Weak Localization in Antidot Arrays: Signature of Classical Chaos

Oleg Yevtushenko,[1,2] Gerd Lütjering,[3,*] Dieter Weiss,[1,3] and Klaus Richter[2]
[1]*Universität Regensburg, D-93040, Regensburg, Germany*
[2]*Max-Planck-Institut für Physik komplexer Systeme, D-01187, Dresden, Germany*
[3]*Max-Planck-Institut für Festkörperforschung, D-70569, Stuttgart, Germany*
(Received 6 April 1999)

We study, experimentally and theoretically, quantum weak localization (WL) corrections to the classical magnetoconductivity of two-dimensional ballistic systems with regular and disordered patterns of dense antidots. We analyze the observed temperature and flux dependences of the WL using different theoretical models for the chaotic dynamics and dephasing rates. The measured resistivity curves, which deviate from those of diffusive systems, reflect chaotic motion and correlations in the classical dynamics of electrons in the antidot landscape. The results support the significance of the Ehrenfest time as a relevant time scale for ballistic WL.

PACS numbers: 72.15.Rn, 05.45.Mt, 73.23.–b

Quantum interference effects in small phase-coherent conductors represent key features of mesoscopic behavior and have given rise to a variety of novel, experimentally observable effects. Among those, *weak localization* (WL)—a decrease in the average conductivity with respect to the classical one—is a prominent example. This quantum effect which was initially observed in disordered samples can be attributed to the constructive interference of diffusive time-reversed trajectories backscattered from impurities [1]. WL in ballistic conductors has been studied, both experimentally and theoretically, mainly for small phase-coherent cavities where the elastic mean free paths exceed the relevant device dimensions considerably [2]. Hence backscattering arises from specular reflections at the boundaries and WL should carry features of the underlying ballistic classical dynamics, namely, chaotic, integrable, or mixed.

Large antidot arrays built from a high-mobility two-dimensional electron gas (2DEG) probably represent the closest ballistic counterpart to impurity scattering in a disordered system. The regular arrangement of antidots on a lattice, which act as artificial potential pillars, has led to a variety of new effects ranging from commensurability peaks [4] in the classical magnetoresistance up to quantum oscillations at subkelvin temperatures [5].

Here we focus on ballistic WL (BWL) due to backscattering at the antidots [6] arranged both on a regular lattice and randomly. In this case the usual diffusive WL (DWL) theory for disordered systems is no longer valid: It is well suited to describe coherent backscattering from (pointlike) impurities where the scattering is regarded as a *quantum* process [1]. This provides a "quantum splitting" of classical trajectories at impurities allowing the formation of pairs of time-reversed backscattered paths. Moreover, an electron rapidly loses memory of its previous state; i.e., the motion can be regarded as a delta-correlated, diffusive process.

Antidots with a diameter a considerably larger than the Fermi wavelength λ_F act as *classical* scatterers. Therefore, WL in antidot structures calls for a generalization of WL theory beyond the diffusion approximation in order to account for correlations in the ballistic classical dynamics. In this context semiclassical methods [2,3,7] are attractive, since they provide a close link between the classical dynamics and quantum effects. However, approaches, which exploit the genuine semiclassical limit $\hbar \rightarrow 0$ by using stationary-phase arguments, turn out to be too crude to correctly account for BWL [8]. According to suggestions by Argaman [9] and Aleiner and Larkin [10], it is the exponential separation of initially close orbits in a chaotic system with classical scatterers like antidots which provides a mechanism for a minimal wave packet of size λ_F to split into two parts, which then follow time-reversed paths before they interfere constructively upon return. This approach goes beyond the standard semiclassical stationary-phase arguments introducing another relevant time scale for WL in a chaotic system: the Ehrenfest time [11] for the spreading of the wave packet over a distance of the size a of the antidots,

$$t_E = \lambda^{-1} \ln(a/\lambda_F); \tag{1}$$

λ is the mean Lyapunov exponent of the classical system.

In this Letter we compare experimental results for BWL in antidot structures with a corresponding theoretical analysis applying to both regular and irregular antidot patterns. The measurements show the role of the antidots for backscattering and, more importantly, signatures of chaotic dynamics and the Ehrenfest time.

