Spectral Flow and Vortex Dynamics in a Temperature Gradient

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In the mixed state of superconductors the spectral flow of fermion zero modes in the vortex core couples the motion of vortices to that of the normal fluid. This gives rise to a heat current perpendicular to the direction of vortex motion and therefore to longitudinal thermomagnetic effects like the thermopower and the Peltier effect. Analysis of vortex motion in a temperature gradient on this basis yields excellent agreement with experimental results.

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The equation of motion governing vortex dynamics in superfluids and superconductors has been the subject of intensive research. Considerable insight has been achieved recently by noting the important role played by the so called spectral flow [1–3]. This effect, known from relativistic quantum field theory, occurs in Fermi superfluids and superconductors, if quantized vortices are present. The spectrum of bound excitations in the vortex core has an asymmetric branch, which crosses zero as a function of angular momentum [4]. Motion of the vortex with respect to the heat bath leads to spectral flow along this branch. In this process the momentum of the quasiparticles (QPs) in the vortex core is not conserved, which implies a transfer of momentum from the vortex to the heat bath and thus an additional force on the vortex. Spectral flow occurs if the excitation spectrum may be regarded as continuous, as, e.g., in the hydrodynamic limit of vortex motion usually explored experimentally in superconductors. In this limit the consequences of spectral flow are important. For example, the Hall angle is small, since the spectral flow almost "cancels" the Magnus effect. Including the spectral flow force into the equation of motion of vortices reproduces the results of the microscopic theory of mutual friction [5]. The spectral flow force has recently been observed experimentally in ^{3}He [6].

We point out here that the spectral flow is also essential for an understanding of thermomagnetic effects in the mixed state of superconductors [7,8]. These effects arise, since the motion of vortices couples to that of the normal fluid and therefore gives rise to a heat current. We argue that the spectral flow provides just such a coupling and that it determines in particular the longitudinal thermomagnetic effects like the thermopower and the Peltier effect. We derive an equation of motion for vortices in a temperature gradient. Analysis of vortex motion based on this equation yields excellent agreement with experimental results.

The thermomagnetic effects which relate the temperature gradient ∇T to the electrical field $\mathbf{E}$ may be defined according to [9]

$$\mathbf{E} = S\nabla T + Q\nabla T \times \mathbf{B}. \tag{1}$$

Here S is the thermopower (or Seebeck coefficient) and Q is the Nernst coefficient. Experiments in the flux flow regime in high temperature superconductors show that the thermal Hall angle defined as $\tan\alpha_{\mathrm{th}} = QB/S$ is of order 1 in magnetic fields of order 1 T; i.e., the Seebeck effect is of the same magnitude as the Nernst effect [8,10]. Since the electrical field is directly related to the vortex velocity $\mathbf{v}_L$ according to $\mathbf{E} = \mathbf{B} \times \mathbf{v}_L$ this implies that vortices move at a large angle with respect to ∇T, as shown in Fig. 1. Note that at the same time the Hall angle α_H obtained from the resistivity (ρ) and the Hall resistivity (ρ_H) is small, of the order $\tan\alpha_H = \rho_H/\rho \approx 10^{-3}$ in magnetic fields of order 1 T; i.e., the large thermal Hall angle occurs in the hydrodynamic limit of vortex motion [8,10,11]. The thermomagnetic effects defined by Eq. (1) are related to another set of effects [9], which describe the coupling between the heat ($\mathbf{j}_h$) and the electrical ($\mathbf{j}$) current densities:

$$\mathbf{j}_h = \Pi\,\mathbf{j} + \epsilon k \mathbf{B} \times \mathbf{j}. \tag{2}$$

Here Π and ϵ are, respectively, the Peltier and Ettingshausen coefficients and k is the thermal conductivity. According to the Onsager relations $\Pi = ST$ and $\epsilon k = QT$. This has been verified experimentally, e.g., for the Peltier coefficient and for the thermopower [11]. It is therefore equivalent to discuss either the electrical voltage resulting from an applied temperature gradient or the heat current resulting from an applied electrical current.

A qualitative understanding of the coupling between vortex motion and the heat current is obtained by considering a current driven resistive state, as shown in Fig. 2. In the hydrodynamic limit vortices move almost perpendicular to the applied electrical current $\mathbf{j}$ with velocity $\mathbf{v}_L$.

