to the carrier orbital motion by virtue of the finite thickness and the low density of the layer (spin plays no role in our explanation). We predict a large anisotropy of resistivity in the 2D plane. Our main approximations are the following: (1) Boltzmann transport theory; (2) harmonic confinement; (3) δ-function random impurity scattering. None of these approximations is qualitatively significant because we predict a very large (factors of 10–1000) and robust effect. One important corollary of our theory is that the same effect should persist on the insulating side as long as the localization length is larger than the magnetic length except for the fact that $d\rho/dT$ should always be negative on the insulating side, which is what is experimentally observed. Two important approximations of our theory are that we have neglected all spin-related effects as well as all crystallographic anisotropy effects, which could, in principle, be added to our theory if future experiments demand such an improvement. The main (and an important) limitation of the theory is that our predicted anisotropy seems *not* to be consistent with the existing data in Si inversion layers [12] where no magnetoresistive anisotropy has been seen. We do not know the reason for this disagreement—one possibility being that there is an additional scattering mechanism, possibly spin related, which also plays a role in the observed magnetoresistance and compensates for the anisotropy arising from orbital effects. The other possibility is that screening could be anisotropic in the presence of a strong parallel field (since the 2D Fermi surface is highly anisotropic), leading to a cancellation of the transport anisotropy in the Si inversion layer where screening effects are typically very strong (our theory neglects screening as we assume short-range impurity scattering). While more work is clearly needed to understand the quantitative details of the observed magnetoresistance (particularly in Si inversion layers) and spin-related effects may very well be playing an additional role in the experimental data, our work compellingly demonstrates the importance of *orbital* magnetoresistance in the presence of a parallel magnetic field in the low density limit which cannot be neglected in future work on the subject.

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