Naturally Large Cosmological Neutrino Asymmetries in the Minimal Supersymmetric Standard Model

John McDonald*
Department of Physics and Astronomy, University of Glasgow, Glasgow G12 8QQ, Scotland
(Received 27 August 1999)

A large neutrino asymmetry could have significant observable consequences for nucleosynthesis and the cosmic microwave background. If the baryon asymmetry originates via the Affleck-Dine mechanism along a $d = 4$ flat direction of the scalar potential in the minimal supersymmetric standard model and if the lepton asymmetry originates via Affleck-Dine leptogenesis along a $d = 6$ direction, corresponding to the lowest dimension directions conserving R parity, then the ratio n_L/n_B is naturally in the range 10^8–10^9. As a result, a potentially observable neutrino asymmetry is correlated with a baryon asymmetry of the order of 10^{-10}.

PACS numbers: 98.80.Cq, 12.60.Jv

Introduction.—It has long been known that there could exist a large cosmological neutrino asymmetry ("degeneracy") [1,2]. This has recently become of particular interest [3,4] due to its effects on the cosmic microwave background (CMB), which will be observed in detail by the MAP and PLANCK satellites. In addition, a large neutrino asymmetry can also affect big bang nucleosynthesis (BBN) [2] and large scale structure (LSS) formation [1]. Present CMB, nucleosynthesis, and LSS bounds can already exclude a range of neutrino asymmetries.

A number of suggestions for the origin of a large asymmetry have been made [5,6]. In the context of supersymmetric (SUSY) models, the most natural possibility is probably the Affleck-Dine (AD) mechanism [7]. However, although it is possible, *in principle*, to account for a large neutrino asymmetry, there is no particular reason to expect a very large asymmetry (or, indeed, a large asymmetry which is nevertheless small enough to be compatible with present observational upper bounds). Recently, an interesting model has been proposed by Casas, Cheng, and Gelmini (CCG) [6], which is based on an Affleck-Dine mechanism in an extension of the minimal SUSY standard model (MSSM) involving right-handed sneutrinos, and which can account for a large lepton asymmetry. In this Letter we wish to show that there is good reason to expect a large (but not too large) neutrino asymmetry from the Affleck-Dine mechanism in the context of the MSSM itself. Our main point is that typically a number of scalar fields will have expectation values along flat directions of the MSSM scalar potential at the end of inflation. If the baryon asymmetry and lepton asymmetry originate from the AD mechanism along different flat directions, then the ratio of the baryon number to the lepton number will be simply determined by the dimension of the nonrenormalizable terms responsible for lifting the flat directions. For the R-parity conserving models on which we concentrate (which both eliminate dangerous renormalizable B and L violating terms from the MSSM superpotential and also allow for neutralino dark matter [8]), the dimension of the nonrenormalizable terms is even and so the lowest dimension flat directions have $d = 4$ and $d = 6$ [9]. If the observed B asymmetry originates along a $d = 4$ direction, then the reheating temperature is fixed to be about 10^8 GeV. In this case, if the L asymmetry originates along a $d = 6$ direction, the ratio of the B to L asymmetry will be 10^8–10^9. As a result, a B asymmetry of about 10^{-10} will naturally result in an L asymmetry in the range 0.01–0.1. This mechanism requires no unusually large flat direction vacuum expectation values (VEVs); it is simply the conventional AD mechanism taking into account the likelihood that more than one flat direction scalar field will have an expectation value at the end of inflation.

Lepton asymmetry and present limits.—A large neutrino asymmetry has a number of effects on cosmology [1,2]. It changes the neutrino decoupling temperature, the primordial production of light elements during BBN, the time of matter-radiation equality, the contribution of relic neutrinos to the present energy density of the Universe, and alters LSS formation and the CMB. The neutrino asymmetry is usually characterized by the neutrino degeneracy parameter $\xi_\nu = \mu/T_\nu$, where μ is the neutrino chemical potential and T_ν is the neutrino temperature. $T_\nu = y_\nu T_\gamma$, where T_γ is the present photon temperature. $y_\nu = (4/11)^{1/3}$ in the absence of a neutrino asymmetry and is smaller in the presence of a neutrino asymmetry, as the neutrinos decouple at a higher temperature [2,4]. The neutrino-to-entropy ratio is related to the degeneracy parameter by (for $(\xi_\nu/\pi)^2 \ll 1$)

