Measurements of Flux-Dependent Screening in Aharonov-Bohm Rings

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(Received 4 October 1999)

In order to investigate the effect of electronic phase coherence on screening we have measured the flux-dependent polarizability of isolated mesoscopic rings at 350 MHz. At low temperatures (below 100 mK) both the nondissipative and the dissipative parts of the polarizability exhibit flux oscillations with a period of one-half a flux quantum in a ring. The sign and amplitude of the effect are in good agreement with recent theoretical predictions. The observed positive magnetopolarizability corresponds to an enhancement of screening when time reversal symmetry is broken. The effect of electronic density and temperature are also measured.

PACS numbers: 73.23.–b, 72.15.Rn

When an electric field **E** is applied to an isolated metallic sample, electron screening gives rise to an induced dipole **d**. In the linear response regime:

$$\mathbf{d} = \alpha \mathbf{E}, \tag{1}$$

where α is the electric polarizability. For a sample of typical size a much larger than the Thomas-Fermi screening length λ_s, α is essentially determined by geometry, with a negative correction of the order of λ_s/a [1]. The measurement of α gives information on the way electrons screen an external electric field. At the mesoscopic scale, when phase coherence through the sample is achieved (i.e., the phase coherence length is of the same order as the typical size of the system) electronic properties are sensitive to the phase of the electronic wave functions, which can be tuned by an Aharonov-Bohm flux in a ring geometry [2]. It has been recently suggested that screening of an electric field may be sensitive to this phase coherence, leading to a flux-dependent mesoscopic correction to polarizability [3–5]. In particular, the polarizability of an Aharonov-Bohm ring is expected to exhibit oscillations as a function of the magnetic flux.

We have inferred the electrical response of an array of rings from the flux dependence of the capacitance C, placed underneath the rings, of an rf superconducting microresonator (Fig. 1, left inset). This experiment has been checked to be sensitive only to the electrical response of the rings [6]. C is modified by the nondissipative response α' of the rings [7]:

$$\frac{\delta C}{C} = kN_s\alpha', \tag{2}$$

where N_s is the number of rings coupled to the resonator and k the electric coupling coefficient between a ring and the capacitance, which depends only on geometry. Since the resonance frequency $f = 1/(2\pi\sqrt{LC})$, with L the inductance of the resonator, the change in C shifts the resonance frequency. In addition the dissipative response α'' of the rings weakens the quality factor Q of the resonator:

$$\frac{\delta f}{f} = -\frac{1}{2}kN_s\alpha'(\omega), \qquad \delta\left(\frac{1}{Q}\right) = kN_s\alpha''(\omega). \tag{3}$$

In order to measure δf and δQ, the rf frequency is modulated at 100 kHz. Using lock-in detection, the reflected signal from the resonator at the modulated rf frequency is measured and used to lock the experimental setup on the resonance frequency. The feedback signal is then proportional to the variation of f, whereas the signal at double the modulation frequency is proportional to the variation of Q. We are careful to inject sufficiently low power ($\approx$10 pW) so as not to heat the sample. The signal is measured as a function of the magnetic field. To improve accuracy, the derivative of the signal is also detected by modulating at 30 Hz a magnetic field of 1 G amplitude. Our precision with this setup is $\delta f/f = 10^{-8}$ and $\delta Q/Q = 10^{-8}$.

The rings are etched in a high-mobility GaAs-AlGaAs heterojunction. Etching strongly decreases the

