Ripples in Tapped or Blown Powder

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We observe ripples forming on the surface of a granular powder in a container submitted from below to a series of brief and distinct shocks. After a few taps, the pattern turns out to be stable against any further shock of the same amplitude. We find that the wavelength of the pattern is proportional to the amplitude of the shocks. Starting from considerations involving air flow through the porous granulate and avalanche properties, we build up a semiquantitative model which satisfactorily fits the set of experimental observations of either tapped or blown powder.

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In recent years, there has been a great deal of interest in the response of granular materials to various types of external perturbations. Up to now, the vast majority of the experimental, theoretical, and simulated works have dealt with model granular solids in the sense that the particles were supposed to be large enough (i.e., typically larger than 100 μm) to avoid significant interaction with the surrounding fluids. In reverse, and rather paradoxically, the understanding of the behavior of fine powders has received much less attention although it is universally recognized as the keystone of an increasing number of high-tech industrial processes. In this spirit, a few recent attempts were made towards the analysis of the behavior of fine cohesive or noncohesive powders (typically in the range from 1 μm to less than 30 μm) (e.g., Refs. [1,2]).

Among others, our present knowledge about instability of layers of granular solids under vertical vibrations is currently firmly established. As a matter of fact, the original paper of Melo *et al.* [3] has stimulated a series of subsequent papers (e.g., Ref. [4]) dealing with several facets of the same problem. In short, all of these works converged towards a description of the observed dynamic behaviors in terms of Faraday instability in liquids combined with the specific dynamics of the inelastic bouncing ball.

This paper reports the observation and analysis of a novel form of surface instability of a thin layer (thickness typically 10 mm) of a fine powder (particle size 10 μm) submitted to a series of vertical shocks from below the container. First, it is observed that, after a few taps, the surface displays a regularly corrugated pattern made of a succession of jointed heaps sitting at the natural avalanche angle. The crucial point here is that any further tap does not induce any significant change in the pattern which thus is a steady state with respect to further vertical shocks. Second, and this is a clue to the understanding of the process, the characteristic wavelength of the pattern is found to be proportional to the amplitude of the taps.

Our analytical model considers the powder-air interaction. It involves two basic features of the fine granular material: firstly, Darcy's law for modeling the air flux through the porous cake of granulate and, secondly, the maximum stability angle of a granulate before generating avalanches. In brief, it is shown that a corrugated surface stands as a more stable state than a horizontal flat surface with respect to air blow from below, because it is easier to eject a particle from a flat surface than from an inclined surface sitting at the avalanche angle. Additionally, the apices of the hills created by air blow are seen to be unstable when compared to both sides of the hills. A couple of further experiments provide a clear view of this last feature.

We start from a simple tabletop experiment using a cylindrical transparent tube made of Lucite or glass. The dimensions of the tube are unimportant. In a typical experiment, this tube can be about 20 cm long and 2.5 cm in diameter. We fill half of the tube with fine powder (e.g., glass beads, diameter 10 μm). We keep the tube horizontal and rigidly fixed at both ends, with the powder initially set flat and horizontal thus giving a granulate thickness of about 12.5 mm in the center. We take a heavy metallic or plastic rod and knock gently and repeatedly at a low pace and at a constant intensity onto the center of the tube from below, applying vertically as brief taps as possible, i.e., letting the rod rebound appreciably after each separate shock. After a few taps (about ten to twenty), the surface, initially flat, smooth, and horizontal, turns out to exhibit ripples similar to those reported in Fig. 1a. Now, tapping *more energetically*, but still keeping the intensity as constant as possible from one tap to the next, induces a pattern where the mean distance between two successive ripples increases significantly (Fig. 1b). Furthermore, under energetic tapping, a careful observation of the surface shows that, at every tap, a limited number of particles may be ejected upwards starting both from the apices of the hills and from the small plateaus which happen occasionally between imperfectly jointed hills. Note that similar results can be obtained using rather large and hollow, i.e., light, particles. Excessive wetness prevents the observation of these surface patterns.

