Intermediate Shocks in Three-Dimensional Magnetohydrodynamic Bow-Shock Flows with Multiple Interacting Shock Fronts

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Simulation results of three-dimensional (3D) stationary magnetohydrodynamic (MHD) bow-shock flows around perfectly conducting spheres are presented. For strong upstream magnetic field a new complex bow-shock flow topology arises consisting of two consecutive interacting shock fronts. It is shown that the leading shock front contains a segment of intermediate 1–3 shock type. This is the first confirmation in 3D that intermediate shocks, which were believed to be unphysical for a long time, can be formed and can persist for small-dissipation MHD in a realistic flow configuration.

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Many phenomena in astrophysical and laboratory plasmas may be described by the equations of magnetohydrodynamics (MHD) [1]. The MHD equations describe the motion of a nonrelativistic conducting fluid, and can be derived by combining the equations of compressible hydrodynamics with the Maxwell equations. MHD allows for three different small-amplitude—or linear—wave modes: the fast, the Alfvén, and the slow wave. These wave modes are anisotropic as the wave speeds strongly depend on the angle between the direction of propagation and the magnetic field $\vec{B}$. Because of nonlinear effects, MHD waves can steepen into shocks. Corresponding to the three types of linear waves, the MHD equations allow for three different types of shocks, namely, the fast, intermediate and slow shocks. All MHD shocks have the property of coplanarity, which means that the downstream magnetic field lies in the plane defined by the upstream magnetic field and the shock normal.

While fast and slow MHD shocks are known to occur in plasma flows, it has been believed for a long time that intermediate MHD shocks are unphysical [1,2], a view that is still expressed in most present-day textbooks on MHD. In the dissipationless—or ideal—MHD system intermediate shocks are indeed unstable as they break up instantaneously upon arbitrary small perturbation of the magnetic field component out of the plane of coplanarity (by Alfvén waves). However, recently it has been shown that intermediate shocks can be stable when dissipation is included [3–9]. The precise influence of dissipation mechanisms and magnitudes on the stability of intermediate MHD shocks is complicated and the analysis remains incomplete. Nevertheless, the following general statements can be made. Intermediate shocks are stable in the dissipative MHD system for wide ranges of the coefficients of resistivity, viscosity, and heat conduction [3–9]. They can be destabilized by Alfvén waves, but only when the integrated amplitude of the perturbation is sufficiently large [4]. The amplitude of the perturbation required for destabilization decreases with decreasing dissipation [9,10]. The stability issues involving intermediate shocks are due to mathematical properties peculiar to MHD, namely, nonstrict hyperbolicity [5,9], nonconvexity [5,8,11], and rotational invariance [9].

The theoretical results on the existence of intermediate MHD shocks have been confirmed in simulations, but only in one dimension (1D) [3,5]. In these simulations the initial left and right states had to be chosen coplanar in order for the stationary intermediate shocks to form and persist. It can be argued that such a situation is exceptional, as one may think that coplanarity of left and right states does generally not occur at many locations and for long times in real flows. The question whether intermediate shocks can be formed and can persist in realistic three-dimensional (3D) flows is thus still vigorously debated [5–14].

MHD shocks.—The equations of single-fluid MHD in conservative form are given by

$$\frac{\partial}{\partial t}\begin{bmatrix} \rho \\ \rho\vec{v} \\ \vec{B} \\ e \end{bmatrix} + \nabla\cdot\begin{bmatrix} \rho\vec{v} \\ \rho\vec{v}\vec{v} + \mathsf{I}(p + \vec{B}\cdot\vec{B}/2) - \vec{B}\vec{B} \\ \vec{v}\vec{B} - \vec{B}\vec{v} \\ (e + p + \vec{B}\cdot\vec{B}/2)\vec{v} - (\vec{v}\cdot\vec{B})\vec{B} \end{bmatrix} = \mathsf{D}, \tag{1}$$

supplemented with the divergence free condition $\nabla\cdot\vec{B} = 0$ as an initial condition. Here ρ and p are the plasma density and pressure, respectively, $\vec{v}$ is the plasma velocity, $\vec{B}$ is the magnetic field, and $e = p/(\gamma - 1) + \rho\vec{v}\cdot\vec{v}/2 + \vec{B}\cdot\vec{B}/2$ is the total energy density of the plasma. I is the unity matrix. The right-hand side D contains dissipative terms [1,5,9], which may include resistivity, viscosity, and heat conduction. The magnetic permeability $\mu = 1$ in our units. We take $\gamma = 5/3$ for the adiabatic index.