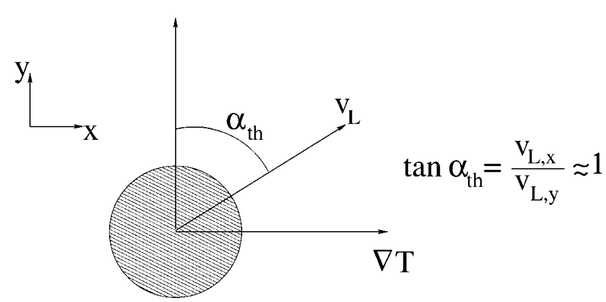

FIG. 1. Vortex motion in a temperature gradient. The magnetic field is in the z direction (see text).

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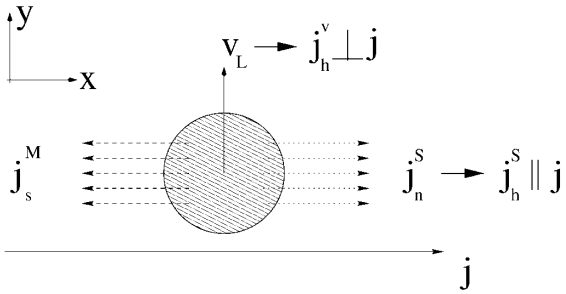

FIG. 2. Current driven vortex motion in the hydrodynamic regime, where $\tan\alpha_H \ll 1$. The magnetic field is in the z direction (see text).

The heat current has two components: (i) It is well known that a heat current $\mathbf{j}_h^v$ parallel to $\mathbf{v}_L$ results directly from the motion of vortices, since vortices transport an entropy s_v (per unit length). s_v arises mainly from the normal excitations in the vortex core. Obviously $\mathbf{j}_h^v$ is perpendicular to the electrical current $\mathbf{j}$ and therefore gives rise to the transverse Ettingshausen effect. This vortex heat current is unique to the mixed state of superconductors and leads to transverse thermomagnetic effects much larger than those in the normal state (see, e.g., [7,8,12]). (ii) We propose that a longitudinal heat current arises from the spectral flow: Because of the Magnus effect a vortex moving with velocity $\mathbf{v}_L$ generates a supercurrent $\mathbf{j}_s^M$ perpendicular to $\mathbf{v}_L$, which takes off momentum. The spectral flow generates in addition a stream $\mathbf{j}_n^S$ of unbound QPs, which carry off momentum (almost) equal, but opposite to that of the induced superflow [2]. This explains why the total force on the vortex perpendicular to $\mathbf{v}_L$ is nearly zero and therefore that the electrical Hall angle α_H is small. However, it implies also a heat current $\mathbf{j}_h^S$ perpendicular to $\mathbf{v}_L$, since obviously the normal QP current $\mathbf{j}_n^S$ carries heat, whereas the induced superflow $\mathbf{j}_s^M$ does not. Since $\mathbf{j}_h^S$ is parallel to the electrical current $\mathbf{j}$ this gives rise to a longitudinal Peltier effect (Fig. 2).

We turn now to a quantitative discussion and consider the motion of vortices in a temperature gradient. We start with the equation of motion for a vortex:

$$0 = \mathbf{F}_M + \mathbf{F}_I + \mathbf{F}_S + \mathbf{F}_{\text{th}} + \mathbf{F}_d \,. \tag{3}$$

The forces in this equation are

$$\mathbf{F}_M = \pi n_s(\mathbf{v}_s - \mathbf{v}_L) \times \mathbf{z}\,, \tag{4}$$

$$\mathbf{F}_I = \pi n_n(\mathbf{v}_n - \mathbf{v}_L) \times \mathbf{z}\,, \tag{5}$$

$$\mathbf{F}_S = -\pi C(\mathbf{v}_n - \mathbf{v}_L) \times \mathbf{z}\,, \tag{6}$$

$$\mathbf{F}_{\text{th}} = -s_v \nabla T\,, \tag{7}$$

$$\mathbf{F}_d = D(\mathbf{v}_n - \mathbf{v}_L)\,. \tag{8}$$

Here $\mathbf{v}_s$ and $\mathbf{v}_n$ are the velocities of the superfluid and normal components of the liquid, respectively, and n_s and n_n are the corresponding densities. The total density is given by $n = n_s + n_n$. $\mathbf{z}$ is a unit vector in the direction of the magnetic field. We use units $\hbar = c = 1$. We assume that vortices move freely in response to the driving forces. The influence of pinning will be discussed below.