More reliable information has been obtained using a more sophisticated device. We set a CCD (charge-coupled device) camera above the tube in order to record and process the successive patterns obtained during the experiments. We also used a magnetically driven tapping device. The acceleration due to the taps has been measured using

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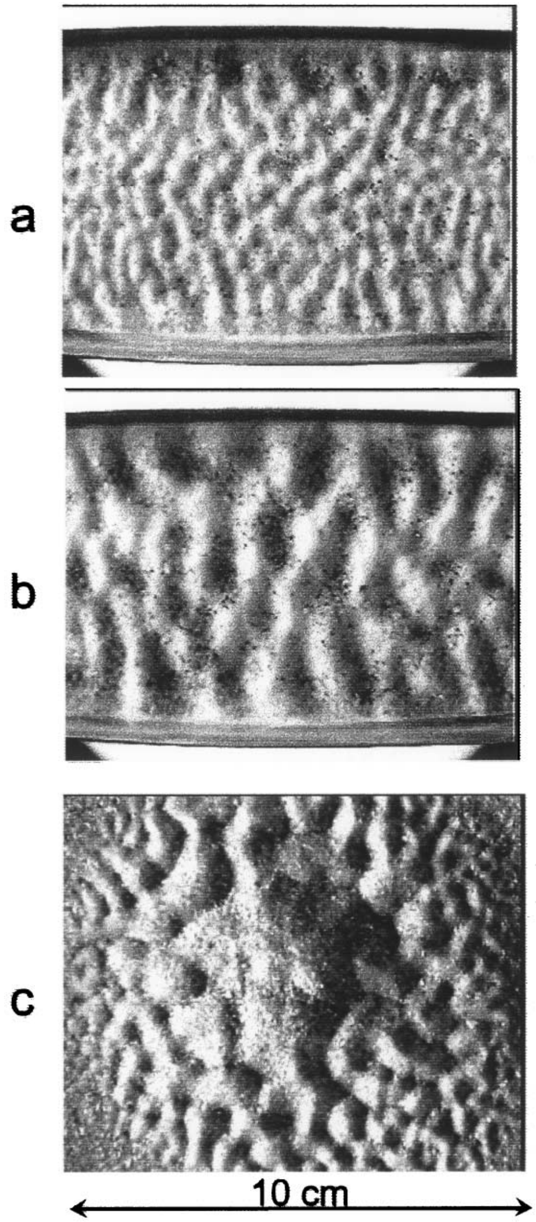

FIG. 1. Three bird's-eye views of the corrugated surface observed after twenty shocks of constant amplitudes onto the underpart of the containers. Snapshots (a) and (b) correspond to the cylindrical horizontal container and are obtained at two different shock amplitudes, larger in (b) than in (a). Measurements are performed in the median part of the pattern. Snapshot (c) is obtained in a rectangular metallic box (size 20×40 cm^2) containing a layer of fine sand beach, tapped under the central part. In this latter case, the pattern reproduces the transient deformation of the underlying metallic sheet.

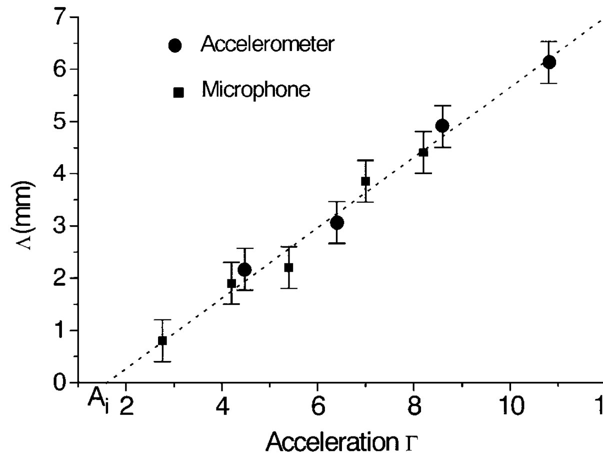

FIG. 2. Characteristic wavelength of the pattern in millimeter versus tap acceleration $(/g)$ measured as the signal delivered by two different detectors. The dashed line shows a linear fit of the experimental results.

a commercial Bruer and Kjaer accelerometer or a microphone stuck on the tube and tared with respect to the accelerometer results. As shown in Fig. 2 both detectors provide a clear linear dependence of the wavelength of the pattern as a function of the tap acceleration.

Considering the peculiar geometry of the pattern made of jointed heaps sitting approximately at avalanche angle, we note that the characteristic wavelength Λ is determined by the contrast of the pattern. We call h_T the altitude (starting from the bottom of the container) of the apices of the corrugated surface, h_B the altitude of the valleys, and h_i the altitude of the initially flat and horizontal surface of the granular layer. We have $h_T + h_B = 2h_i$. θ is the avalanche angle of the powder, which is about 30° in our glass bead powder. The wavelength Λ is given by $\Lambda = 2(h_T - h_B)\cot\theta$.