The samples were prepared from high-mobility GaAs/AlGaAs heterostructures with an electron mobility $\mu \simeq 110\ \mathrm{m^2/V\,s}$ and a charge carrier density of $n_s \simeq 4.9 \times 10^{15}\ \mathrm{m^{-2}}$. This gives estimates for the Fermi velocity V_F, the transport time τ, and the related transport mean free path $l_{\mathrm{tr}} = V_F \tau$ of the unpatterned sample as $V_F = \hbar\sqrt{2\pi n_s}/m^* \simeq 2.95 \times 10^5$ m/s, $\tau = \mu \mathrm{m}^*/e \simeq 4.3 \times 10^{-11}$ s, $l_{\mathrm{tr}} \simeq 13\ \mu$m. The antidots were defined by electron beam lithography and reactive ion etching [12]. An electron micrograph of a

 0031-9007/00/84(3)/542(4)$15.00

disordered antidot distribution (DA) with a mean period of $d \simeq 300$ nm is shown in the upper inset in Fig. 1. The diameter of the antidots is $a \simeq 200$ nm, significantly larger than the Fermi wavelength $\lambda_F \simeq 36$ nm. Hence the antidots are classical scatterers.

The major experimental result to be explained theoretically is displayed in Fig. 1. The lower inset shows the negative quantum correction, $\Delta\sigma$, to the classical conductivity, σ_2, as a function of magnetic field B. It was extracted from the experimental data by subtraction from the conductivity at high temperatures where quantum effects are suppressed. As shown in Fig. 1 the temperature dependence of $\Delta\sigma$ is well fitted by an exponential law:

$$\Delta\sigma(T)|_{B=0} \sim \exp(-T/T_c) \tag{2}$$

for a wide range of 1.2 K $\leq T \leq$ 44 K with $T_c \simeq 14.5$ K [13]. We find the same exponential dependence with $T_c \simeq$ 12.5 K for a similar device with a *regular array* (RA) of antidots with period $d = 300$ nm. These measured exponential dependences can hardly be explained within the standard DWL theory which predicts in 2D [1,14]

$$\Delta\sigma = -\mathcal{G}\ln\left(1 + \frac{\tau_\phi}{\tau}\right) \overset{\tau_\phi \gg \tau}{\simeq} -\mathcal{G}\ln\left(\frac{\tau_\phi}{\tau}\right), \qquad \mathcal{G} \equiv e^2/\pi h. \tag{3}$$

According to Eq. (3), the temperature dependence, which is mainly determined by the properties of the phase relaxation time $\tau_\phi(T)$, is expected to be rather logarithmical than exponential. The observation of WL up to anomalously high temperatures $T \simeq 40$ K clearly suggests that reflection at the antidots with mean distance $d \ll l_{\mathrm{tr}}$ is the dominant mechanism for backscattering and momentum relaxation and calls for a theory of BWL.

In the following we analyze the anomalous T dependence based on the approach by Aleiner and Larkin mentioned above. It accounts for correlations in the chaotic ballistic dynamics in the "Lyapunov region" for time scales up to the Ehrenfest time t_E by replacing the diffusion operator through the regularized Liouville operator, the Frobenius-Perron operator [10]. For times larger than t_E the classical mechanics is assumed to be uncorrelated and is treated as diffusive again. The result for BWL then reads [10]

$$\Delta\sigma = -\mathcal{G}\exp\{-(t_E/\tau_\phi)[1 - \lambda_2/(\lambda^2\tau_\phi)]\}\ln(\tau_\phi/\tau). \tag{4}$$

Here, $\lambda_2 \sim \langle\delta\lambda(t_1)\delta\lambda(t_2)\rangle$ characterizes fluctuations in the Lyapunov exponent λ. Correlations in the chaotic dynamics are incorporated in the exponential prefactor, while the diffusive motion on longer time scales is reflected in the logarithm. Equation (4) holds for $a^2 \geq \lambda_F d$, which is fulfilled in the experiment.

To estimate λ for the antidot arrays we use results for 3- and 4-disk problems neglecting corrections to λ from scattering at distant antidots. We obtain [15]