We discuss the forces in Eq. (3) briefly. (i) $\mathbf{F}_M$ and $\mathbf{F}_I$ are the Magnus and Iordanskii forces, which describe the coupling of vortex motion to the superfluid and normal components of the liquid [13]. (ii) $\mathbf{F}_S$ is the spectral flow force [1–3]. It provides an additional coupling between the motion of vortices and that of the normal fluid. The coefficient C depends on the regime of vortex dynamics and is given by [3,6]

$$\frac{C}{n} = 1 - \frac{\omega_0^2\tau^2}{1 + \omega_0^2\tau^2}\tanh\left(\frac{\Delta(T)}{2k_BT}\right). \tag{9}$$

Here ω_0 is the level spacing in the vortex cores, τ is the scattering time of the QPs, and $\Delta(T)$ is the energy gap. The hydrodynamic limit is characterized by the condition $\omega_0\tau \ll 1$. In this limit $C/n \approx 1$ and $(n - C)/n \simeq (\Delta/E_F)^2 \ll 1$ [3,14]. In the collisionless limit characterized by $\omega_0\tau \gg 1$ Eq. (9) yields $C/n \to 0$ for $T \to 0$ and $C/n \approx 1$ for $T \to T_c$. Apparently, spectral flow is suppressed in this limit with decreasing temperature and vanishes for $T \to 0$ [15]. (iii) $\mathbf{F}_{\text{th}}$ in Eq. (3) describes the force on a vortex by a temperature gradient [16]. The existence of this force is well known from experiment [7,8,12]. It is due to the finite transport entropy s_v: If a moving vortex transports entropy it experiences, vice versa, a force in a temperature gradient. s_v is temperature dependent and vanishes for $T \to 0$ and for $T \to T_c$ [7,8,12]. (iv) $\mathbf{F}_d$ is a dissipative friction force. The friction coefficient D is of the order $D/n \simeq \omega_0\tau/(1 + \omega_0^2\tau^2)$, i.e., $D/n \ll 1$ in both the hydrodynamic and the collisionsless limits.

Summing all contributions the equation of motion may be written as

$$\pi n_s \mathbf{v}_s \times \mathbf{z} + (\gamma - \pi n_s)\mathbf{v}_n \times \mathbf{z} + D\mathbf{v}_n - s_v \nabla T = D\mathbf{v}_L + \gamma \mathbf{v}_L \times \mathbf{z}\,, \tag{10}$$

where $\gamma = \pi(n - C)$. Note that $\gamma \ll 1$ in the hydrodynamic limit and $\gamma \approx \pi n$ in the collisionless limit at low temperatures, where spectral flow is suppressed. Solving Eq. (10) for $\mathbf{v}_L$ and using $\mathbf{E} = \mathbf{B} \times \mathbf{v}_L$ we obtain the electric field

$$\mathbf{E} = \frac{B}{D^2 + \gamma^2}\{D\pi n_s \mathbf{v}_s - \gamma\pi n_s \mathbf{v}_s \times \mathbf{z} - D\pi n_s \mathbf{v}_n - [D^2 + \gamma(\gamma - \pi n_s)]\mathbf{v}_n \times \mathbf{z} + \gamma s_v \nabla T + D s_v \nabla T \times \mathbf{z}\}. \tag{11}$$

Expressions for the resistivity and the Hall angle are obtained from Eq. (11) by requiring $\nabla T = 0$ and $\mathbf{v}_n \approx 0$ as usual for current driven vortex motion. Using $\mathbf{E} = \rho \mathbf{j}_s - \rho_H \mathbf{j}_s \times \mathbf{z}$ with $\mathbf{j}_s = n_s e \mathbf{v}_s$, we find

$$\rho = \frac{B\Phi_0 D}{D^2 + \gamma^2} \quad \text{and} \quad \rho_H = \frac{B\Phi_0\gamma}{D^2 + \gamma^2}\,. \tag{12}$$

The Hall angle α_H is given by $\tan\alpha_H = \gamma/D$. In the hydrodynamic limit $\tan\alpha_H$ is small. As discussed above the spectral flow force almost cancels the $\mathbf{v}_L$ dependent part of the Magnus (and Iordanskii) force, and vortices move approximately at right angles with respect to the supercurrent. In contrast, in the collisionless limit at low temperatures spectral flow is suppressed yielding $\gamma \approx \pi n \gg D$ and we obtain $\rho \to 0$ and $\rho_H \simeq B\Phi_0/\pi n$. In this limit vortices move with the superfluid, i.e., $\mathbf{v}_L \simeq \mathbf{v}_s$.

