Dark-State Polaritons in Electromagnetically Induced Transparency

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We identify form-stable coupled excitations of light and matter ("dark-state polaritons") associated with the propagation of quantum fields in electromagnetically induced transparency. The properties of dark-state polaritons such as the group velocity are determined by the mixing angle between light and matter components and can be controlled by an external coherent field as the pulse propagates. In particular, light pulses can be decelerated and "trapped" in which case their shape and quantum state are mapped onto metastable collective states of matter. Possible applications of this reversible coherent-control technique are discussed.

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Dark states and electromagnetically induced transparency (EIT) [1,2] can be used to make a resonant, opaque medium transparent by means of quantum interference. Associated with the induced transparency is a dramatic modification of the refractive properties. This can result, for instance, in very slow group velocities [3]. In the present contribution we study the propagation of quantum fields in EIT media. We demonstrate the existence of form-stable quantum excitations associated with such propagation, which we term "dark-state polaritons." The polaritons are mixtures of photonic and Raman-like matter branches. We show that their group velocity is directly related to the ratio of the two contributions and can be externally manipulated by adiabatically changing a coherent control field. In particular, dark-state polaritons can be stopped and reaccelerated in such a way that their shape and quantum state are preserved. In this process the quantum state of light is ideally transferred to collective atomic excitations and vice versa.

The possibility to coherently control the propagation of quantum light pulses via dark-state polaritons opens up interesting applications involving the generation of nonclassical states of atomic ensembles (in squeezed or entangled states), reversible quantum memories for light waves [4–6], and high resolution spectroscopy [7]. Furthermore, the combination of the present technique with studies on few-photon nonlinear optics [8–12] can be used, in principle, for processing of quantum information stored in collective excitations of matter. It may also provide an interesting tool to study quantum scattering phenomena in systems involving coherent cold collisions. In this regard this work opens a link between nonlinear optics for light waves and nonlinear atom optics. For example, an interaction (or entanglement) between light waves can be induced by a collisional interaction of atoms (s-wave scattering) or vice versa. Finally it may be interesting to combine the adiabatic mapping technique with methods to excite solitons or vortex states of the atomic center-of-mass motion in double Bose-Einstein condensates [13]. In this way the quantum properties of these excitations could be manipulated and analyzed.

We consider a medium consisting of Λ-type 3-level atoms with two metastable lower states, as shown in Fig. 1. A quantum field described by the slowly varying dimensionless operator

$$\hat{E}(z,t) = \sum_k a_k(t) e^{ikz} e^{-i(\nu/c)(z-ct)} \tag{1}$$

couples resonantly the transition between the ground state $|b\rangle$ and the excited state $|a\rangle$. $\nu = \omega_{ab}$ is the carrier frequency of the optical field. The upper level $|a\rangle$ is furthermore coupled to the stable state $|c\rangle$ via a coherent control field with the slowly varying, real Rabi frequency $\Omega(t)$. For the purposes of the present discussion the external field can be treated classically. We assume that initially (i.e., before the quantum pulse arrives) all atoms are in their ground states $|b_j\rangle$. To describe the quantum properties of the medium, we use collective, slowly varying atomic operators, appropriately averaged over small but macroscopic volumes containing $N_z \gg 1$ particles at position z,

$$\hat{\sigma}_{\alpha\beta}(z,t) = \frac{1}{N_z} \sum_{j=1}^{N_z} |\alpha_j\rangle\langle\beta_j| e^{-i\omega_{\alpha\beta}t}. \tag{2}$$

The interaction between light and atoms is governed by the Hamiltonian

$$\hat{V} = -N \int \frac{dz}{L} \left(\hbar g \sum_k a_k e^{ikz} \hat{\sigma}_{ab}(z) + \hbar\Omega \hat{\sigma}_{ac}(z) \right) + \text{H.c.} \tag{3}$$

Here $g = \wp\sqrt{\frac{\nu}{2\hbar\epsilon_0 V}}$ is the atom-field coupling constant with $\wp$ being the dipole moment of the a–b transition and V being the quantization volume. N is the number of atoms in this volume and L is its length in the z direction.

