In Search of the Elusive Zonal Flow Using Cross-Bicoherence Analysis

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(Received 27 December 1999)

We show that the modulational instability growth rate of zonal flows is determined directly from the quasilinear wave kinetic equation. We also demonstrate the relation between zonal-flow growth and the cross bispectrum of the high-frequency drift-wave-driven Reynolds stress and the low-frequency plasma potential by explicit calculation. Experimental measurements of the spatiotemporal evolution of the spectrum integrated bicoherence at the $L \rightarrow H$ transition near the edge shear layer indicate a modification in the nonlinear phase coupling, which might be linked to the generation of sheared $\mathbf{E} \times \mathbf{B}$ flows.

PACS numbers: 52.25.Fi, 52.25.Gj, 52.40.Hf, 52.35.Ra

Zonal flows [1], which are toroidally and azimuthally symmetric shear layers with a spectrum of radial scales ($n = m = 0$, $k_r \rho_i$ finite), have recently become the subject of intense interest and investigation in magnetic confinement physics. This surge in activity is due to new theoretical and computational results which have pinpointed the crucial role of zonal-flow shear in the self-regulation of drift-wave turbulence levels and transport scalings [2,3]. As a consequence of their toroidal and poloidal symmetry, zonal flows are intrinsically unable to tap expansion free energy available in the plasma by any means, since $\tilde{V}_{r,\mathbf{E}\times\mathbf{B}} = 0$. Thus, the *only* means for exciting zonal flows is via nonlinear energy transfer from unstable drift waves with finite k_θ and $k_\parallel$.

In this Letter, we discuss aspects of the theory of zonal-flow shearing-as-mode coupling and also present a theory of the cross bispectrum of the zonal-flow fluctuation with the two short-wavelength perturbations. Recent progress in the characterization of the nonlinear nature of broadband fluctuations has provided a new bridge between experimental measurements and turbulence models to explain the redistribution of energy supplied to the fluctuation spectrum by multiple instabilities [4,5]. Relevant experimental results [6–8] on the link between fluctuations and poloidal flows are also presented.

It is now widely appreciated that zonal-flow shearing plays an important role in regulating drift-wave turbulence levels. Recently [9], it was noted that, in comparison to coherent shearing by the mean electric field, the broad spectrum of zonal flows regulates turbulence by the process of *random shearing*, represented, in the spirit of quasilinear theory, by diffusion in $\underline{k}$ space. Averaging and quasilinearly closing the wave kinetic equation yields

$$\frac{\partial\langle N\rangle}{\partial t} + \frac{\partial}{\partial k_r}\Gamma_{k_r} = C(N), \tag{1a}$$

$$\Gamma_{k_r} = -D\,\frac{\partial\langle N\rangle}{\partial k_r}, \tag{1b}$$

$$D = \sum_q k_\theta^2 q^2 |\tilde{V}_{Eq}|^2 R(\underline{k}, q), \tag{1c}$$

$$R(\underline{k}, q) = \gamma_k / [(\Omega_q - q v_g)^2 + \gamma_k^2]. \tag{1d}$$

The notation is standard and follows that of Ref. [9]. Here, Γk_r is the $\underline{k}$-space flux (with diffusion coefficient D) which drives evolution of the mean drift-wave spectrum $\langle N\rangle$. Note that validity of the assumption of random shearing (and therefore of the applicability of quasilinear theory) requires overlap of resonances between drift-wave group velocity (v_g) and zonal-flow phase velocity (Ω_q/q), i.e., a state of drift-wave *ray chaos.* The effect of random shearing on drift-wave energy $\langle\varepsilon\rangle$ may then be ascertained by multiplying Eq. (1a) by ω_k and summing over $\underline{k}$ to obtain

$$\frac{\partial\langle\varepsilon\rangle}{\partial t} = \sum_{\underline{k}} \omega_{\underline{k}}\langle C(N)\rangle + \sum_{\underline{k}}\sum_q \frac{2q^2 c_s^2 k_\theta^2 |\tilde{V}_{Eq}|^2 R(\underline{k}, q) k_r \,\partial\langle\eta\rangle/\partial k_r}{(1 + k^2\rho_s^2)^2}. \tag{2}$$