$$\lambda \simeq (V_F/d)\ln(4.276d/a) \simeq 1.8\ \mathrm{ps}^{-1}. \tag{5}$$

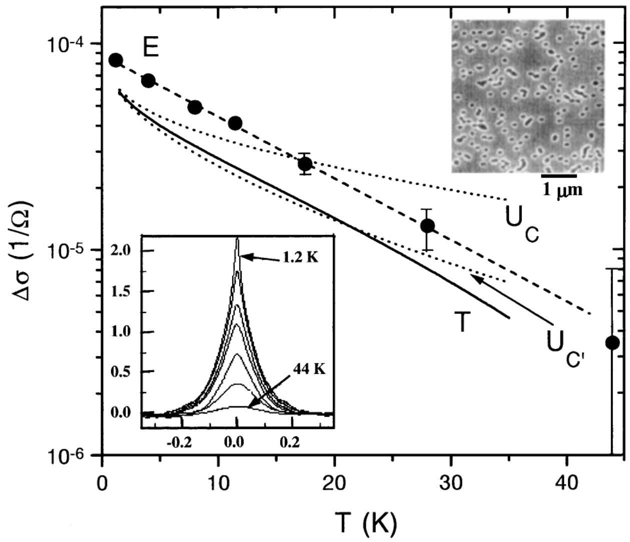

FIG. 1. Comparison of the experimental (filled dots and dashed line) and theoretical (solid line T and dotted lines $U_C, U_{C'}$) temperature dependence of the weak localization correction $\Delta\sigma$. The line T is the result of ballistic WL, Eq. (9), and the lower and upper dotted curves, U_C and $U_{C'}$, show results of the usual WL theory, Eq. (3), for diffusive systems. For $T > 15$ K the slope of the curve T is adjusted by choosing $C = 2.3$ ns K^2. The curves U_C and $U_{C'}$ correspond to the same C and to $C' = 0.13C$, respectively (see text). Upper inset: Electron micrograph of a disordered antidot array with average period of 300 nm. Lower inset: $\Delta\sigma$ in units of e^2/h as a function of magnetic field (in tesla) for temperatures $T = 1.2, 4, 8, 11.5, 17.5, 28$, and 44 K.

This estimate agrees with numerical simulations for RAs [16]. From Eq. (1) we find $t_E \simeq 0.94$ ps corresponding to a length scale for the Lyapunov region of $V_F t_E \simeq d$.

We continue with the discussion of the temperature dependence $\tau_\phi(T)$ in a 2DEG with antidots. Two main mechanisms are responsible for the decay of phase correlation, prevailing at different temperatures: *electron-electron* (e-e) interactions ($T < 10$ K) and *electron-phonon* (e-ph) interactions ($T > 10$ K), giving an overall dephasing rate $\tau_\phi^{-1} = \tau_{e\text{-}e}^{-1} + \tau_{e\text{-ph}}^{-1}$.

At lower temperatures $V_F\tau_\phi(T) \simeq V_F\tau_{e\text{-}e}(T) \gg d$ and the electronic motion can be considered as diffusive over the distance of the coherence length while estimating the role of e-e interactions. Following this hypothesis we apply the theory by Altshuler and Aronov on dephasing in 2D dirty systems [14]:

$$\tau_{e\text{-}e}^{-1} = (k_B T/\hbar g_2)\ln(g_2/2); \qquad g_2 = h/e^2\sigma_2. \tag{6}$$

The classical conductivity $\sigma_2 = e^2 n_s \tau_{\mathrm{ant}}/m^*$ is given by the Drude formula, and $\tau_{\mathrm{ant}} \leq d/V_F \simeq 1$ ps is the antidot scattering time. Considering the geometry of the antidot billiards, a more precise value of $\tau_{\mathrm{ant}} \simeq 0.3$ ps can be found for RAs. This estimate holds also true for DAs, at least for a slightly disordered arrangement, with the same antidot density. Correspondingly, one finds $\sigma_2^{\mathrm{RA}} \simeq \sigma_2^{\mathrm{DA}}$, in accordance with experiment when comparing σ_2 for the DA and RA samples. The Drude formula gives $g_2 \simeq 15$

and $\rho_2 = \sigma_2^{-1} \simeq 1.73$ kΩ for the parameters of the experiment. This is in reasonable agreement with the measured (classical) resistivity of 1.89 kΩ for the DA sample at $B = 0$ and high $T \simeq 44$ K where $\Delta\sigma \rightarrow 0$. Equation (6) then gives $\tau_{e\text{-}e} \simeq 57$ ps$/T$ (K).

From the 2D diffusion coefficient, $D_2 = V_F^2 \tau_{\text{ant}}/2 \simeq 0.013$ m^2/s, we obtain at low T, where $\tau_\phi \simeq \tau_{e\text{-}e}$, the mean square displacement during the time τ_ϕ: $\langle r \rangle = \sqrt{D_2 \tau_{e\text{-}e}(1\text{ K})} \simeq 2.5d \simeq 10 V_F \tau_{\text{ant}}$. Thus the electrons can be regarded as moving diffusively over the coherence length justifying our *a priori* assumption.