The evolution of the Heisenberg operator corresponding to the optical field can be described in a slowly varying amplitude approximation by the propagation equation

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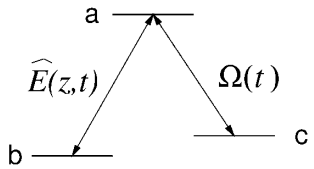

FIG. 1. 3-level Λ-type medium resonantly coupled to a classical field with Rabi frequency $\Omega(t)$ and quantum field $\hat{E}(z,t)$.

$$\left(\frac{\partial}{\partial t} + c\,\frac{\partial}{\partial z}\right)\hat{E}(z,t) = igN\hat{\sigma}_{ba}(z,t)\,. \tag{4}$$

The atomic evolution is governed by a set of Heisenberg-Langevin equations

$$\frac{\partial}{\partial t}\,\hat{\sigma}_{\mu\nu} = -\gamma_{\mu\nu}\sigma_{\mu\nu} + \frac{i}{\hbar}\,[\hat{V},\hat{\sigma}_{\mu\nu}] + F_{\mu\nu}\,, \tag{5}$$

where $\gamma_{\mu\nu}$ are the transversal decay rates and $\hat{F}_{\mu\nu}$ are δ-correlated Langevin noise operators.

We now assume that the Rabi frequency of the quantum field is initially much smaller than Ω and that the number of photons in the input pulse is much less than the number of atoms. In such a case the atomic equations can be treated perturbatively in $\hat{E}$. In zeroth order only $\hat{\sigma}_{bb} = 1$ is different from zero and in first order one finds

$$\hat{\sigma}_{ba} = -\frac{i}{\Omega(t)}\,\frac{\partial}{\partial t}\,\hat{\sigma}_{bc}\,, \tag{6}$$

$$\hat{\sigma}_{bc} = -\frac{g\hat{E}}{\Omega} - \frac{i}{\Omega}\left[\left(\frac{\partial}{\partial t} + \gamma_{ba}\right) \times \left(-\frac{i}{\Omega}\,\frac{\partial}{\partial t}\,\hat{\sigma}_{bc}\right) + \hat{F}_{ba}\right]. \tag{7}$$

In the above equations we disregarded a (small) decay of the Raman coherence (γ_{bc}).

The propagation equations simplify considerably if we assume a sufficiently slow change of Ω, i.e., adiabatic conditions [8,10]. By introducing a normalized time $\tilde{t} = t/T$, where T is a characteristic time scale, and expanding the right-hand side (rhs) of (7) in powers of $1/T$, we find in lowest nonvanishing order

$$\hat{\sigma}_{bc}(z,t) = -g\,\frac{\hat{E}(z,t)}{\Omega(t)}\,. \tag{8}$$

Note that $\langle \hat{F}_x(t)\hat{F}_y(t')\rangle \sim \delta(t - t') = \delta(\tilde{t} - \tilde{t}')/T$. Thus in the perturbative and the adiabatic limit the propagation of the quantum light pulse is governed by the equation

$$\left(\frac{\partial}{\partial t} + c\,\frac{\partial}{\partial z}\right)\hat{E}(z,t) = -\frac{g^2N}{\Omega(t)}\,\frac{\partial}{\partial t}\,\frac{\hat{E}(z,t)}{\Omega(t)}\,. \tag{9}$$

If Ω is constant, the term on the rhs simply leads to a modification of the group velocity of the quantum field according to $v_g = c/(1 + \frac{g^2N}{\Omega^2})$. In the general case, the field equation will acquire an additional term proportional to $(\dot{\Omega}/\Omega)\hat{E}$ which describes reversible changes in quantum amplitudes due to stimulated Raman scattering.