Here, $\langle\eta\rangle = (1 + k^2\rho_s^2)\langle\varepsilon\rangle$ is the wave potential enstrophy density. Equation (2) may then be rewritten as

$$\frac{\partial\langle\varepsilon\rangle}{\partial t} + \sum_q \gamma_q |\tilde{V}_{Eq}|^2 = \sum_{\underline{k}} \omega_{\underline{k}}\langle C(N)\rangle, \tag{3a}$$

$$\gamma_q = -2q^2 c_s^2 \sum_{\underline{k}} \frac{k_\theta^2\rho_s^2}{(1 + k^2\rho_s^2)^2} R(\underline{k}, q) k_r \frac{\partial\langle\eta\rangle}{\partial k_r}. \tag{3b}$$

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It is interesting to note that the expression for γ_q given in Eq. (4b) agrees with that given for the zonal-flow growth rate in Eq. (6a) of Ref. [9]. This observation, together with Eq. (4), shows that *drift waves and zonal flows together conserve total energy.* Hence, "shear suppression of drift-wave turbulence" is simply the process whereby drift-wave energy is coupled to damped zonal flows. Equivalently, "zonal-flow generation" refers to the concomitant gain in flow energy which accompanies the reduction in drift-wave intensity due to energy transfer. Thus, the often-invoked impact of shear suppression of turbulence by zonal flows is not due to enhanced dissipation or stabilization, but rather to nonlinear coupling of fluctuation energy to benign, axisymmetric modes which are intrinsically incapable of causing transport. The mode coupling process responsible for drift-wave $\rightarrow$ zonal-flow energy transfer is one mediated by nearly isosceles triads with a thin vertex angle. These triads have legs $\underline{k}_1 = \underline{k} - \underline{q}$, $\underline{k}_2 = -\underline{k}$, $\underline{k}_3 = \underline{q}$, where $\underline{k}$ and $\underline{q} = q\hat{r}$ are the drift-wave and zonal-flow wave vectors, respectively (with $|\underline{k}| \gg |\underline{q}|$). It is also interesting to observe here that the zonal-flow growth rate γ_q may be calculated directly from the quasilinear wave kinetic equation [Eq. (1)] using the additional proviso of energy conservation.

Having established that zonal flows are generated by energy-conserving triad interactions with drift waves, we now consider the theory of their cross bispectrum. In general, the cross bispectrum of three functions $x(t)$, $y(t)$, and $z(t)$ is given by

$$\hat{B}_{xyz}(\omega_1, \omega_2) = \langle xyz \rangle = x(\omega_1)y(\omega_2)z(\omega_1 + \omega_2), \quad (4a)$$

so that the related cross bicoherence,

$$\hat{b}^2_{xyz}(\omega_1, \omega_2) = \frac{|\hat{B}_{xyz}(\omega_1, \omega_2)|}{[P_{zz}(\omega_1, \omega_2)]^{1/2} x(\omega_1) y(\omega_2)}, \quad (4b)$$

is a normalized measure of the strength of nonlinear energy transfer between frequencies ω_1 and ω_2 as mediated by triad interactions (see Ref. [10] and references therein). Since zonal flows are generated by drift-wave-induced momentum transport, the cross bispectrum of interest is $\langle \tilde{V}_r \tilde{V}_\theta \overline{\phi} \rangle$, namely, that of the radial and poloidal $\mathbf{E} \times \mathbf{B}$ velocities of two high-frequency drift waves $(\underline{k}, \omega_{\underline{k}})$ with the potential perturbation of the low-frequency zonal flow (q, ω_q). Here, it is understood that $\overline{\phi}$ refers to $e\overline{\phi}/T$. Since $|\underline{k}| \gg |\underline{q}|$, $\omega_{\underline{k}} \gg \Omega_q$, it follows that

$$\begin{aligned}\langle \tilde{V}_r \tilde{V}_\theta \overline{\phi} \rangle &= B_{\tilde{V}_r, \tilde{V}_\theta, \overline{\phi}}[\omega_1 = \omega_{\underline{k}}, \omega_2 = -\omega_{\underline{k}}, \omega_1 + \omega_2 = \Omega_q \rightarrow 0] \\ &= -2[(k_r \rho_s)(k_\theta \rho_s)] \frac{c_s^2 \omega_{\underline{k}}}{(1 + k_s^2 \rho_s^2)} \left(\frac{\delta N}{\delta \overline{\phi}_q} \right) |\overline{\phi}_q|^2. \end{aligned} \quad (5)$$