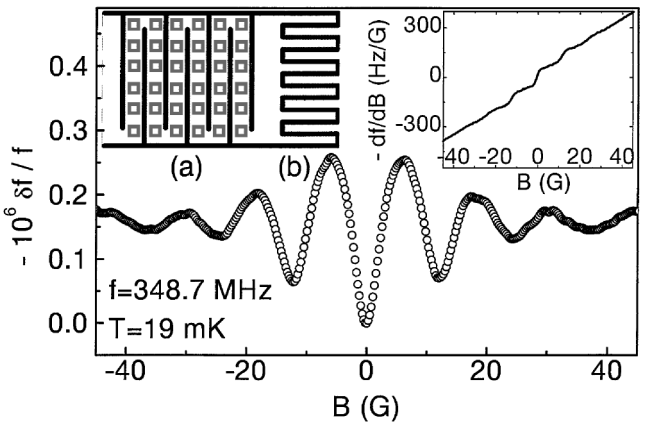

FIG. 1. Typical relative variation of the resonance frequency (proportional to $\delta\alpha'$) versus magnetic field (illumination 3). $\delta f/f$ is periodic with a period of 12.5 G corresponding to one-half a flux quantum in a ring. Left inset: schematic picture of the resonator and the rings on the capacitance of the resonator. The capacitance [comblike structure (a)] is well separated from the inductance [meander line (b)]. Right inset: derivative of the resonance frequency versus magnetic field before substraction of base line.

conductivity of the rings, but the nominal conductivity is recovered by illuminating the sample with an infrared diode [8]. We have checked this on a connected sample. The characteristics of the rings, deduced from transport measurements on wires of the same width etched in the same heterojunction, are the following: at nominal electronic density ($n_e = 3 \times 10^{11}$ cm^{-2}) the mean free path $l_e = 3$ μm, the etched width is 0.5 μm, whereas the effective width $W = 0.2$ μm (estimated from weak localization experiments [9]) is much smaller due to depletion, the coherence length is $L_\Phi = 6.5$ μm, and the effective perimeter $L = 5.2$ μm. The rings are thus ballistic in the transverse direction and diffusive longitudinally. The mean level spacing $\Delta = h^2/(2\pi mWL) \simeq 80$ mK $\simeq 1.66$ GHz and the Thouless energy $E_c = hD/L^2 = 450$ mK, with D the diffusion coefficient and m the effective mass of electrons. The Thomas-Fermi screening length is $\lambda_s = \pi a^\star/2 = 16$ nm, with $a^\star$ the effective Bohr radius. There are $N = 10^5$ rings on one sample. The resonator is made by optical lithography in niobium on a sapphire substrate. The length of the capacitance is $l = 20.5$ cm and the inductance is 5 cm long. It has a single resonance frequency $f_0 = 385$ MHz and a quality factor of 10 000. The distance between the capacitance and the inductance is 300 μm. A 0.9 μm thick mylar film is inserted between the detector and the ring substrate in order to reduce inhomogeneities of the magnetic field due to the Meissner effect in the vicinity of the superconducting resonator. The quality factor of the system constituted by the resonator and the rings is reduced to 3000, probably due to dielectric losses in the GaAs substrate, and the resonance frequency is 350 MHz. The system is cooled in a dilution refrigerator down to 18 mK.

The typical field dependence of the rings contribution to $\delta f/f$ is shown in Fig. 1. This signal is superimposed on the diamagnetic response of the niobium resonator (Fig. 1, right inset) which we subtract in the following way: the base line of the derivative of the resonance frequency is removed and the signal is then integrated. One thus obtains the curves of Figs. 1 and 2, which are directly proportional to the flux dependence of the polarizability of the rings. The resonance frequency is periodic in field with a period of 12.5 G which corresponds to one-half a flux quantum $\Phi_0/2 = h/2e$ in a ring, consistent with an Aharonov-Bohm effect averaged over many rings [10]. The resonance frequency decreases by about 100 Hz between 0 and 6 G. The magnetic field reduces the amplitude of the oscillations, which cannot be detected for a field higher than 40 G, consistent with the finite width of the ring. Illuminating the sample with the electroluminescent diode strongly affects the measured signal. At low illumination the amplitude of the oscillations increases and decreases at higher illuminations (see Fig. 3). The extremum amplitude of $\delta_\Phi f/f = [f(\Phi = \Phi_0/4) - f(\Phi = 0)]/f$, is -2.8×10^{-7}. In our geometry the coupling coefficient reads [7]

$$k = \frac{1}{\pi \epsilon_0 \epsilon a l d} \frac{\ln(\frac{d+a}{d-a})}{\ln(\frac{d}{r})} \tag{4}$$