$$\eta_L = \frac{15}{4\pi^4 g(T_\gamma)}\, y_\nu^3(\pi^2\xi_\nu + \xi_\nu^3), \tag{1}$$

where $g(T)$ is the number of light degrees of freedom in thermal equilibrium [$g(T_\gamma) = 2$]. BBN imposes a constraint on ξ_{ν_e} [2],

$$-0.06 \lesssim \xi_{\nu_e} \lesssim 1.1\,,$$
$$(-4 \times 10^{-3} \lesssim \eta_{L_e} \lesssim 9 \times 10^{-2})\,. \tag{2}$$

The upper limit assumes a large asymmetry for ν_μ or ν_τ. In the absence of such an asymmetry, the upper bound becomes $\xi_{\nu_e} \lesssim 0.14$ ($\eta_{L_e} \lesssim 9.6 \times 10^{-3}$) [2]. A more

 0031-9007/00/84(21)/4798(4)$15.00

recent analysis tightens this to $\xi_{\nu_e} \lesssim 0.09$ ($\eta_{L_e} \lesssim 6.2 \times 10^{-3}$) [10]. LSS imposes the bound, from the requirement of a sufficiently long matter-dominated epoch,

$$|\xi_{\nu_\mu,\nu_\tau}| \lesssim 6.9, \qquad (|\eta_L| \lesssim 2.8). \tag{3}$$

A danger for any mechanism generating a large lepton asymmetry is that anomalous $B + L$ violating processes acting on the thermalized lepton number could generate a baryon asymmetry of a similar order of magnitude [11]. This is suppressed if the lepton number is large enough to prevent electroweak symmetry restoration [12], such that the sphalerons gain a mass much larger than T [11,13]. This possibility is also important as a way to eliminate dangerous topological defects such as domain walls or monopoles [13]. The most recent estimate for the case of the three generation MSSM is that $SU(2)_L \times U(1)_Y$ nonrestoration occurs if the lepton asymmetry is larger than $n_L^c = 0.72T^3$ at T_{ew} [14]. This translates into a lepton-to-entropy ratio

$$\eta_L^c = \frac{45 n_L^c}{2\pi^2 g(T) T^3} = 0.016, \tag{4}$$

where we have used $g(T_{ew}) \approx 100$. Noting that $y_\nu \lesssim (4/11)^{1/3}$, this imposes a lower bound on ξ_{ν_i} ($i = e, \mu, \tau$),

$$\Sigma_i \xi_{\nu_i} \gtrsim 0.23, \tag{5}$$

assuming entropy conservation throughout [15]. We see that, with $\xi_{\nu_{\mu,\tau}}$ of the same order of magnitude of ξ_{ν_e}, this can be well within the range of ξ_{ν_e} allowed by BBN. This is very important, as in general we would expect the ν_e asymmetry to be of the same magnitude as that of $\nu_{\mu,\tau}$. Therefore compatibility of the BBN constraint with the symmetry nonrestoration constraint is essential for the existence of a model which can naturally generate a large L asymmetry. (We also note that, independent of the details of the model, the lepton asymmetry can only exist in two ranges if it is to be compatible with the observed baryon asymmetry, anomalous $B + L$ violating processes, and LSS: $\eta_L \lesssim 10^{-10}$ and $0.016 \lesssim \eta_L \lesssim 2.8$.) Future observations by the MAP and PLANCK satellites are expected to probe the lepton asymmetry down to $\xi_\nu \approx 0.5$ and $\xi_\nu \approx 0.25$ ($\eta_L \approx 0.035$ and $\eta_L \approx 0.017$), respectively [3].