Note here that $N = N(\overline{\phi}) \cong (\delta N / \delta \overline{\phi}_q) \overline{\phi}_q$ follows from the fact that the nonzero contribution to the cross bispectrum of Eq. (5) is due *only* to that part of N which is coherent with $\overline{\phi}$. Using the linearized wave kinetic equation to compute $\mathrm{Re}\,\delta N / \delta \overline{\phi}_q$ and noting that $\omega_k \partial \langle N \rangle / \partial k_r = (1 + k_\perp^2 \rho_s^2)^{-1} \partial \langle \eta \rangle / \partial k_r$ then yields

$$\langle \tilde{V}_r \tilde{V}_\theta \overline{\phi} \rangle = 2c_s^2 \frac{k_\theta^2 \rho_s^2}{(1 + k_\perp^2 \rho_s^2)^2} q_r^2 c_s^2 \frac{R_{\underline{k},q}}{\Omega_i} k_r \frac{\partial \langle \eta \rangle}{\partial k_r} |\overline{\phi}_q|^2. \quad (6)$$

It is interesting to note that the result of Eq. (6) is very closely related to the expression for zonal-flow growth rate given in Eq. (8) of Ref. [6], namely,

$$\begin{aligned}\frac{\partial}{\partial t} \left| \frac{e\overline{\phi}_q}{T} \right|^2 &= -2q_r^2 c_s^2 \sum_{\underline{k}} \frac{k_\theta^2 \rho_s^2}{(1 + k_\perp^2 \rho_s^2)^2} \\ &\quad \times R_{\underline{k},q} k_r \frac{\partial \langle \eta \rangle}{\partial k_r} \left| \frac{e\overline{\phi}_q}{T} \right|^2. \end{aligned} \quad (7)$$

Indeed, when dedimensionalized, and the right side of Eq. (7) is summed over $\underline{k}$, the results of Eqs. (6) and (7) are identical. Moreover, it is clear that the (wave) spectrum integrated cross bispectrum $\sum_{\underline{k}} B_{\tilde{V}_r, \tilde{V}_\theta, \overline{\phi}} = \sum_{\underline{k}} \langle \tilde{V}_r \tilde{V}_\theta \overline{\phi} \rangle$ is essentially equivalent to the evolution rate for zonal-flow energy. This is not too surprising, as triad mode coupling involving $\tilde{V}_{r+\underline{k}}$, $\tilde{V}_{\theta-\underline{k}}$ and $e\overline{\phi}/T$ is, in fact, the underlying physical mechanism for pumping zonal-flow energy. Note also that Eq. (6) gives a "first principles" theoretical prediction of the cross bispectrum which is suitable for direct comparison with experimental data and the results of numerical simulations.

Since measurement of plasma potential fluctuations is, in general, quite difficult, it is natural to investigate the cross bispectrum of the associated *density* perturbations. In that case, the quantity of interest is $\langle (\tilde{n}/n_0)^2 (\overline{n}/n_0) \rangle$, where $\tilde{n}/n_0$ are the short-wavelength, high-frequency drift-wave perturbations at $\underline{k}$, ω, and $\overline{n}/n_0$ is the zonal-flow density perturbation at Ω_q. A short, straightforward calculation using the methodology given above then yields

$$\begin{aligned}\left\langle \frac{\tilde{n}}{n_0}, \frac{\tilde{n}}{n_0}, \frac{\overline{n}_q}{n_0} \right\rangle &= 2\Omega_i R_{\underline{k},q} \frac{k_\theta}{(1 + k_\perp^2 \rho_s^2)^2} \\ &\quad \times \left| \frac{\overline{n}_q}{n_0} \right|^2 \frac{\partial}{\partial k_r} \langle \eta \rangle. \end{aligned} \quad (8)$$

Here, $\eta = (1 + k_\perp^2 \rho_s^2) |\tilde{n}_{\underline{k}}/n_0|^2$ (i.e., $\tilde{n}/n \cong e\tilde{\phi}/T$ for drift waves) and $|\overline{n}_q/n_0|^2$ is the zonal-flow density perturbation intensity. Since, for zonal flows $\overline{n}/n_0 = -q^2 \rho_s^2 e\overline{\phi}/T$ (with $q^2 \rho_s^2 < 1$), the cross bispectrum $\langle |\tilde{n}_k/n_0|^2 \overline{n}_q/n_0 \rangle$ is smaller in magnitude than the corresponding result given in Eq. (6). Also, note that a cross bispectrum constructed from Eq. (8) should employ a sum over positive frequencies only, to avoid spurious cancellation. Otherwise, however, the correspondence between cross bispectrum and zonal-flow growth rate persists.