with the relative dielectric constant of GaAs $\epsilon = 12.85$, the size of the rings $a = 1.3$ μm, the length of the capacitance $l = 20.5$ cm, the distance between one lead of the capacitance and a ring $d/2 \simeq 3.15$ μm, and the width of the lead $r = 1$ μm. Since all the rings are not identically coupled to the resonator, k has to be understood as an average capacitive coupling between one ring and the capacitance. Note that approximatively one-half the rings are well coupled to the capacitance. Because of these approximations the experimental value is given within a 50% error range. One obtains $\delta_\Phi \alpha'/\alpha_{1D} = 0.7 \times 10^{-3}$, where $\alpha_{1D} = \epsilon_0 \pi^2 R^3/\ln(R/W)$ is the polarizability of a quasi-one-dimensional (quasi-1D) circular ring of radius R.

The dissipative part of the polarizability is obtained from the field dependence of Q at different illuminations

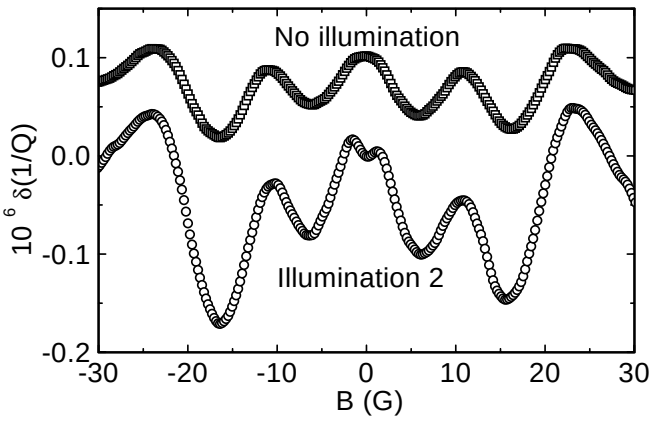

FIG. 2. Variation of $1/Q$ (proportional to $\delta\alpha''$) versus B at two different illuminations. The period of the oscillations is 12.5 G. At high illumination a dip appears in the zero field region. The curve at illumination 0 is shifted for clarity.

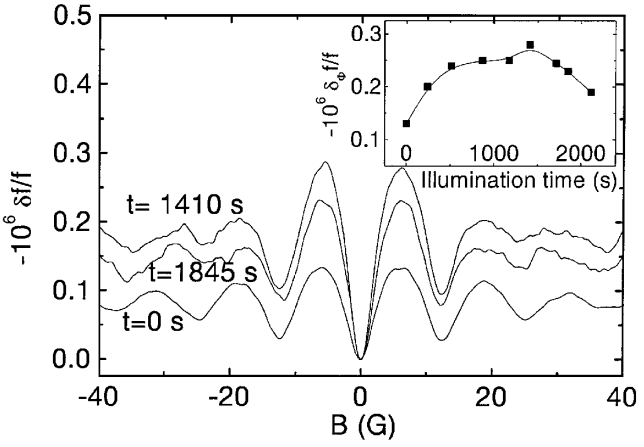

FIG. 3. $\delta f/f$ versus B at different illuminations. Inset: Relative variation of resonance frequency versus illumination time (continuous line is only a guide to the eye).

(Fig. 2). It exhibits a periodic behavior with the same period as the resonance frequency. At low illumination $1/Q$ decreases with magnetic field for small field whereas at higher illumination a dip in the zero field region, which is not understood, appears indicating an increase of $1/Q$ with field. The typical amplitude of $\delta_\Phi(1/Q)$ is -10^{-7}, hence $\delta_\Phi\alpha''/\alpha_{1D} = -1.3 \times 10^{-4}$.