One can obtain a very simple solution of Eq. (9) by introducing a new quantum field $\hat{\Psi}(z,t)$ via the canonical transformation

$$\hat{\Psi}(z,t) = \cos\theta(t)\hat{E}(z,t) - \sin\theta(t)\sqrt{N}\,\hat{\sigma}_{bc}(z,t)\,,$$

$$\cos\theta(t) = \frac{\Omega(t)}{\sqrt{\Omega^2(t) + g^2N}}\,, \tag{10}$$

$$\sin\theta(t) = \frac{g\sqrt{N}}{\sqrt{\Omega^2(t) + g^2N}}\,.$$

$\hat{\Psi}$ obeys the following equation of motion:

$$\left[\frac{\partial}{\partial t} + c\cos^2\theta(t)\,\frac{\partial}{\partial z}\right]\hat{\Psi}(z,t) = 0\,, \tag{11}$$

which describes a shape-preserving propagation with velocity $v = v_g(t) = c\cos^2\theta(t)$:

$$\hat{\Psi}(z,t) = \hat{\Psi}\left(z - c\int_0^t d\tau\cos^2\theta(\tau), t = 0\right). \tag{12}$$

Several interesting properties of the new field should be noted. By introducing a plane-wave decomposition $\hat{\Psi}(z,t) = \sum_k \hat{\Psi}_k(t)e^{ikz}$, one finds that the mode operators $\hat{\Psi}_k$ and $\hat{\Psi}_k^\dagger$ obey the commutation relations

$$[\hat{\Psi}_k, \hat{\Psi}_{k'}^+] = \delta_{k,k'}\left[\cos^2\theta + \sin^2\theta\,\frac{1}{N}\sum_j (\hat{\sigma}_{bb}^j - \hat{\sigma}_{cc}^j)\right]. \tag{13}$$

In the linear limit considered here, where the number density of photons is much smaller than the density of atoms, $\hat{\sigma}_{bb}^j \approx 1, \hat{\sigma}_{cc}^j \approx 0$. Thus the new field possesses bosonic commutation relations and we can associate with it bosonic quasiparticles (polaritons). Furthermore, one immediately verifies that all number states created by $\hat{\Psi}_k^\dagger$ are dark states [2,5]:

$$|D_n^k\rangle = \frac{1}{\sqrt{n!}}\,(\hat{\Psi}_k^\dagger)^n|0\rangle\,|b_1 \ldots b_N\rangle\,, \tag{14}$$

where $|0\rangle$ denotes the field vacuum. The states $|D_n^k\rangle$ do not contain the excited atomic state and are thus immune to spontaneous emission. Moreover, they are eigenstates of the interaction Hamiltonian with eigenvalue zero, $\hat{V}|D_n^k\rangle = 0$. For these reasons we call the quasiparticles dark-state polaritons.

In summary, we have found a shape-preserving, polaritonlike mixture of an electromagnetic field and collective Raman coherences. This excitation is not of the soliton-type since no special pulse shape or pulse area is required. It is related to the classical adiabaton solutions of pulse-pair propagation in Λ-type media [14,15] in the limit of one strong and one weak field. We emphasize, however, that the field can here be in any quantum state.

One of the most interesting aspects of dark-state polaritons is the possibility to coherently control their properties by changing $\Omega(t)$. For example, by adiabatically rotating $\theta(t)$ from 0 to $\pi/2$, one can decelerate and stop

an input light pulse. In this process, pulse shape and the quantum state of the light pulse are mapped onto collective states of matter in which they are stored. Likewise, the polariton can be reaccelerated to the vacuum speed of light; in this process, the stored quantum states are transferred back to the field. This is illustrated in Fig. 2, where we have shown the coherent amplitude of a dark-state polariton which results from an initial light pulse as well as the corresponding field and matter components. One recognizes that the pulse shape is preserved and that the stopping corresponds to a transfer from field to atomic Raman excitations:

$$\hat{a}_k^\dagger \hat{a}_l^\dagger \hat{a}_m^\dagger \ldots |0\rangle |b_1 \ldots b_N\rangle \longleftrightarrow \sqrt{N}\,\hat{\sigma}_{cb}^k \sqrt{N}\,\hat{\sigma}_{cb}^l \sqrt{N}\,\hat{\sigma}_{cb}^m \ldots |b_1 \ldots b_N\rangle |0\rangle , \tag{15}$$

where the plane-wave decomposition $\hat{\sigma}_{cb}(z) = e^{i(\nu/c)z} \times (\sum_k \hat{\sigma}_{cb}^k e^{-ikz})$ has been used.