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The phase speeds of the three MHD waves, fast, Alfvén, and slow, in propagation direction x are denoted by c_{fx}, c_{Ax}, and c_{sx}, and the respective Mach numbers are defined by $M_{fx} = |v_x|/c_{fx}$, $M_{Ax} = |v_x|/c_{Ax}$, and $M_{sx} = |v_x|/c_{sx}$. Three types of shocks are described by the MHD equations, connecting plasma states which are labeled from 1 to 4, with state 1 superfast ($v_n > c_{fn}$ in the shockframe, with n the direction of the shock normal), state 2 subfast but super-Alfvénic, state 3 sub-Alfvénic but superslow, and state 4 subslow. The fast 1–2 shock refracts the magnetic field away from the shock normal. A limiting case of the fast shock is the $1-2=3$ switch-on shock, where $2=3$ means that $v_n = c_{An}$ downstream, and for which the upstream magnetic field is parallel to the shock normal, while the magnetic field makes a finite angle with the shock normal in the downstream state. The tangential component of the magnetic field is thus switched on. Intermediate shocks (1–3, 1–4, 2–3, and 2–4) bring a super-Alfvénic upstream plasma to a sub-Alfvénic downstream state, while the magnetic field is flipped over the shock normal as the tangential component of the magnetic field changes sign. The slow 3–4 shock refracts the magnetic field towards the shock normal.

Intermediate shocks in 3D MHD bow-shock flows.—In this paper we investigate 3D numerical simulation results of MHD bow-shock flows around a perfectly conducting sphere (Fig. 1) [14]. A uniform superfast plasma flow falls in on the sphere and a stationary bow shock is formed. This problem has three free parameters, for which we choose the upstream plasma $\beta = 2p/B^2$, the Alfvénic Mach number M_{Ax} along the upstream magnetic field lines (we choose the direction x along the upstream magnetic field), and the angle θ_{vB} between the upstream velocity field and magnetic field. We simulate the 3D bow-shock flows starting from a uniform initial condition and by advancing the time-dependent MHD equations until a steady state solution is reached. We solve the ideal MHD equations using a conservative finite volume shock capturing scheme which is second order accurate in space and time, employing a slope-limiter approach [14,15]. Numerical dissipation plays a role analogous to a small physical dissipation, with effective resistivity, viscosity, and heat conduction of roughly similar strength.

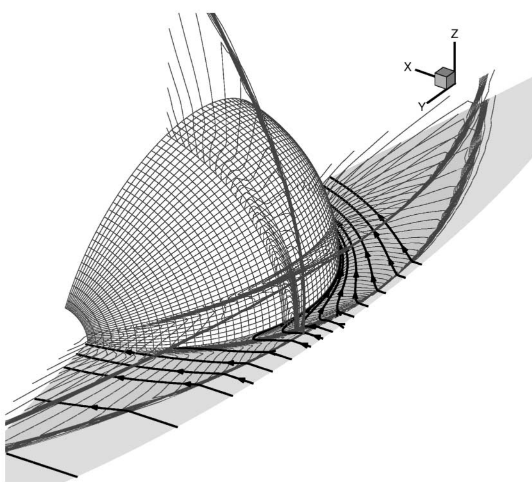

FIG. 1. Magnetically dominated 3D bow-shock flow around a sphere with inflow $M_{Ax} = 1.49$, $\beta = 0.4$, and $\theta_{vB} = 5°$. The flow comes in from the right. Density contours and magnetic field lines are shown in the xy plane, which is a plane of symmetry parallel to the upstream magnetic field and velocity vectors and going through the center of the sphere. Density contours are shown in two additional planes. In the upstream flow the magnetic field is aligned to the x axis ($40 \times 80 \times 40$ grid).

Figure 2a shows that for an upstream flow with a *weak* magnetic field (to be made more specific below) a bow-shock flow is obtained with a single shock front. This is the classical bow-shock topology which is well known from hydrodynamic bow shocks, and which until now was believed to arise for all fast MHD bow-shock flows as well. Figures 1 and 2b, however, show that for an upstream flow with a *strong* magnetic field the leading bow-shock front is followed by a secondary shock front. Extensive simulations [14] show that this previously unknown complex bow-shock topology arises for bow-shock flows when the uniform upstream plasma state satisfies the conditions that

$$B^2 > \gamma p \qquad (\text{equivalent to } \beta < 2/\gamma \text{ or } c_{Ax} > c_{\text{sound}}) \tag{2}$$

and

$$\rho v_x^2 > B^2 > \rho v_x^2 \frac{\gamma - 1}{\gamma(1-\beta)+1}$$
$$\left(\text{equivalent to } 1 < M_{Ax} < \sqrt{\frac{\gamma(1-\beta)+1}{\gamma - 1}}\right), \tag{3}$$