Flat directions and the Affleck-Dine mechanism.—The scalar potential along a flat direction during inflation has the form [16]

$$V(\Phi) \approx (m^2 - cH^2)|\Phi|^2 + \frac{\lambda^2 |\Phi|^{2(d-1)}}{M_*^{2(d-3)}} + \left(\frac{A_\lambda \lambda \Phi^d}{d M_*^{d-3}} + \text{H.c.}\right), \tag{6}$$

where H is the expansion rate of the Universe, m is the usual gravity-mediated SUSY breaking scalar mass term ($m \approx 100$ GeV), cH^2 is the order H^2 correction to the scalar mass due to the energy density of the early Universe [17] (with c positive for Affleck-Dine baryogenesis and typically of order one), $A_\lambda = A_{\lambda o} + a_\lambda H$ (where $A_{\lambda o}$ is the conventional gravity-mediated SUSY breaking term), and the natural scale of the nonrenormalizable terms is M_*, where $M_* = M_{Pl}/\sqrt{8\pi}$ is the supergravity (SUGRA) mass scale. The baryon asymmetry forms when the Affleck-Dine scalar begins to oscillate coherently about zero, which happens at $H \approx m$.

The field Φ is a linear combination of squark, slepton, and Higgs fields such that the F- and D-term contributions to the renormalizable SUSY scalar potential vanish. The flat directions are characterized by the lowest dimension scalar operators which have nonzero VEV along the flat direction; these also correspond to the nonrenormalizable superpotential terms responsible for lifting the flat directions and supplying the CP violation responsible for generating the asymmetry. The possible R-parity conserving $d = 4$ and $d = 6$ operators are given in Table 2 of Ref. [9]. The magnitude of the asymmetry generated once the AD field begins to oscillate coherently is then

$$n \approx m\phi^2 \sin\delta_{CP}, \tag{7}$$

where $\phi/\sqrt{2}$ is the amplitude of the AD field when it begins to coherently oscillate at $H \approx m/c^{1/2}$, m is the mass of the AD scalar, and δ_{CP} is the CP violating phase responsible for generating the asymmetry. δ_{CP} can originate in one of two ways. If the A terms have order H corrections, the phase corresponds to the phase difference between a_λ and $A_{\lambda o}$. This is expected in F-term inflation models. In minimal D-term inflation models there are no order H corrections to the A terms [18]. In this case, the phase is essentially the random initial phase of the AD scalar relative to the A terms. In both cases, the most natural possibility is that $\delta_{CP} \approx 1$. We will assume this throughout. The initial value of the AD scalar field is given by

$$\phi \approx \left(\frac{2^{d-2}}{(d-1)\lambda^2}\right)^{1/(2d-4)} (m^2 M_*^{2(d-3)})^{1/(2d-4)}. \tag{8}$$

The present charge-to-entropy ratio is then

$$\eta_Q = \frac{2\pi n T_R}{H^2 M_{Pl}^2} \equiv \frac{T_R}{2} \frac{c \sin\delta_{CP}}{\lambda^{2/(d-2)}} \frac{m^{(4-d)/(d-2)} M_*^{-2/(d-2)}}{(d-1)^{1/(d-2)}}. \tag{9}$$

The asymmetries for the $d = 4$, $d = 6$, and $d = 8$ directions are then given by

$$\eta_4 \approx 7 \times 10^{-11} c_4 \left(\frac{T_R}{10^8 \text{ GeV}}\right)\left(\frac{1}{3!\,\lambda_4}\right) \sin\delta_{CP4}, \tag{10}$$

$$\eta_6 \approx 2 \times 10^{-2} c_6 \left(\frac{T_R}{10^8 \text{ GeV}}\right)\left(\frac{1}{5!\,\lambda_6}\right)^{1/2} \times \left(\frac{100 \text{ GeV}}{m_6}\right)^{1/2} \sin\delta_{CP6}, \tag{11}$$

$$\eta_8 \approx 22 c_8 \left(\frac{T_R}{10^8 \text{ GeV}} \right) \left(\frac{1}{7! \, \lambda_8} \right)^{1/3} \times \left(\frac{100 \text{ GeV}}{m_8} \right)^{2/3} \sin\delta_{CP8} \,, \quad (12)$$

where we have used as a typical value of the nonrenormalizable self-coupling of the AD field $\lambda_d \approx 1/(d-1)!$, such that the strength of the physical Φ interaction is determined purely by the mass scale M_* [18]. Thus