Regarding the experimental evidence for zonal flows, a reversal in the poloidal phase velocity of fluctuations has been observed in the edge of plasma tokamaks, stellarators, and reversed field pinch confinement devices [11–13]. In this region, the plasma potential has a maximum (i.e., the radial electric field reverses direction) and the phase velocity is generally controlled by the $\mathbf{E} \times \mathbf{B}$ velocity. During the transition to improved confinement regimes (i.e., *L*-*H* transition), there is a change in the radial electric field, an increase in the $\mathbf{E} \times \mathbf{B}$ shearing rate and a reduction of plasma turbulence which is consistent with the framework of the $\mathbf{E} \times \mathbf{B}$ shear turbulence reduction model [14]. Thus, such edge plasmas are natural venues for studies of zonal-flow physics. Indeed, nonlinear properties of plasma turbulence have been investigated in the boundary of fusion plasmas, providing a link between experimental measurements and turbulence models [4–8]. Bispectral analysis tools have been used to study the strength of nonlinear mechanisms in the proximity of the velocity shear layer, using data from the Advanced Toroidal Facility (ATF) [12] and during *L*-*H* transition in the W7-AS stellarator [6,7]. During *L*-*H* transition scenarios in the W7-AS stellarator, there is a relatively small enhancement of the stored energy with a concomitant reduction (but without disappearance) in the level of fluctuations [6]. This transitioning plasma provides a unique opportunity to investigate the interplay between plasma flows and fluctuations in improved confinement regimes.

The radial profile of the total integrated bicoherence for frequencies lower than 500 kHz for ion saturation current fluctuations, as well as the poloidal phase velocity of fluctuations in the edge region of the ATF stellarator, are shown in Fig. 1. Note that there is a substantial radial variation in the strength of nonlinear interactions near the shear location. The frequency resolved bicoherence shows that the maxima in bicoherence are mainly due to sum frequencies in the range 50–200 kHz.

The radial structure of the total integrated bicoherence ($f < 500$ kHz) during the *L* to *H* transition has also been investigated in the W7-AS stellarator using reflectometry measurements [6,7]. During the transition to the improved confinement regime, there is an abrupt increase of the total bicoherence [Fig. 2(a)] simultaneous with the reduction in fluctuation intensity. The level of bicoherence [Fig. 2(b)] increases at all radial positions in the plasma bulk side of the last closed flux surface (LCFS) position [6] [Fig. 2(b)]. The high level of bicoherence is due to the enhanced nonlinear interaction in a wide range of frequencies (10–500 kHz) [7]. Interestingly, similar results have been reported in the *L*-*H* transition in the continuous current tokamak using probe measurements [8].

From previous experiments, we have shown that Reynolds stress driven flows can play a significant role in the plasma boundary region, providing a coupling between fluctuations and edge shear flow [15]. In the framework of this interpretation, the modification of the bicoherence in the proximity of the velocity shear layer in ATF and during the *L*-*H* transition in W7-AS might reflect a modification in the nonlinear energy transfer between fluctuations and $\mathbf{E} \times \mathbf{B}$ flows. Indeed, the simultaneous increase in bicoherence at the transition, along with the peaking of the profile of integrated bicoherence in the shear layer, together are tantalizingly suggestive of an