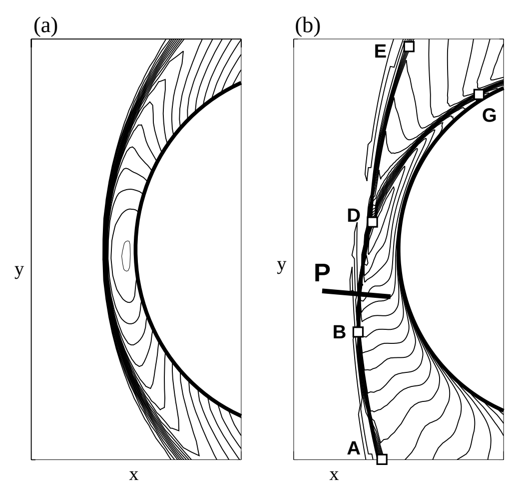

FIG. 2. Bow-shock flows over a sphere (thick solid). The flow comes in from the left. Density contours (thin solid) in the xy symmetry plane are shown. The incoming magnetic field is aligned with the x axis. (a) Pressure-dominated flow: $M_{Ax} = 3.985$, $\beta = 0.4$, $\theta_{vB} = 5°$. (b) Magnetically dominated flow: $M_{Ax} = 1.5$, $\beta = 0.4$, $\theta_{vB} = 3.8°$.

with x the direction along the magnetic field. These conditions are satisfied when the magnetic field is strong and thermal and dynamical pressure effects are dominated by magnetic effects. We call a state satisfying these conditions magnetically dominated, as opposed to pressure dominated. The magnetically dominated topology of Fig. 2b is found not only for flows around spheres but also around, e.g., paraboloid surfaces [14,16], not only for small values of θ_{vB} as in Figs. 1 and 2 but for any value of θ_{vB} [14,16], and using various grid resolutions and numerical schemes, i.e., various effective dissipations.

Figure 3 shows plots, along cut P perpendicular to the leading shock front in Fig. 2b, representing the Mach numbers in the direction of the shock normal. The shock along cut P is of 1–3 intermediate type (close to $1-3 = 4$, where $3 = 4$ means that $v_n = c_{sn}$ downstream). The 1–3 shock segment extends out of the xy plane (Fig. 1). Intermediate shocks thus exist and persist in this 3D MHD flow. In the following section we explain, in terms of the geometrical properties of MHD shocks [17], why the topology of Fig. 2b with intermediate shocks has to arise when the upstream flow is magnetically dominated.

Explanation in terms of the properties of MHD shocks.—We consider the flow topology in the xy plane (Fig. 1). We call a point on a shock front where the upstream magnetic field is perpendicular to the shock front a perpendicular point (e.g., point B in Fig. 4). Switch-on shocks and intermediate shocks exist only for certain parameter ranges of the upstream plasma [18]. The conditions (2) and (3) are precisely the conditions under which switch-on shocks exist. Given a fast 1–2 shock segment AB with upward deflection of the magnetic field (Fig. 4) in a magnetically dominated upstream flow—conditions (2) and (3) are satisfied—the shock at the perpendicular point B bordering the segment AB is necessarily a switch-on shock with upward deflection. In this case the topology of Fig. 4a with a single shock entirely of fast type is impossible, because the lower fast shock segment, which deflects the magnetic field downwards, cannot be linked continuously to the switch-on shock at point B [17].

Instead, the complex topology of Fig. 4b arises. Only a segment of 1–3 intermediate type (BD) can be linked continuously to the switch-on shock at point B. The curved 1–3 intermediate segment BD can have only a limited extent, because the MHD Rankine-Hugoniot relations (Fig. 5) show that, for increasing angle θ between the upstream magnetic field and the shock normal, the 1–3 shock first becomes a $1-3 = 4$ shock and then ceases to exist [17,18]. At this point the leading shock front splits up into two consecutive fronts. The leading segment DE is of the 1–2 fast type, and detailed analysis of the simulation results (see [14]) shows that the secondary segment DG is 2–4 intermediate close to point D, evolving into 3–4 slow along the front [16]. This complex topology with intermediate shock segments is obtained in the simulation of Fig. 2b. In contrast, for pressure-dominated flows, for which conditions (2) and (3) are not satisfied, the magnetic field is not refracted at a perpendicular point: the angle ϕ in Fig. 4a vanishes and the shock front can be entirely of fast shock type. This single-front topology is indeed obtained in the simulation of Fig. 2a.

Intermediate shocks in physical plasmas.—The simulation results and analysis discussed above show that an intermediate shock segment necessarily and naturally arises and persists for the realistic 3D configuration of uniform magnetically dominated MHD flow falling in on a sphere. During the time-dependent formation of the bow-shock solution in the simulation, the flow automatically imposes the coplanarity condition for the intermediate shocks along a segment of the leading shock front due to the nature of the 3D problem. We have performed simulations (not shown) which confirm that noncoplanar perturbation of the inflow, e.g., by rotating the inflow velocity around the x axis over a certain angle θ in Fig. 1, makes the intermediate shock segment split up into two nonintermediate shocks [4,10], but in this driven problem the intermediate shock segment *reforms* in a different location. This shows that intermediate shocks can be present in driven MHD flows, also when there are perturbations.