The transfer of quantum states between light and matter opens interesting prospectives for the generation of nonclassical atomic ensembles in squeezed and entangled states, high-precision spectroscopy with resolution beyond the standard quantum limit [7] as well as reversible quantum memories. Furthermore, by trapping correlated photons in separate media, entangled states of separated atomic ensembles can be created. With respect to these applications this paper is complementary to our earlier studies in which we showed that quantum states of light can be mapped onto Dicke-like collective states of an EIT medium in an optical resonator [4,5]. The quantum states of matter generated in the case of this paper are more complicated, however, trapping the light in a traveling-wave geometry does not require special shaping of the classical driving pulses (quantum impedance matching), which is necessary in a cavity configuration.

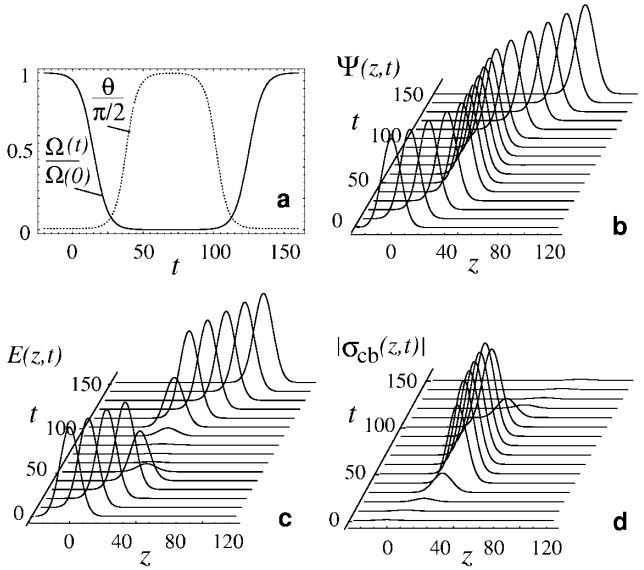

FIG. 2. Propagation of a dark-state polariton with envelope $\exp\{-(z/10)^2\}$. The mixing angle is rotated from 0 to $\pi/2$ and back according to $\cot\theta(t) = 100\{1 - 0.5\tanh[0.1(t - 15)] + 0.5\tanh[0.1(t - 125)]\}$ as shown in (a). The coherent amplitude of the polariton $\Psi = \langle\hat{\Psi}\rangle$ is plotted in (b), and the electric field $E = \langle\hat{E}\rangle$ and matter components $|\sigma_{cb}| = |\langle\hat{\sigma}_{cb}\rangle|$ in (c) and (d), respectively. Axes are in arbitrary units with $c = 1$.

We also note related studies on quantum memories for light involving mapping the quantum state of the field onto atoms by dissipative absorption [6,16]. In contrast to these approaches the adiabatic passage technique [17] used here allows for a complete and reversible excitation transfer of arbitrary quantum wave packets.

Finally, our approach is also different from the mechanism suggested recently in [18], in which "freezing" of the light pulse in a laboratory frame was proposed using moving atoms.

The above analysis involves a perturbation expansion, an adiabatic approximation, and disregards the decay of Raman coherence. In what follows the validity of these approximations is discussed. We note that, by making use of (8), one finds $g^2\hat{E}^+\hat{E}/|\Omega|^2 = \hat{\sigma}_{cb}\hat{\sigma}_{bc}$. In other words, the ratio of the average intensities of quantum and control fields is proportional to that of the matter field $\langle\hat{\sigma}_{cc}\rangle$. If the initial number of photons in the quantum field is much less than the number of atoms, $\langle\hat{\sigma}_{cc}\rangle$ is always much smaller than unity. Therefore the mean intensity of the quantum field remains small compared to that of the control field even when the latter is turned to zero.