The sensitivity of the electrostatic properties of mesoscopic systems to quantum coherence has been emphasized by Büttiker for connected geometries [11]. The phase coherent correction to the polarizability of isolated systems was recently theoretically investigated. Efetov found that it is possible to relate self-consistently this correction to the flux dependence of the screened potential [3]. Noat *et al.* calculated this effect in the diffusive regime [4] and found no flux correction in the canonical ensemble. In the grand canonical (GC) ensemble no effect is predicted if the rf pulsation ω is much smaller than the inverse relaxation time γ. However when $\omega \gg \gamma$ the correction to the polarizability is positive. In particular, for quasi-1D rings, one has [12]

$$\frac{\delta_\Phi\alpha'}{\alpha_{1D}} = \frac{\epsilon}{16\pi^2 \ln(R/W)} \frac{\Delta}{E_c} \frac{\lambda_s}{W}. \quad (5)$$

Using supersymmetry techniques Blanter and Mirlin essentially confirmed these results [5]. Since the rings in our experiment are completely isolated, the results concerning the canonical ensemble, in which the effect is predicted to be zero or very small, should apply. However our experiment shows unambiguously a decrease of the resonance frequency as we increase the magnetic field at low field: this corresponds to a *positive* magnetopolarizability. The GC result [Eq. (5)] leads to $\delta_\Phi\alpha'/\alpha_{1D} = 0.8 \times 10^{-3}$, which is close to the experimental value. Therefore the value and the sign of the effect are consistent with rings considered in the GC case in the limit $\gamma \ll \omega$. This discrepancy can be related to an ensemble averaging intermediate between canonical and GC, a situation called GC-canonical by Kamenev and Gefen [13]: if the system is brought to a GC equilibrium at a certain value of the magnetic flux and then submitted to a time dependent field whose time scale is faster than the particle equilibration time the response of the system can be identical to the GC case in the limit $\gamma \ll \omega$. Another possibility is that the mathematical cancellation responsible for the absence of magnetopolarizability in the canonical ensemble disappears when one does not have the condition $\omega \ll \Delta$.

We find that the polarizability is greater in a magnetic field than in zero field. This can be related to the flux dependence of screening within a simple model in which the Thomas-Fermi screening length is flux dependent. As the correction to the polarizability due to screening is negative and of the order of λ_s/a, an increase of polarizability corresponds to a decrease of the screening length. Hence the charges are more concentrated on the edges of the sample in the presence of a magnetic flux. This phenomenon has to be related to the disappearance of weak localization and thus enhancement of the metallic character of the sample in the presence of a magnetic field.

Concerning the quality factor our results are, at least for the small electronic density, in agreement with what is predicted for the dissipative part of the magnetopolarizability [14]. For low magnetic field we observe a negative $\delta_\Phi(1/Q)$ which corresponds to a negative $\delta_\Phi\alpha''$ according to (2). The measured ratio $\delta_\Phi\alpha''/\delta_\Phi\alpha' = -0.29$ for the illumination 0. Note that this ratio does not depend on the electric coupling coefficient and hence can be determined with a good accuracy. In the GC ensemble, $\delta_\Phi\alpha''/\delta_\Phi\alpha'$ is related to the level spacing distributions function [15,14] which obeys universal rules of random matrix theory [16,17]:

$$\frac{\delta_\Phi\alpha''}{\delta_\Phi\alpha'} = \frac{2\pi\omega}{\Delta}\left[R^{\mathrm{GUE}}\left(\frac{\pi\omega}{\Delta}\right) - R^{\mathrm{GOE}}\left(\frac{\pi\omega}{\Delta}\right)\right]. \quad (6)$$

R^{GUE} is the two level correlation function in the Gaussian unitary ensemble and R^{GOE} in the Gaussian orthogonal ensemble. This formula is valid in the limit $\gamma \ll \omega$ and $\gamma \ll \Delta$ and yields $\delta_\Phi\alpha''/\delta_\Phi\alpha' = -0.26$ which is close to the experimental value. This is a good indication that we are effectively in the regime $\gamma \ll \omega$ and $\gamma \ll \Delta$.

