Parametric Instability of Dust Lattice Waves in a Turbulent Plasma Sheath

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It is shown that dust lattice waves (DLWs) become parametrically unstable in a turbulent dusty plasma sheath that contains random-phase ion plasma waves. Physically, the parametric instability arises because of the nonlinear coupling between ion plasma waves and DLWs. The parametric instability is identified as the possible reason for the observed "sublimation" transition of the plasma crystal from a solid to a gaslike state.

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About five years ago, Melandsø [1] theoretically predicted the existence of the dust lattice wave (DLW) in a strongly coupled dusty plasma. In the DLW, the restoring force comes from the Debye-Hückel (or Yukawa) interaction between the nearest charged dust neighbors, while the dust mass provides the inertia. The dust lattice waves have been spectacularly observed [2–6] in low-temperature laboratory dusty plasma discharges. It has been reported that the DLWs can be either spontaneously excited [2–4] in the dusty plasma sheath or they may be triggered by low-power laser beams [5,6]. Furthermore, Morfill *et al.* [7] have theoretically shown that stochastic dust charge fluctuations can cause instability of the DLWs. On the other hand, Schollmeyer *et al.* [8] have experimentally observed the excitation of monolayer plasma crystals due to a periodic modulation of the potential well in the dusty plasma sheath.

In this Letter, we present a theory for the excitation of the DLWs in a turbulent dusty plasma sheath which contains random-phase ion plasma waves (IPWs). It is found that the nonlinear interaction between the latter and the DLWs produces modulated IPWs whose ponderomotive force reinforces the DLWs. This selfconsistent nonlinear interaction between the ion plasma and dust lattice waves produces instabilities, which are responsible for melting and sublimation [3,9–13] of dusty plasma crystals in strongly coupled Coulomb systems.

Let us consider a dusty plasma sheath in the presence of a dc electric field $\mathbf{E}_0 = E_0\hat{\mathbf{z}}$, where E_0 is the strength of the electric field and $\hat{\mathbf{z}}$ is the unit vector in the z (vertical) direction. The sheath electric force $Q_0\mathbf{E}_0$ levitates the dust grains in a dusty plasma because it overcomes the gravity force $m_d\mathbf{g}$; the latter pulls charged dust grains vertically downward. Thus, at equilibrium, we have $Q_0\mathbf{E}_0 = m_d\mathbf{g}$, where $Q_0(m_d)$ is the unperturbed dust charge (the dust mass) and $\mathbf{g} = g\hat{\mathbf{z}}$ is the gravity. The constituents of our dusty plasma are singly charged positive ions and extremely massive micron-sized charged dust particulates. Accordingly, at equilibrium, we also have $n_{i0} \approx Z_{d0}n_{d0}$, where $n_{i0}(n_{d0})$ is the unperturbed ion (the dust) number density and Z_{d0} is the number of charges residing on the surface of negatively charged dust grains. We suppose that almost all of the plasma electrons in the sheath are attached onto the dust grain surface during the dust charging, and, accordingly, there is a complete electron number density depletion. Furthermore, we assume that the dust sizes are much smaller than the ion Debye radius $\lambda_{Di} = (T_i/4\pi n_{i0}e^2)^{1/2}$, where T_i is the ion temperature and e is the magnitude of the electron charge, and that there are sufficient numbers of dust particles (which are shielded by the ions) in the ion Debye sphere so that the collective interaction, as discussed here, remains intact.

In the dusty plasma sheath, there is an influx of supersonic ion flow from the ambient plasma. The streaming ions generate ion plasma wave turbulence due to the two-stream instability. The frequency (much larger than the dust plasma frequency) of the ion plasma oscillations is $\omega_{\mathbf{k}} = (\omega_{pi}^2 + 3k^2v_{ti}^2)^{1/2}$, where $\omega_{pi} = (4\pi n_i e^2/m_i)^{1/2}$ is the ion plasma frequency, $v_{ti} = (T_i/m_i)^{1/2}$ is the ion thermal velocity, n_i is the ion number density, and m_i is the ion mass. The question now arises as to how the ion wave turbulence affects the dynamics of ultralow frequency (ULF) (in comparison with ω_{pi}) dust lattice waves, which are spontaneously excited in the dusty plasma sheath.

The presence of ultralow frequency density ripples produces a force $\mathbf{F}_D = -\nabla_{\mathbf{l}}\omega_{\mathbf{k}}$, which acts on ion quasiparticles (or ion plasmons). As a result, a change appears in the number density ($N_{\mathbf{k}} = \sum_{\mathbf{k}} |\mathbf{E}_{\mathbf{k}}|^2/\omega_{\mathbf{k}}$) of ion plasmons. The dynamics of the perturbed ion plasmon number density $\delta N_{\mathbf{k}}$ ($= N_{\mathbf{k}} - N_{\mathbf{k}0}$, where $N_{\mathbf{k}0} \approx |\mathbf{E}_{\mathbf{k}0}|^2/4\pi\omega_{p0}$ is the unperturbed ion plasmon number density) in the presence of ULF density ripples is governed by a wave kinetic equation [14,15]