$$\frac{\eta_6}{\eta_4} \approx 3.4 \times 10^8 \left(\frac{3! \, \lambda_4}{(5! \, \lambda_6)^{1/2}} \right) \left(\frac{c_6}{c_4} \right) \times \left(\frac{100 \text{ GeV}}{m_6} \right)^{1/2} \frac{\sin\delta_{CP6}}{\sin\delta_{CP4}} \equiv 3.4 \times 10^8 f_6 \,, \quad (13)$$

where f_6 is typically of the order of 1. Similarly, $\eta_8/\eta_4 \approx 3.0 \times 10^{11} f_8$.

From these we see that, firstly, if the observed baryon asymmetry ($\eta_{B\,\mathrm{obs}} \approx (3\text{–}8) \times 10^{-11}$ [19]) comes from a $d = 4$ direction, then the parameters must be close to their maximal or upper bound values; T_R must be close to the thermal gravitino upper bound $\sim 10^8$ GeV [19–21], while δ_{CP4} must be close to 1. Secondly, assuming the observed asymmetry is due to $d = 4$ baryogenesis, the asymmetry from a $d = 6$ direction is given by

$$\eta_6 = (1\text{–}3) \times 10^{-2} f_6 \,. \quad (14)$$

Thus an asymmetry of 0.01–0.1 is expected in this case. Therefore if the L asymmetry originates from a $d = 6$ AD mechanism and the B asymmetry from a $d = 4$ AD mechanism, a large L asymmetry will exist today, which is naturally within the range of η_L permitted by nucleosynthesis. $SU(2)_L \times U(1)_Y$ symmetry nonrestoration is also a natural feature, and the expected range of values is potentially observable by MAP and PLANCK. On the other hand, if the L asymmetry came from a $d = 8$ flat direction, the asymmetry would be

$$\eta_8 = (9\text{–}25) f_8 \,. \quad (15)$$

Although this can be compatible with the LSS upper bound, $|\eta_L| \lesssim 2.8$, it would not naturally be compatible with the BBN upper bound on the ν_e asymmetry, $\eta_{L_e} \lesssim 0.006$. Thus $d = 6$ AD leptogenesis is favored.

Affleck-Dine cosmology along multiple flat directions.—Usually, the AD mechanism is studied for the case of a single flat direction. However, it is likely that more than one flat direction scalar will have a negative order H^2 correction to its mass squared term. As a result, we can expect several flat direction fields to be nonzero at the end of inflation and to begin to oscillate coherently once $H \lesssim m$.

The directions which can be simultaneously flat are those characterized by operators which do not share any field in common. This can be seen by considering the D-term contribution to the scalar potential [8],

$$V_D = \frac{g_\alpha^2}{2} |\Phi_i T_\alpha \Phi_i|^2, \quad (16)$$

where α is the gauge group with generators T^α and Φ_i are the MSSM scalar fields. For flat directions to be independent, the expectation values of the MSSM fields which form the flat direction must cancel independently in the D term. This is possible only if they do not have any field in common, since otherwise they would have to be varied simultaneously to keep the D term zero, implying a single AD field with a single expectation value. Thus, as long as the squark and slepton fields have different gauge indices or are orthogonal in generation space, the corresponding flat directions can simultaneously have nonzero values at the end of inflation. Additional constraints arise from the requirement of vanishing renormalizable F-term contributions to the scalar potential. This requires that no more than one field in each trilinear term in the MSSM superpotential gains an expectation value.

As an explicit example, consider the case where the B asymmetry comes from the AD mechanism along the $d = 4$ $u^c u^c d^c e^c$ and $QQQL$ directions, and the L asymmetry comes from the $d = 6$ $(e^c LL)^2$ and $(d^c QL)^2$ directions. Suppose that *all* scalar fields have negative order H^2 corrections to their mass terms. Suppose also that the reheating temperature is $T_R \approx 10^8$ GeV. We then cannot allow a B violating AD scalar to be nonzero along a flat direction with $d > 4$, since it would result in a too large B asymmetry. Most combinations of "orthogonal" flat directions of the MSSM scalar potential will have such $d > 4$ flat directions. However, on scales larger than the horizon during inflation we can expect domains with different combinations of flat directions to exist, so that anthropic selection will imply that we live in a domain with only $d = 4$ B violating flat directions. To prevent a B violating $d = 6$ flat direction, all of the $9u^c$, $9d^c$, $3e^c$, $6L$, and $18Q$ fields (including color, $SU(2)_L$ and generation indices) must be employed in flat directions in such a way that no $d > 4$ AD baryogenesis occurs. For example, the D term allows the following set of 12 flat direction scalars