Equation (6) holds true and the theory of dephasing in dirty systems is applicable, if the inequality $\sqrt{(k_B T)/(\hbar D_2)} L_z \ll 1$ is satisfied [14]. For a thickness $L_z \simeq 50$ nm and the value for D_2 of the 2DEG we find that the condition is valid up to $T \simeq 25$ K. At $T > 25$ K one deals with the intermediate case between e-e interactions in a dirty system and the pure Coulombic interaction [17]. However, a detailed analysis of this intermediate regime is beyond the scope of the present paper and we restrict ourselves to the expression (6).

Finally we discuss the dephasing due to the e-ph interactions which gets important at high temperatures. Keeping the hypothesis of diffusive motion the T dependence of the e-ph contribution is similar to

$$\tau_{e\text{-ph}}(T) = C/T^2. \tag{7}$$

Equation (7) reflects the interaction of electrons in 2D with transverse 3D phonons in the presence of the vibrating walls of the antidots [18]. In general, we cannot *a priori* use any standard model to calculate the constant C. An estimate of C is particularly complicated for antidot structures: in the temperature range of interest the phonons have no sufficient free space between the antidots to propagate in the usual manner, since their wavelength is of the order of several d. In the high-T regime we thus treat C as *the only adjustable* parameter.

Consider now the exponent in Eq. (4) in view of the values for λ, Eq. (5), $\tau_{e\text{-}e}$, Eq. (6), and $\tau_{e\text{-ph}}$, Eq. (7). Given $\lambda \simeq \lambda_2$ [10], we have $\lambda^2 \tau_{e\text{-}e}/\lambda_2 \simeq \lambda \tau_{e\text{-}e} \gg 1$ for $T < 30$ K. Based on this inequality and including $\tau_{e\text{-ph}}$ [C in Eq. (7) will be specified below] we can assume

$$\lambda \tau_\phi \gg 1\,, \tag{8}$$

up to high temperature [Eq. (8) is verified later]. Equation (8) shows that the effect of fluctuations of λ is negligible in the context of the experiment. We hence can omit them in Eq. (4) using the simplified expression for BWL,

$$\Delta\sigma \simeq -\mathcal{G} \exp(-t_E/\tau_\phi) \ln(\tau_\phi/\tau_{\text{ant}})\,, \tag{9}$$

where we further have replaced τ by τ_{ant}.

We now compare the measured dependence $\Delta\sigma(T)$ with our calculation based on Eq. (9) accounting for the estimates of λ, t_E, and $\tau_\phi(T)$. By variation of C we adjust the slope of the theoretical line to the experimental one at 15 K $\le T \le$ 35 K (see Fig. 1). The adjustment yields $C \simeq 2.3$ ns K^2. Up to $T \simeq 15$ K, the temperature dependence is predominantly given by e-e interaction [see case (i) in the upper inset in Fig. 2]. In this T regime the slope of the theoretical curve (full line in Fig. 1), which shows considerable agreement with experiment, is rather insensitive to $\tau_{e\text{-ph}}$ and, consequently, to the constant C. Summarizing, we have found a reasonably good agreement of the experimental and theoretical slopes over nearly the entire wide temperature range. Simultaneously, our presumption (8) is verified [see case (ii) in the upper inset in Fig. 2].

In contrast, the usual theory for DWL, Eq. (3), which corresponds to $t_E = 0$, cannot adequately describe the experiments. There exists no choice for C in Eq. (7) for $\tau_{e\text{-ph}}$ which would lead to a reasonable agreement with the experimental data over the whole temperature range. The lower dotted curve $U_{C'}$ in Fig. 1 shows the result for $C' = 0.13C$ which comes closest to the experimental slope. However, the C' chosen then leads to unrealistically small values of the dephasing time. For $T = 20$ K one has $\tau_\phi \simeq 2\tau_{\text{ant}}$ and for $T > 20$ K follows $\tau_\phi < 2\tau_{\text{ant}}$, resulting in destruction of the coherence for the shortest trajectories upon return and, in contrast to the experiment, excluding any possibility to observe WL at such high temperatures. Also the application of the alternative law $\tau_{e\text{-ph}} = \tilde{C}/T^3$ cannot at all restore the exponential dependence of $\Delta\sigma(T)$ [19].