We start from the first part of an assertion initially put forth by Faraday [5] who stated that in the course of vertical vibration of a granular powder, "It forms a partial vacuum into which the air round the heap enters with more readiness than the heap itself,...," and we consider here that, during the fall following the upward launching of the sand by the taps, the trapped air is forced upwards through the porous material. Then, supposing that the air flux through the granulate obeys a Poiseuille flow, we use Darcy's law. It gives the velocity v of a fluid flux emerging from the porous medium whose permeability is K and where the air is pushed by a uniform pressure difference ΔP acting over a thickness h of the granulate as $v_h = K\Delta P/h$.

In the course of our experiments ΔP is expected to be proportional to the tap acceleration applied on the underpart of the tube A because the granulate undergoes a ballistic flight. Thus, the velocity of the air emerging from the surface at altitude h can be written as $v_h = \alpha A/h$, where α is the coefficient of proportionality given by Darcy's law which involves the permeability of the granular material. This can be considered as a crude first order approximation since the particle density is not homogeneous during the rising and the falling. Actually the dynamics of the process is complex but it is seen that the effect resulting from the upcoming air flux through the granular cake rather occurs at the end of the fall, i.e., when the granulate is already being compacted.

A single spherical bead (diameter D) is lifted up by the air flow if the air velocity is larger than a minimum velocity v_f (f standing for free fall velocity) given by $v_f = \frac{D^2}{18\eta}\rho g$, where η is the viscosity of the gas (air), ρ is the volumic density of the particles, and g is the gravitational acceleration. Thus a particle deposited on the surface at altitude h can be expelled from a horizontal surface of the layer if the outgoing air velocity is $v_h \geq v_f$. We find that, for glass particles of diameter 10 μm, this threshold air velocity is about 1 cm/s. Thus the threshold quantity $\alpha A_i = h_i v_f$ required for external taps able to blow up particles from the surface of the granulate is about 1 cm^2/s.

In view of our experimental observations, we easily realize that the air slowing-down process through the granular layer is unable to account alone for the above-described results (Fig. 2). If we imagine that the incoming air pulse is unable to eject particles when reaching the apices of the hills, we have $\alpha A_T \leq h_T v_f$. By calculating the ratio of the required initial velocity to induce hills of height h_T to the required initial velocity for the onset of the corrugation, we get $A_T/A_i = h_T/h_i$. Our experiments show that the ratio h_T/h_i is only marginally larger than 1 while the observed amplitude ratio is about 8. We conclude that another process should be taken into account to explain the observed features.

We can get an estimated value of the product αA in the course of our experiments. We take profit from our observation that, in the bottom of the valleys when a small flat surface survives or on the top of the hills when the steady state pattern is obtained, a small number of particles are expelled at every tap (See Fig. 3). Near the maximum tap intensity, the upward jump of these particles is on the order of a 0.2 mm. There, the velocity of the emerging particles is given by $\sqrt{2h_m g} \simeq 6$ cm/s which can be considered as approximately measuring the velocity of the emerging air flux at altitude h_m (about 1.2 cm). Thus, we find $\alpha A \simeq 7.2$ cm^2/s which, in view of Fig. 2, stands as a correct order of magnitude. Note again that, due to the fact that h_T/h_B is only slightly larger than 1, the outgoing air flux is practically the same at any point on the corrugated surface. It ensues that on both sides of the hills the velocity of the air flux propagating upwards is not sufficiently damped when reaching the inclined surface to ensure the stability of the pattern. There, another process must come into play. We put forward the following considerations.

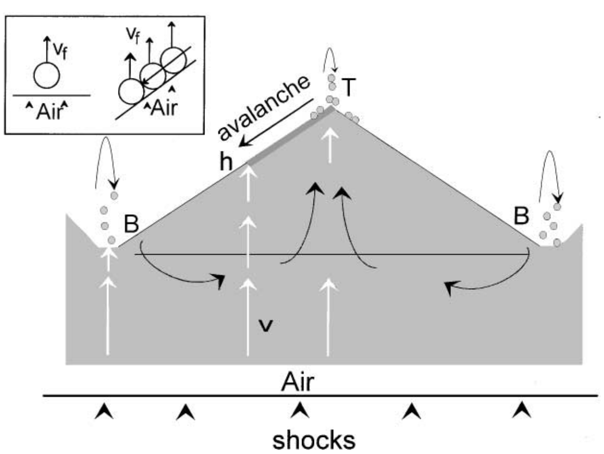

FIG. 3. Sketch of the ripple buildup showing the screening effect of the inclined sides of the hills with respect to air blown from below. The white arrows correspond to air trajectory while the black ones show the movement of the particles. Inset: the balance of forces on a horizontal and on an inclined surface.