$$7(u^c d^c QQ) + u^c u^c d^c e^c + QQQL + 2(e^c LL)^2 + (d^c QL)^2, \quad (17)$$

where each flat direction is implicitly characterized by a different combination of gauge and generation indices. (The $d = 6$ terms in brackets should be thought of as $d = 3$ terms squared, so that they only involve three fields each.) This set of flat directions exhausts all of the MSSM squark and slepton fields.

This shows that it is possible to have a complete set of flat directions with no $d \geq 6$ B violating directions. However, not all of these D-flat directions can be consistent with F-flatness conditions, so it is necessary to check that a complete set of D- *and* F-flat directions with no $d \geq 6$ B violating directions exists. An example is given by

$$u_2^{c\,1} u_3^{c\,2} d_3^{c\,3} e_3^c + (d_2^{c\,3} u_1^3 l_2)^2, \tag{18}$$

corresponding to a $d = 4$ B violating direction from a gauge-invariant monomial of the form [22] $u^c u^c d^c e^c$ and a $d = 6$ L violating direction from $(d^c QL)^2$ (where superscripts denote color indices and subscripts denote flavor indices). Choosing a flavor basis with the down quark and lepton Yukawa matrices diagonalized, the squark fields which can form flat directions in addition to those in (18) once F-flatness conditions are imposed are of the form $u^{c\,i}$, $d^{c\,i}$ $(i = 1, 2)$, and Q^3. Since there is no gauge-invariant monomial which can be constructed using these fields, there are no other B violating flat directions.

Once the B and L asymmetries are established at $H \approx m$, the Universe is dominated by coherently oscillating scalar field condensates, corresponding to the inflaton and the various AD scalars. One difference between the MSSM AD mechanism presented here and that of CCG based on right-handed sneutrinos is that the AD condensate in our case carries an asymmetry of left-handed sneutrinos and so serves as a source term for finite-density $SU(2)_L \times U(1)_Y$ breaking [12,15]. Thus $SU(2)_L \times U(1)_Y$ nonrestoration occurs throughout. In the CCG model, $SU(2)_L \times U(1)_Y$ breaking occurs only once the right-handed sneutrino condensate decays to create a density of left-handed neutrinos. This must occur before the inflaton has thermalized, otherwise sphaleron processes would convert the lepton asymmetry from the decaying sneutrino field into a large baryon asymmetry. This leads to additional constraints on the model [6]. In our model there are no constraints from inflaton thermalization. In addition, in the CCG model an observably large asymmetry can be achieved only if the right-handed sneutrino field expectation value at the end of inflation is very large, $\tilde{\nu}_R \gtrsim 10^{17}$ GeV [6]. This is difficult to achieve, since it requires a heavy suppression of nonrenormalizable operators along the flat direction. In our case a large neutrino asymmetry is generated via natural expectation values for the AD scalars after inflation.

Conclusions.—We have discussed the possibility of the AD mechanism being responsible for both baryogenesis and leptogenesis, with the B asymmetry originating along a $d = 4$ direction of the MSSM scalar potential and the L asymmetry along a $d = 6$ direction. This naturally correlates a lepton asymmetry $\eta_L \approx 0.01-0.1$ with the observed baryon asymmetry $\eta_B \approx 10^{-10}$. This is compatible with present nucleosynthesis and structure formation constraints and is in the range detectable by the MAP and PLANCK satellites in the future. It also implies finite-density $SU(2)_L \times U(1)_Y$ nonrestoration, which is essential for the consistency of the model. The model is dependent upon anthropic selection to eliminate dangerous B violating flat directions; however, a domain structure of the Universe, with different combinations of flat directions on scales much larger than the present horizon, will be a natural feature of the MSSM in the context of inflation models. Therefore it is quite natural that we find ourselves in a domain determined by anthropic selection.