Experiments on vortex dynamics in a temperature gradient are usually performed such that $\nabla T \neq 0$ and $\mathbf{j} = \mathbf{j}_s + \mathbf{j}_n = 0$. It is, however, important to realize that although the total current $\mathbf{j}$ vanishes the components $\mathbf{j}_s$ and $\mathbf{j}_n$ are finite in general if $\nabla T \neq 0$. The reason is that a temperature gradient sets up a normal (quasiparticle) current [9,17]

$$\mathbf{j}_n = en_n\mathbf{v}_n = -L_{QP}\nabla T\,, \tag{13}$$

which is compensated by a supercurrent $\mathbf{j}_s = -\mathbf{j}_n$. This is well established for a superconductor in a temperature gradient *without* an applied magnetic field. However, on average it should be valid also in the mixed state [18]. The coefficient L_{QP} in a superconductor for elastic scattering is given by [9,17]

$$L_{QP} = L_n G\left(\frac{\Delta}{k_B T}\right) \simeq L_n \frac{n_n(T)}{n}\,, \tag{14}$$

where $L_n = S_n/\rho_n$. Here S_n and ρ_n are the normal state thermopower and resistivity, respectively. The function $G(\Delta/k_BT)$ is the same function which governs the behavior of the electronic thermal conductivity in the superconducting state. Its main temperature variation is due to the decrease of the numbers of quasiparticles with decreasing temperature (at least in the case of elastic impurity scattering) [9]. From Eqs. (13) and (14) and using that $\mathbf{j} = 0$ requires $n_n\mathbf{v}_n = -n_s\mathbf{v}_s$ we obtain $\mathbf{v}_n = -L_n\nabla T/en = -n_s\mathbf{v}_s/n_n$. Inserting this into Eq. (10) we obtain the equation of motion of a vortex in a temperature gradient:

$$\Phi_0 L_n\left(1 - \frac{\gamma}{\pi n}\right)\nabla T \times \mathbf{z} - \left(s_v + \frac{DL_n}{en}\right)\nabla T = D\mathbf{v}_L + \gamma\mathbf{v}_L \times \mathbf{z}\,. \tag{15}$$

Using $\gamma = \pi(n - C)$ the force perpendicular to ∇T may be written as $\Phi_0 L_n(C/n)\nabla T \times \mathbf{z}$. Apparently, the spectral flow plays the central role for this force. In particular, the force is absent in the collisionless limit at low temperatures, where $C \to 0$. Note, however, that in this limit the Hall term $\propto \gamma\mathbf{v}_L \times \mathbf{z}$ is large.

The electric field is obtained from Eq. (11) as

$$\mathbf{E} = \frac{B}{D^2 + \gamma^2}\left\{(D\Phi_0 L_n + \gamma s_v)\nabla T + \left[Ds_v + \frac{D^2L_n}{en} - \gamma\Phi_0 L_n\left(1 - \frac{\gamma}{\pi n}\right)\right]\nabla T \times \mathbf{z}\right\}. \tag{16}$$

This yields for the thermopower

$$S = \rho\,\frac{S_n}{\rho_n} + \tan\alpha_H\,\rho\,\frac{s_v}{\Phi_0}\,, \tag{17}$$

where Eq. (12) has been used. The interpretation is straightforward: The first contribution to S results from the motion of the normal fluid along the temperature gradient. The second contribution is due to the heat carried by the vortices which has a component perpendicular to ∇T because of the finite Hall angle α_H. For the Nernst coefficient Eq. (16) yields

$$QB = \rho\left(\frac{s_v}{\Phi_0} + \frac{D}{\pi n}\frac{S_n}{\rho_n}\right) - \tan\alpha_H\,\rho\,\frac{S_n}{\rho_n}\left(1 - \frac{\gamma}{\pi n}\right). \tag{18}$$

The three contributions to Q have the following origin: The first term arises from the motion of the vortex along the temperature gradient. The second term arises since the vortices have a component of motion perpendicular to the temperature gradient due to the thermopower: The coupling of this motion to the normal fluid via $D\mathbf{v}_n$ drags normal fluid in this direction and therefore contributes to the transverse voltage. The third term is the Nernst effect of the normal fluid, as is apparent from comparison with the result for the thermopower.