An important question is where this new MHD bow-shock topology with intermediate shocks could be observed in physical plasmas. Several requirements have to be fulfilled. First, conditions (2) and (3) have to be satisfied upstream from a bow shock. These conditions are, e.g., fulfilled in front of fast solar coronal mass ejections (CMEs) [14,17], and may occasionally occur in the solar

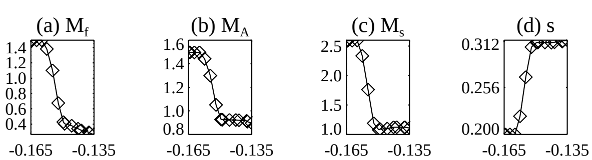

FIG. 3. Normal Mach numbers and entropy along cut P in Fig. 2b.

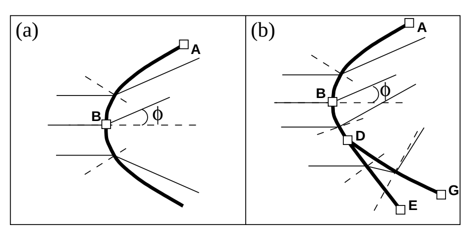

FIG. 4. Two proposed topologies for a shock front with an upper segment AB of 1–2 fast type for the case of a magnetically dominated upstream flow. Thick lines are shock fronts, thin lines are magnetic field lines, and shock normals are dashed. (a) The shock front cannot entirely be of the 1–2 fast type because the two shock segments cannot be linked continuously at point B. (b) A complex shock topology is necessary to channel the flow.

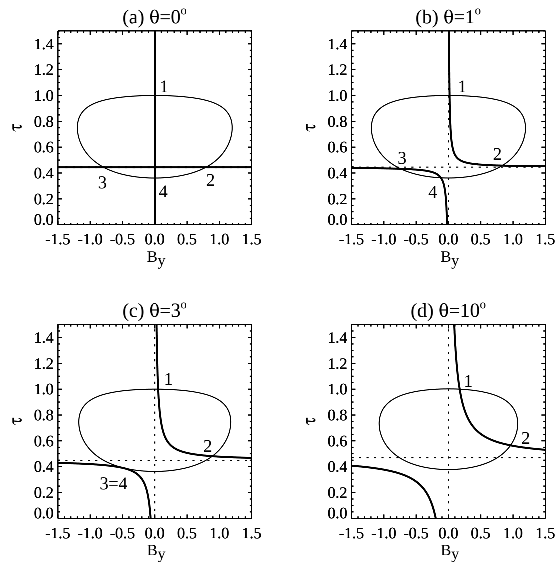

FIG. 5. Intersection points 1, 2, 3, and 4 indicate solutions of the MHD Rankine-Hugoniot relations for the upstream parameters of Fig. 2b (state 1) and for various values of the angle θ between the upstream magnetic field and the shock normal. τ is the inverse density, and B_y is the component of the magnetic field tangential to the shock front. The maximum angle θ for which 1–3 intermediate shocks can occur is approximately 3°.

wind upstream from the earth's bow shock, when the solar wind magnetic field is stronger than average, e.g., during magnetic cloud events [14,16]. Second, the dissipation mechanisms and magnitudes in the physical plasma under consideration have to match MHD dissipation for which intermediate shocks are stable. Third, the perturbations in the physical plasma have to be small and infrequent enough not to prevent the formation of intermediate shocks. Unfortunately, quantitative assessment of the two latter conditions seems prohibitively difficult at present due to the complex and not fully understood relationship between MHD intermediate shock stability and dissipation. Moreover, in the case of the earth's bow shock, kinetic effects and the collisionless nature of the plasma complicate the stability of shocks [19,20]. There is evidence from observations and simulations that intermediate shocks may form in collisionless plasmas [19,20].

The ultimate test for the applicability of our predictive theoretical result is confrontation with observations. Claims have been made of intermediate shock observations in interplanetary space [21] and in front of fast CMEs [14,17]. Our results seem especially relevant for the observation of intermediate shocks in Venus' bow shock as reported by Kivelson *et al.* [20]. Satellites to be launched in the near future (CLUSTER II and STEREO) may provide observations in our solar system of the new bow-shock topology with intermediate shock segments. This topology may also arise in some of the many other astrophysical flows in which MHD shock phenomena occur, including shocks induced by astrophysical jets [22].

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