In order to check the validity of the adiabatic approximation we consider the first correction to $\hat{\sigma}_{bc}$:

$$\hat{\sigma}_{bc} \approx -\frac{g\hat{E}}{\Omega} + \frac{1}{\Omega}\left(\frac{\partial}{\partial t} + \gamma_{ba}\right)\frac{1}{\Omega}\,\frac{\partial}{\partial t}\,\frac{g\hat{E}}{\Omega} + \ldots . \tag{16}$$

The nonadiabatic correction in (16) leads to a dissipative spectral narrowing (pulse spreading) of the quantum field due to the finite bandwidth of the transparency window [10]. This results in a "pulse" matching of quantum and classical control fields [15,19]. Using the adiabatic solution (12), one can verify that these corrections are small for propagation distances:

$$z \ll z_{\max} = \frac{g^2 N}{\gamma_{ab}} \times \frac{L_p^2}{c} , \tag{17}$$

where L_p is the length of the input pulse. Hence, in order to trap a pulse with negligible losses, it is required that

$$\frac{g^2 N L_p}{c\,\gamma_{ab}} \gg 1 . \tag{18}$$

This condition contains the number of atoms which is a signature of collective interactions. It should be con-

trasted to the strong-coupling condition corresponding to a quantum state transfer in cavity QED [20,21]. We note, in particular, that in the optically dense medium the adiabatic condition (18) is much easier to implement.

The effect of the Raman-coherence decay can be easily estimated using the explicit expression for the generated matter states (15). It is clear that the collective states containing n_e atomic excitations will dephase at a rate $\gamma_{bc} n_e$. Hence, the time of the storage should be limited to $t_s \ll (\gamma_{bc} n_e)^{-1}$ to avoid decoherence [5].

In the discussion above we have considered the case where the control field depends only on time. This can be realized, e.g., when the control field propagates in a direction perpendicular to that of the quantum field. In experiments involving hot atomic vapors, copropagation is required, however, in order to cancel Doppler broadening. Therefore propagation effects of the control field need to be considered. If the quantum field is weak, the control field propagates as in free space and thus $\Omega(z,t) = \Omega(t - z/c)$. In this case, one finds

$$\left(\frac{\partial}{\partial t} + c\cos^2\theta(z,t)\,\frac{\partial}{\partial z}\right)\frac{\hat{E}(z,t)}{\Omega(z,t)} = 0\,. \qquad (19)$$

Since the group velocity is now also z dependent, trapping of the pulse does not preserve the shape exactly. Nevertheless it is evident that trapping and a reversible transfer of the quantum state from light to atoms are still possible. In experiments, however, a more practical approach can be taken in which a light pulse enters the medium already with $v_g^0 \ll c$. In such a case retardation of the control field can be ignored and one has $\Omega(t - z/c) \approx \Omega(t)$. Since the index of refraction is close to unity there will be no reflection losses at the entrance plane. However, the polariton pulse becomes spatially compressed according to $L_p/L_p^0 = v_g^0/c$, and its amplitude grows according to the boundary condition $\hat{\Psi}(0,t) = \sqrt{c/v_g^0}\,\hat{E}(0,t)$. In this way, the total number of polaritons inside the medium is equal to the number of photons outside.

In conclusion we have shown that it is possible to control the propagation of quantum pulses in optically thick Λ-type media. This coherent control mechanism is based on dark-state polaritons associated with EIT. In particular, a quantum light pulse can be "trapped," in which case its shape and quantum state are preserved in stationary atomic excitations. The matterlike polariton can then be reaccelerated and converted back into a photon pulse. These properties of dark-state polaritons can be used for squeezing and entanglement transfer from light to atoms. Furthermore, we anticipate interesting applications involving nonlinear interactions between such polaritons.

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