In order to derive results appropriate for the hydrodynamic and collisionless limits we estimate the magnitude of the various contributions to S and QB. Using $s_v \approx 10^{-14}$–10^{-13} J/Km [7,8] we find $s_v/\Phi_0 \approx 10$–100 A/Km. Using $S_n \approx 1\ \mu$V/K and $\rho_n \approx 10^{-2}\ \mu\Omega$ m we obtain $S_n/\rho_n \approx 100$ A/K m, comparable in magnitude to s_v/Φ_0. $D/n \simeq \omega_0\tau/(1 + \omega_0^2\tau^2)$ is small in both the hydrodynamic and the collisionless limits.

(a) *Hydrodynamic limit* ($\omega_0\tau \ll 1$).—In this limit $\tan\alpha_H \ll 1$ and we find

$$\frac{S}{S_n} \simeq \frac{\rho}{\rho_n}\,, \tag{19}$$

$$QB \simeq \rho\,\frac{s_v}{\Phi_0}\,. \tag{20}$$

Equation (19) is in very good agreement with experimental results on high-T_c superconductors [8,10,11]. Minor deviations are related to the finite Hall angle [19]. Equation (20) is the usual expression relating the Nernst coefficient, the flux flow resistivity, and the transport entropy. It is frequently used to extract the transport entropy from the experimental data [7,8].

(b) *Collisionless limit* ($\omega_0\tau \gg 1$).—New results are obtained in this limit at low temperatures, where the spectral flow is suppressed so that $\gamma \approx \pi n$. This yields $\tan\alpha_H \gg 1$ and we obtain

$$S \simeq \tan\alpha_H QB \simeq \tan\alpha_H\,\rho\,\frac{s_v}{\Phi_0} = \frac{Bs_v}{\pi n}\,. \tag{21}$$

Apparently, the absence of spectral flow at low temperatures leaves the heat carried by the moving vortices as the only source of thermomagnetic effects. Note, however, that, since $\tan\alpha_H \gg 1$, vortices move almost perpendicular to ∇T and the thermopower is larger than the Nernst coefficient. It is instructive to consider also the heat flow in this limit in the case of current driven vortex motion. The heat current associated with the moving vortices is given by $\mathbf{j}_h^v \approx n_v s_v T \mathbf{v}_L$, where $n_v = B/\Phi_0$ is the vortex density. At low temperatures we have $n_s \approx n$ and $\mathbf{v}_L \approx \mathbf{v}_s = \mathbf{j}_s/ne$. This yields $\mathbf{j}_h^v \approx B s_v T \mathbf{j}_s/\pi n = \Pi \mathbf{j}_s$, where Π is the Peltier coefficient. Comparison to Eq. (21) shows that $\Pi = ST$, as is required by the Onsager relations [9].

It has been suggested that the thermopower can be understood by assuming that no force perpendicular to vortices results from the presence of the normal current, whereas the supercurrent provides such a force [18]. However, repeating the above calculation with this assumption it is straightforward to show that $S = (\rho/\rho_n) S_n G(\Delta/k_B T)$. This does not agree with experiment [8,10,11]. Therefore, our considerations show clearly that the forces between vortices and the normal fluid are essential for an understanding of the longitudinal thermomagnetic effects. In particular, since the spectral flow depends on the electron density n in the vortex core it leads to a thermopower independent of $G(\Delta/k_B T)$.

We finally comment on the influence of pinning. In high temperature superconductors pinning effects are weak above the so called irreversibility line and vortex motion induced by a temperature gradient can be observed experimentally in a broad range of temperatures and magnetic fields (see e.g., [8,10–12]). In conventional superconductors pinning effects are usually much more pronounced so that thermomagnetic effects have been observed only in a narrow temperature range (see, e.g., [7,20]). We expect that the main effect of pinning is a reduction of the flux flow resistivity, which leads to a corresponding decrease of the Nernst and Seebeck voltages according to Eqs. (19) and (20). On the other hand, the transport entropy, the thermal Hall angle, as well as the ratio of S and ρ should be unaffected by pinning. This is consistent with recent experimental results [21].

In summary, the spectral flow associated with moving vortices in superconductors couples the motion of vortices and of normal quasiparticles and gives rise to a heat current perpendicular to the direction of vortex motion. This leads to longitudinal thermomagnetic effects like the thermopower and the Peltier effect. An analysis of vortex motion in a temperature gradient on this basis yields excellent agreement with experimental results. Vice versa, the observation of large longitudinal thermomagnetic effects provides strong experimental evidence for the relevance of the spectral flow effect for the motion of vortices in superconductors.

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