The fact that a large lepton asymmetry is a very natural feature of the Affleck-Dine mechanism in the MSSM, in which several flat direction scalar condensates typically form after inflation, should be a source of encouragement to those interested in the cosmological effects of a large neutrino asymmetry.

This research has been supported by the PPARC.

*mcdonald@physics.gla.ac.uk

[1] K. Freese, E. W. Kolb, and M. S. Turner, Phys. Rev. D **27**, 1689 (1983).
[2] H. Kang and G. Steigman, Nucl. Phys. **B372**, 494 (1992).
[3] W. H. Kinney and A. Riotto, Phys. Rev. Lett. **83**, 3366 (1999).
[4] J. Lesgourgues and S. Pastor, Phys. Rev. D **60**, 103521 (1999).
[5] R. Foot, M. J. Thomson and R. R. Volkas, Phys. Rev. D **53**, 5349 (1996).
[6] A. Casas, W. Y. Cheng, and G. Gelmini, Nucl. Phys. **B538**, 297 (1999).
[7] I. A. Affleck and M. Dine, Nucl. Phys. **B249**, 361 (1985).
[8] H. P. Nilles, Phys. Rep. **110**, 1 (1984); G. Jungman, M. Kamionkowski, and K. Greist, Phys. Rep. **267**, 195 (1996).
[9] M. Dine, L. Randall, and S. Thomas, Nucl. Phys. **B458**, 291 (1996).
[10] K. Kohri, M. Kawasaki, and K. Sato, Astrophys. J. **490**, 72 (1997).
[11] J. Liu and G. Segre, Phys. Lett. B **358**, 259 (1994).
[12] A. D. Linde, Phys. Rev. D **14**, 3345 (1976); A. D. Linde, Rep. Prog. Phys. **42**, 389 (1979).
[13] A. Riotto and G. Senjanovic, Phys. Rev. Lett. **79**, 349 (1997); B. Bajc, A. Riotto, and G. Senjanovic, Phys. Rev. Lett. **81**, 1355 (1998); G. Senjanovic, hep-ph/9805361.
[14] B. Bajc and G. Senjanovic, Phys. Lett. B **472**, 373 (2000).
[15] J. McDonald, Phys. Lett. B **463**, 225 (1999).
[16] J. McDonald, hep-ph/9901453.
[17] M. Dine, W. Fischler, and D. Nemeschansky, Phys. Lett. **136B**, 169 (1984); G. D. Coughlan, R. Holman, P. Ramond, and G. G. Ross, Phys. Lett. **140B**, 44 (1984); E. Copeland, A. Liddle, D. Lyth, E. Stewart, and D. Wands, Phys. Rev. D **49**, 6410 (1994); E. D. Stewart, Phys. Rev. D **51**, 6847 (1995); M. Dine, L. Randall, and S. Thomas, Phys. Rev. Lett. **75**, 398 (1995); G. Dvali, hep-ph/9503259.
[18] C. Kolda and J. March-Russell, hep-ph/9802358.
[19] S. Sarkar, Rep. Prog. Phys. **59**, 1493 (1996).
[20] J. Ellis, J. E. Kim, and D. V. Nanopoulos, Phys. Lett. **145B**, 181 (1984).
[21] Recently it has been suggested that nonthermal production of gravitinos at the end of inflation can impose a much tighter upper bound on T_R, depending on the details of the inflation model [G. F. Giudice, I. Tkachev, and A. Riotto, J. High Energy Phys. **08**, 009 (1999)].
[22] The $d = 4$ B violating operator responsible for lifting this flat direction need not be additionally suppressed in order to avoid rapid proton decay, as the operator is composed purely of second- and third-generation fields [23].
[23] L. E. Ibanez and G. G. Ross, Nucl. Phys. **B368**, 3 (1992); T. Goto and T. Nihei, Phys. Rev. D **59**, 115009 (1999); K. Benakli and S. Davidson, Phys. Rev. D **60**, 025004 (1999).