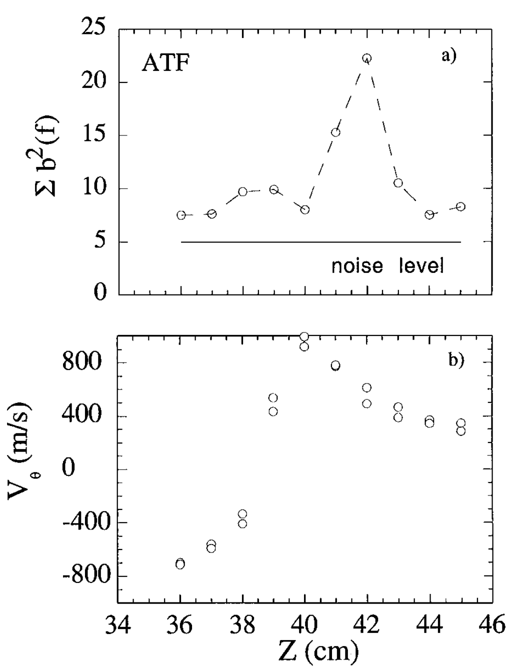

FIG. 1. (a) Radial profile of integrated bicoherence in ATF shear layer; and (b) fluctuation phase velocity profile in shear layer.

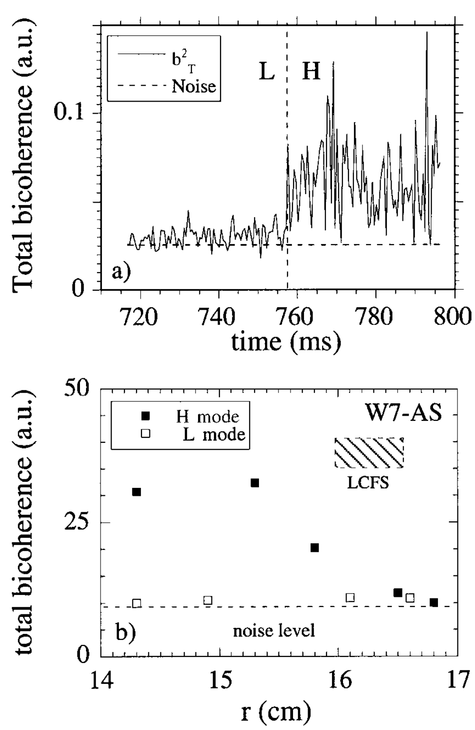

FIG. 2. (a) Evolution in integrated bicoherence during $L \rightarrow H$ transition; (b) comparison of total bicoherence in *L* and *H* modes in W7-AS.

$L \rightarrow H$ transition process which is initiated by the mode coupling-induced generation or enhancement of zonal flows in the plasma edge. This mechanism may also play an important role in explaining the change in the radial electric field during the initial phase of the L-H transition in some tokamaks, where the radial electric field is dominated by the poloidal rotation [16].

However, it is important to stress that there are other possible explanations of the modification of the bicoherence near $\mathbf{E} \times \mathbf{B}$ sheared flows in the plasma boundary region. Because of the complex interpretation of reflectometry measurements [17–19], the modification of the bicoherence may reflect a change in the size and amplitude of the turbulence structure, as compared with the antenna pattern. The phase response depends on turbulence characteristics (amplitude and wavelengths), plasma profiles, and diagnostic geometry. This is specially clear when the diagnostic axis is not exactly parallel to the plasma density gradient as in the case of Wendelstein 7-AS and many other machines. In this situation, the phase signal shows a drift (usually referred to as phase runaway [19]), that disappears only for very low turbulence levels, and phase fluctuations would have a sawtoothlike shape. With this basis, the modification in the bicoherence of the phase signal observed at the L-H transition may have been caused by the modification in the nonlinear coupling of fluctuations and/or by the variation of the amplitude and spatial scales of plasma turbulence.

The relative role of these mechanisms should be clarified by the investigation of the nonlinear coupling between radial and poloidal velocity (high-frequency) fluctuations and low-frequency fluctuations in the plasma potential. The comparison of measurements from different diagnostics (i.e., probes, reflectometry, spectroscopy) would also be very useful. Detailed results from such comprehensive investigations will be reported in future publications.

P. H. Diamond would like to thank Z. Lin, T. S. Hahm, F. L. Hinton, and A. Smolyakov for interesting discussions. This research was supported by U.S. Department of Energy Grant No. DE-FG03-88ER53275 and by DGICYT (Dirección General de Investigaciones Científicas y Técnicas) of Spain under Project No. PB96-0112-C02-02.

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