$$\frac{\partial \delta N_{\mathbf{k}}}{\partial t} + \mathbf{v}_g \cdot \nabla_{\mathbf{l}}\delta N_{\mathbf{k}} - \frac{\omega_{p0}}{2n_{d0}}\nabla_{\mathbf{l}} n_{d1} \cdot \frac{\partial N_{\mathbf{k}0}}{\partial \mathbf{k}} = 0, \quad (1)$$

where $\mathbf{v}_g \approx (3v_{ti}^2/\omega_{p0})\mathbf{k}$ is the group velocity of the ion plasmons and $\omega_{p0} = (4\pi n_{i0}e^2/m_i)^{1/2}$ is the unperturbed ion plasma frequency. In (1), we have replaced the ion number density perturbation n_{i1} by $Z_{d0}n_{d1}$ and have used $n_{i0} = Z_{d0}n_{d0}$. Here, n_{d1} is the density perturbation associated with the DLWs. The dust continuity equation yields $n_{d1} = -n_{d0}\nabla_{\mathbf{l}}\mathbf{r}_n$, where $\mathbf{r}_n$ is the displacement of the $n - th$ dust particulate in the linear chain. The dynamics of the dust particulates is governed by

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$$\frac{\partial^2 \mathbf{r}_n}{\partial t^2} + \nu \frac{\partial \mathbf{r}_n}{\partial t} = \frac{q_{d0}}{m_d} \mathbf{E}_s + \frac{F_c}{m_d} (\mathbf{r}_{n-1} - 2\mathbf{r}_n + \mathbf{r}_{n+1}), \tag{2}$$

where ν is the dust-neutral collision frequency [$\approx 2\sqrt{\pi}\, n_g a^2 c_g$ with the Epstein drag law; n_g is the neutral gas density, a (much smaller than the mean free path for the gas molecules) is the dust radius, and c_g is the thermal velocity of the gas molecules] and

$$F_c = \frac{Q_0^2}{\Delta^3} (2 + 2\kappa + \kappa^2) \exp(-\kappa) \tag{3}$$

represents the force associated with the screened Debye-Hückel (Yukawa) interaction potential between the nearest dust neighbors. Here, $\kappa = \Delta/\lambda_{Di}$ is the ratio between the interparticle distance and the ion Debye radius, and $\mathbf{E}_s$ is the sheath electric field perturbation caused by the ponderomotive force of the ion plasma oscillations. The force balance gives

$$e\mathbf{E}_s = \frac{1}{2} m_i \nabla_{\mathbf{l}} \sum_{\mathbf{k}} \langle |\mathbf{v}_{ik}|^2 \rangle, \tag{4}$$

where $\mathbf{v}_{ik} = i(e/m_i \omega_{\mathbf{k}})\mathbf{E}_{\mathbf{k}}$ is the ion quiver velocity in the electric field of the ion plasma oscillations. In (4), the angular bracket denotes the ensemble average over the period $2\pi/\omega_{p0}$, and we have neglected the ion pressure gradient force because here we are supposing that $\sum_{\mathbf{k}} \langle |\mathbf{E}_{\mathbf{k}}|^2 \rangle / 8\pi n_{i0} T_i \gg n_{d1}/n_{d0}$.

From (2) to (4) we readily obtain

$$\frac{\partial^2 n}{\partial t^2} + \nu \frac{\partial n}{\partial t} + \omega_{DL}^2 n \approx \frac{Z_{d0}\omega_{p0}}{2n_{i0} m_d} \nabla_{\mathbf{l}}^2 \sum_{\mathbf{k}} \langle |\delta \mathbf{E}_{\mathbf{k}}|^2 \rangle, \tag{5}$$

where $n = n_{d1}/n_{d0}$ is the relative dust number density perturbation, $\omega_{DL}^2 = (\omega_{pd}^2/f)(2 + 2\kappa + \kappa^2)\exp(-\kappa)\sin^2(q\Delta/2)$ is the squared dust lattice frequency, $\omega_{pd} = (4\pi n_{d0} Q_0^2/m_d)^{1/2}$ is the dust plasma frequency, $f = \pi n_{d0} \Delta^3$ represents the dust fugacity parameter, q is the wave number of the DLWs, and $\delta \mathbf{E}_{\mathbf{k}}$ is the electric field of the modulated ion plasma waves.

We now assume that n_{d1} and $\delta \mathbf{E}_{\mathbf{k}}$ are proportional to $\exp(i\mathbf{q} \cdot \mathbf{l} - i\Omega t)$, where $\mathbf{q}$ and Ω are the wave vector and the frequency of the DLWs, respectively. Accordingly, from (1) and (5) we obtain

$$\Omega^2 + i\nu\Omega - \omega_{DL}^2 = -\frac{q^2 Z_{d0}\omega_{p0}}{4n_{i0} m_d} \frac{V}{(2\pi)^3} \times \int \frac{d\mathbf{k}\mathbf{q} \cdot (\partial I_{\mathbf{k}0}/\partial \mathbf{k})}{\Omega - 3\omega_{p0}\lambda_{Di}^2 \mathbf{q} \cdot \mathbf{k}}, \tag{6}$$

where V is the volume of the dusty plasma and $I_{\mathbf{k}0} = |\mathbf{E}_{\mathbf{k}0}|^2$. Equation (6) is the dispersion relation of the DLWs in the presence of IPWs in a dusty plasma sheath.