Hence the observed exponential temperature behavior shows signatures of BWL. It offers moreover the principal possibility to extract t_E and thereby the *classical* Lyapunov exponent of the electronic antidot billiard from the *quantum* WL correction. The difference in the absolute

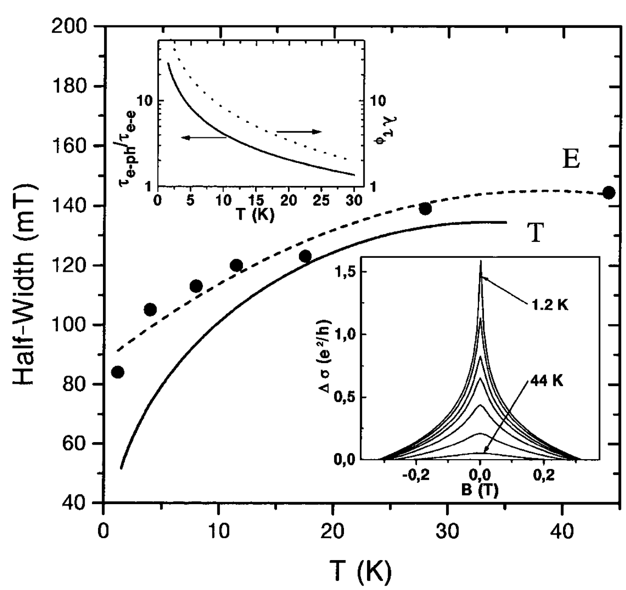

FIG. 2. Experimental (filled dots and dashed line E) and theoretical (solid line T) temperature dependences of the half-width of the ballistic-WL magnetoconductivity peak. The lower inset depicts its magnetic field dependence calculated from Eq. (9) at the same temperatures as in the lower inset in Fig. 1. Upper inset: Ratios of (i) the electron-phonon [Eq. (7), $C = 2.3$ ns K^2] to the electron-electron [Eq. (6)] dephasing rates (left axis, solid line), and (ii) the total dephasing rate to the inverse of the mean Lyapunov exponent (right axis, dotted line).

theoretical and experimental peak heights in Fig. 1 is about 30%. This gives a hint that the ratio $\tau_\phi(1\ \mathrm{K})/\tau_{\mathrm{ant}}$ is underestimated which we consider as an acceptable error of the rough estimates. However, the theoretical models used yield values for τ_ϕ and coherence length in agreement with other experimental data on GaAs/AlGaAs samples [20].

Besides the analysis of WL at zero field we also studied experimentally and theoretically the B-field dependence. Our theoretical results are based on the expression [10]

$$\Delta\sigma \simeq -\mathcal{G}\exp(-t_E/\tau_\phi)[\ln(\lambda_B^2/4D_2\tau_{\mathrm{ant}}) - \Psi(1/2 + \lambda_B^2/4D_2\tau_\phi)], \quad (10)$$

in the same framework as above. In Eq. (10), $\lambda_B = \sqrt{c\hbar/eB}$ denotes the magnetic length and Ψ is the digamma function. The theoretically obtained WL profiles are depicted in the lower inset in Fig. 2. They agree with the experimental profiles in the lower inset in Fig. 1. The T dependence of the half-width of $\Delta\sigma(B)$ is shown as the full curve in Fig. 2 in comparison with experimental results (marked as dots) for the DA sample. Besides the behavior at low temperature the theoretical half-width is in quantitative agreement with the experiment.

Finally we discuss the experimental finding that the temperature dependence of the WL is rather independent of the arrangement (ordered or disordered) of the antidots. We emphasize that we can explain these similar results of RA and DA structures in the framework of the same theory. Both systems are similar insofar as their antidot density is the same, as they exhibit ergodic dynamics and show self-averaging behavior due to the effects of dynamical chaos and residual impurities. Therefore it is natural to assume that the mean Lyapunov exponent, which enters into the expression for $\Delta\sigma(T)$, is not sensitive to the precise formation of the antidots [16]. We believe that the antidot arrangement (DA or RA) affects the fluctuations λ_2 in λ. However, as shown above, these fluctuations can be neglected in the WL expression (4) under the experimental conditions. Therefore Eq. (9), which is based on the mean λ only, serves as a common footing to explain the same dependence $\Delta\sigma(T)$ in RA and DA structures.