The $\Phi_0/2$ component of the Fourier transform of $\delta f/f$ decays exponentially with temperature with a typical scale of 90 mK, independent of illumination (Fig 4). Taking the temperature dependence of this first harmonic to be $\exp(-2L/L_\Phi)$, like in weak localization [18], and supposing that all the temperature dependence comes from L_Φ, one has $L_\Phi \propto 1/T$ without saturation. We deduced $\gamma = 1/\tau_\Phi = D/L_\Phi^2 \simeq 0.8$ mK at 18 mK, consistent with $\gamma \ll \Delta$. γ increases like T^2, in agreement with theoretical predictions on the broadening of single electron energy levels due to electron-electron interaction in a quantum dot [19], and remains below Δ up to $T = 180$ mK. At 50 mK, $L_\Phi \simeq 18$ μm which is larger

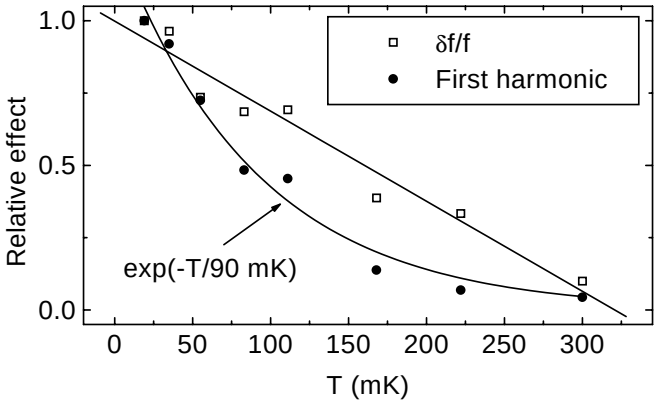

FIG. 4. Temperature dependence of $\delta_\Phi f/f$ and the $\Phi_0/2$ component of the Fourier transform of $\delta f/f$ for illumination 6. $\delta_\Phi f/f$ decreases linearly with temperature, whereas the first harmonic can be fitted by an exponential decay with a temperature scale of 90 mK, independent of illumination.

than the coherence length deduced from weak-localization measurements on connected samples ($L_\Phi \simeq 6.5\ \mu$m at the same temperature). In this latter case one has a 1D geometry and a broadening of the energy levels due to the coupling with reservoirs whereas in an isolated ring the energy spectrum is discrete.

Illumination increases the electronic density in the rings. We start from a situation where the rings are empty or in an electronic localized state. The increase of the signal observed at low illumination corresponds to the repopulation of these depleted rings. The subsequent decay of the signal can be attributed to the increase of the average conductance of the rings and the $1/g$ dependence of the magnetopolarizability in the diffusive regime (cf. formula 4; g is the dimensionless conductance, defined as $g = E_c/\Delta$).

It is instructive to compare these results with previous measurements [14,20] on a similar array of rings coupled to a multimode strip line resonator sensitive to both electric and magnetic responses. Similar amplitudes of the flux dependence of the resonance frequency are found in both experiments indicating that the electric part of the response of the rings is at least of the same order of magnitude as the magnetic one. We plan to measure this latter quantity by coupling the rings to the inductive part of the resonator.

To conclude we have measured the flux-dependent part of the ac polarizability of mesoscopic rings down to 18 mK. Both the nondissipative and the dissipative parts of the polarizability exhibit a small correction periodic in flux with a period of one-half a flux quantum in a ring. The correction of the nondissipative part is positive in low magnetic field in agreement with theoretical predictions in the GC ensemble in the limit $\gamma \ll \omega$. It indicates a better screening of the electric field in the presence of magnetic flux. The correction to the dissipative part is negative for low field, at least for low electronic density. The effect on the polarizability is qualitatively consistent with a $1/g$ dependence over the dimensionless conductance g. These corrections are sensitive to temperature, with a typical scale of 90 mK. It would be interesting to pursue these studies in the low frequency regime.

We thank B. Etienne for the fabrication of the heterojunction and acknowledge fruitful discussions with M. Nardone, L. P. Lévy, L. Malih, A. Mac Farlane, S. Guéron, and L. Limot.

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[8] For each illumination a current of 10 μA is run through the diode during roughly 4 min. Then measurement is done at least 1 h after the illumination in order to ensure good stability of the sample. Curves presented here correspond to illumination 0 (no illumination) to illumination 8 (2000 s of light). An upper value of the estimated illumination power coupled to the sample is 600 photons/s with a wavelength of 766 nm.

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