On both sides of the hills, the ascending air flux meets an inclined sheet of particles which is on the verge of avalanching. Under these circumstances, one particle subjected to the vertical incoming air flux bears a fraction of the weight of the above-lying particles involved in the avalanche layer (see inset of Fig. 3). This additional mass opposes the blowing up of the particles near the surface and thus stabilizes the inclined lateral surfaces against the incoming air flow. We can build up a simplified equation for this screening effect considering that the mass of the concerned particle is increased by a factor $Np\sin\theta$, N being the number of the above-lying particles pertaining to a single sheet of the inclined granulate and p being the unknown number of sheets possibly involved in the avalanche process. Considering that all of the particles sitting above the particle at altitude h participate in the screening effect, we have $Np \simeq (h_T - h)p/D$. Thus the required air velocity v_{ah} needed to eject the considered particle sitting at altitude h is given by $v_{ah} = v_f(h_T - h)p\sin\theta/D$.

As expected, this screening effect determines an altitude h_C under which all of the particles sitting on the inclined lateral surface cannot be expelled by the air flux. This altitude h_C is given by the following equation:

$$\alpha\,\frac{A - A_i}{h_i v_f}\,\frac{1}{\frac{h_T - h_C}{D}\,p\sin\theta} \simeq 1\,.$$

In other words, the upper part of the hill (when $h_C \leq h \leq h_T$) is unstable, whereas the lowest part (when $h_B \leq h < h_C$) is stable against vertical air blow from below.

Now, in view of several preceding observations of convective processes in vibrational sand-heaping experiments, we note that the steady state of the pattern should result from the balance between the small number of expelled particles near the apices and the number of particles which are reinjected into the bulk of the hills at every tap (see black arrows in Fig. 3). We conjecture that this sort of trapping-detrapping process should be independent of the size of the hills. Thus, we write $h_T - h_C = C(h_T - h_B)$, where C is the proportion of the unstable part of the hills. It is an adimensional constant independent of the height of the hills and of the amplitude of the shocks. With this extra assumption, we get the characteristic wavelength Λ of the pattern which is proportional to the amplitude of the shocks, in agreement with the measurements

FIG. 4. The top photograph shows ripples obtained by using fast transient air pulses. The bottom photograph obtained with a continuous air flow exhibits small craters (black points) sitting at the center of the myriad of small fixed volcanoes.

reported in Fig. 2,

$$\Lambda \simeq 2 \frac{\alpha}{C} \frac{A - A_i}{h_i v_f} \frac{D}{p \sin\theta} \cot\theta \, .$$

A numerical estimate using our experimental results is illustrative. We find that $C = 25\%$ of the hills are unstable if only one single layer of powder is involved in the process. Now, if five layers of the superficial sheet participate in the process as has been repeatedly observed in avalanche experiments, we see that only 5% of the upper part of the hills is unstable against the incoming air flux. This percentage is in fair agreement with our direct observation of the pattern (see Fig. 3).

The delicate question of the stability of the particles sitting near the apices of the pattern has motivated a further experiment which we performed, firstly, in order to prove directly the validity of our model based on air-powder interaction and, secondly, to provide visual insight into the question of the stability of the apices of the air-built pattern.

We use a millipore filter, commonly used in chemistry for filtering, in a reverse manner. A plastic filter (pores, 3 μm) is placed at the bottom of a commercial cylindrical glass vessel which allows direct observation or image processing with a CCD camera. In contact with the horizontal filter and above it, we lay a thin layer of powder (about 8 mm thick). Instead of sucking up through the filter as is usually done, we blow from below, using either brief air pulses or a gentle and continuous air flux. The photographs of the resulting surface corrugation are reported in Fig. 4. The upper snapshot shows a surface corrugation made up of triangular-shaped ripples quite similar to that previously reported in our tapping experiments (Fig. 1). We again observe qualitatively that the stronger the air pulses, the larger the characteristic wavelength of the pattern.

The lowest snapshot corresponding to a gentle and continuous air flow going through the powder cake is quite informative. The surface corrugation then exhibits a different aspect because the system has no time to relax between separate successive perturbations as in the preceding experiments. This experiment shows up a myriad of stable small volcanoes organized around small craters (seen as black spots in the snapshot) which spew out powder particles. This can be seen as consistent support to our preceding picture which involved the relative weakness of the apices of the patterns against air blow from below.

Furthermore, we noted that applying successively a series of taps at different and constant amplitudes gives rise to complex patterns which depend on the order of the series. Thus we were able to reproduce wrinkled volcano patterns or herringbone-shaped structures which strikingly remind one of their large scale counterparts in mountainous landscapes. We postpone the description of these findings to a forthcoming paper.

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