To summarize, we have studied the weak localization correction to the classical conductivity of ballistic arrays of large and dense antidots. We find a manifestation of classical correlations in the chaotic electronic motion of the experimentally observed weak localization for regular as well as disordered patterns of antidots. The experiments presented indicate a way to evaluate the Ehrenfest time and the Lyapunov exponent of the antidot arrays from the dependence $\Delta\sigma(\tau_\phi(T))$. Thus our results can also be of broader interest in the context of chaotic billiards and other systems exhibiting chaotic transport.

This work was supported by the Deutsche Forschungsgemeinschaft (GRK 176 and IN 36/8-1) and by the BMBF (Contract No. 01BM622/5). We acknowledge the support of the experimental work by K. von Klitzing. O. Y. is grateful to A. Sergeev, U. Rössler, and M. Suhrke for useful discussions and acknowledges the hospitality of the *Institut für Theoretische Physik,* University Regensburg.

*Present address: Boston Consulting, D-80331, München, Germany.

[1] G. Bergmann, Phys. Rep. **107**, 1 (1984); S. Chakravarty and A. Schmid, *ibid.* **140**, 194 (1986).

[2] For reviews, see, e.g., contributions to the special issue: Chaos **3**(4) (1993), or Ref. [3].

[3] H. U. Baranger, in *Nano-Science and Technology,* edited by G. Timp (Springer, Berlin, 1997).

[4] For reviews, see D. Weiss, in Festkörperprobleme **31**, 341 (1991); R. Schuster and K. Ensslin, *ibid.* **34**, 195 (1994).

[5] D. Weiss *et al.*, Phys. Rev. Lett. **70**, 4118 (1993); F. Nihey and K. Nakamura, Physica (Amsterdam) **184B**, 398 (1993); R. Schuster *et al.*, Phys. Rev. B **47**, 6843 (1993).

[6] For previous work on WL in finite and macroscopic antidot lattices, see G. Lütjering *et al.*, Surf. Sci. **361/362**, 709 (1996); T. Ihn *et al.*, Mo-P90: in *Proceedings of ICPS24, Jerusalem, 1998* (World Scientific, Singapore, 1998), respectively.

[7] K. Richter, *Semiclassical Theory of Mesoscopic Quantum Systems* (Springer, Heidelberg, 2000).

[8] This current issue reflects nonunitarity of the semiclassical evolution. BWL line shapes in cavities could be interpreted *qualitatively* in terms of coherent backscattering [3]. However, a semiclassical evaluation of the *full* resistance leads to a *vanishing* WL correction [3,7].

[9] N. Argaman, Phys. Rev. Lett. **75**, 2750 (1995); Phys. Rev. B **53**, 7035 (1996).

[10] I. L. Aleiner and A. I. Larkin, Phys. Rev. B **54**, 14 423 (1996); Chaos Solitons Fractals **8**, 1179 (1997).

[11] B. V. Chirikov, F. M. Izrailev, and D. L. Shepelyanskii, Sov. Sci. Rev. C **2**, 209 (1981).

[12] D. Weiss *et al.*, Appl. Phys. Lett. **58**, 2960 (1991).

[13] In our discussion we do not rely on the experimental data above 30 K (where $\Delta\sigma \sim 0.05\sigma_2$), as the errors increase significantly. The error bars are based on the assumption that the classical background to be subtracted is known within $\pm 15\ \Omega$.

[14] B. L. Altshuler and A. G. Aronov, in *Electron-electron Interactions in Disordered Systems*, edited by A. L. Efros and M. Pollak (North-Holland, Amsterdam, 1985).

[15] P. Gaspard, *Chaos, Scattering and Statistical Mechanics* (Cambridge University Press, Cambridge, England, 1998).

[16] K. Richter, R. Hennig, M. Suhrke, and O. Yevtushenko, Physica E (Amsterdam) (to be published).

[17] M. Reizer and J. W. Wilkins, Phys. Rev. B **55**, R7363 (1997).

[18] See experimental data and theoretical discussion in N. G. Ptitsina *et al.*, Phys. Rev. B **56**, 10 089 (1997).

[19] O. Yevtushenko, K. Richter, and D. Weiss, Ann. Phys. (Leipzig) **8**, SI-297 (1999).

[20] Some experimental data can be found in J. A. Katine *et al.*, Superlattices Microstruct. **16**, 211 (1994); C.-T. Liang, *et al.*, Phys. Rev. B **49**, 8518 (1994).