We analyze (6) for the one-dimensional problem so that we replace $V/(2\pi)^3$ by $L/2\pi$, and take

$$I_{\mathbf{k}0} = \frac{I_0}{\sqrt{2\pi}\, k_w L} \exp[-(K - k_0)^2/2k_w^2], \tag{7}$$

where $I_0 = \langle |\mathbf{E}_{\mathbf{k}0}|^2 \rangle$ is the maximum IPW intensity corresponding to a mean ion plasmon mean wave number k_0, L is the length of the system, k_w represents the width of the random-phase IPW spectrum, and K is the IPW wave number along the direction of the DL wave number q. By using (7) into (6), we obtain

$$\Omega^2 + i\nu\Omega - \omega_{DL}^2 = -\omega_{p0}^2 \frac{q^2}{k_w^2} \alpha S_0 W(\eta), \tag{8}$$

where $\alpha = Z_{d0} m_i/m_d$, $S_0 = I_0/24\pi n_{i0} T_i$, $W(\eta) = (2\pi)^{-1/2} \int d\xi\, \xi(\xi - \eta)^{-1} \exp(-\xi^2/2)$ is Ichimaru's W function [16] with the argument $\eta = (k_q - k_0)/k_w$, where $k_q = \Omega\, \omega_{p0}/3q v_{ti}^2$.

Equation (8) admits novel instabilities. Specific results follow in two limiting cases. First, for $\eta \ll 1$, we have $W(\eta) \approx 1 + i(\pi/2)^{1/2}\eta$. By replacing $W(\eta)$ in (8) by the latter, letting $\Omega = \Omega_r + i\Omega_i$, we obtain for $\Omega_r \approx \omega_{DL} \gg \nu$, the growth rate $\Omega_i < \Omega_r$,

$$\Omega_i \approx \sqrt{\frac{\pi}{8}} \frac{q^2}{k_w^2} \frac{\omega_{p0}^2}{\omega_{DL}} \frac{k_0}{k_w} \left(1 - \frac{\omega_{DL}}{q v_0}\right) \alpha S_0, \tag{9}$$

where $v_0 = 3 v_{ti} k_0 \lambda_{Di}$. Equation (9) predicts a kinetic instability when the Cherenkov condition $v_0 > \omega_{DL}/q$ is satisfied. Second, for $\eta \gg 1$, we have $W(\eta) \approx -1/\eta^2$, so that (8) takes the form

$$(\Omega^2 + i\nu\Omega - \omega_{DL}^2)(\Omega - \Omega_0)^2 = 9q^4 v_{ti}^4 \alpha S_0, \tag{10}$$

where $\Omega_0 = 3qk_0\lambda_{Di}^2 \omega_{p0}$. Equation (10) is a fourth order polynomial in Ω, and it can be numerically analyzed. However, for $\Omega \gg \nu, \omega_{DL}$, Eq. (10) reduces to

$$\Omega(\Omega - \Omega_0) = \pm 3q^2 v_{ti}^2 \alpha^{1/2} S_0^{1/2}. \tag{11}$$

Equation (11) predicts hydrodynamic instabilities for $S_0^{1/2} > 3k_0^2 \lambda_{Di}^2/\alpha^{1/2}$. The latter inequality is somewhat stringent. On the other hand, for $|\Omega(\Omega + i\nu)| \ll \omega_{DL}^2$, we obtain from (10)

$$\Omega - \Omega_0 = \pm i3(q^2 v_{ti}^2/\omega_{DL})\alpha^{1/2} S_0^{1/2}, \tag{12}$$

which exhibits an oscillatory instability whose growth rate is $\gamma = 3(q^2 v_{ti}^2/\omega_{DL})\alpha^{1/2} S_0^{1/2}$. The growth time ($\gamma^{-1}$) for some typical laboratory dusty plasma parameters [3] (viz., $n_{i0} \sim 10^8$ cm^{-3}, $T_i \sim 0.1$ eV, $Z_0 \sim 10^4$, $\Delta \sim \lambda_{Di} \sim 200\ \mu$m, $\omega_{DL} \sim 10$ s^{-1}, $\omega_{p0} = 2 \times 10^6$ s^{-1}, $q\lambda_{Di} \sim 0.1$, and $S^{1/2} = 10^{-5}$) turns out to be a fraction of a second.

In summary, we have considered the nonlinear propagation of DLWs in a turbulent dusty plasma bath which contains a broad-band spectrum of random-phase ion plasma waves. It is found that free energy of the turbulent ion wave spectrum is parametrically coupled to the DLWs due to kinetic and oscillatory instabilities. As a result, the lattice chain becomes unstable. The "sublimation" of the chain and subsequent melting of dusty plasma crystals are then associated with the instability of the DLWs in the dusty plasma sheath, as discussed here.

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