Thermal Effects on the Casimir Force in the 0.1–5 μm Range

M. Boström and Bo E. Sernelius

Department of Physics and Measurement Technology, Linköping University, S-581 83 Linköping, Sweden
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The vacuum stresses between a metal half-space and a metal sphere were recently measured at room temperature, in the 0.6–6 μm range, with an estimated accuracy of 5%. In the interpretation it was assumed that the accuracy was not good enough for observing any thermal effects. We claim that thermal effects are important in this separation range and back up this claim with numerical calculations of the Casimir force at zero temperature and at 300 K, based on tabulated optical data of gold, copper, and aluminum. The effects of dissipation and temperature are investigated and we demonstrate the importance of considering these two corrections together.

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When two objects are brought together the mutual electric polarizations of the materials result in an attractive force. At short distances, this is the van der Waals force; at large distances, retardation becomes important and the result is the Casimir force. For idealized perfect conductors there is no van der Waals region. Casimir already predicted the attraction between a pair of parallel, closely spaced, perfect conductors a distance d apart in 1948 [1]. He found for zero temperature the interaction energy (per unit area): $F(d) = -\pi^2\hbar c/720d^3$. Casimir and Polder [2] predicted a similar attraction between an atom and a metal half-space. Sukenik *et al.* [3] measured the Casimir-Polder force between ground state sodium atoms and gold mirrors. They treated the mirrors as perfect conductors and found excellent agreement with 0 K theory. We recently investigated the retarded free energy of attraction between atoms and thin metal films [4]. In the case of hydrogen atoms separated 3 μm from thin silver films, thermal corrections were found to be less than 5% at 300 K. The good agreement with the zero temperature result is in part due to cancellations of different thermal corrections. Whereas the transverse magnetic energy contribution increases with temperature, the opposite is true for the transverse electric energy contribution. Lamoreaux [5] performed the first high accuracy measurement of the Casimir force between macroscopic objects. The measured force in the 0.6–6 μm separation range between a gold-coated half-space and a gold-coated sphere was found to be in good agreement with theoretical zero temperature results. The estimated level of accuracy in this experiment was 5%. Measurements with even higher level of accuracy have since been performed [6,7]. At the smallest separations (0.1 μm) a 1% level of accuracy has been claimed. These high precision measurements are very important for the verification of the theoretically predicted Casimir force. These experiments can also be used to produce constraints for hypothetical interactions predicted by unified gauge theories [8]. In the interpretation of the experimental results, discussed above, thermal corrections have been assumed small. There are several corrections to the ideal zero temperature Casimir force, e.g., finite temperature, finite conductivity, surface roughness, and curvature. Traditionally the different corrections to the Casimir force have been treated separately. Recently Klimchitskaya *et al.* [9] investigated surface roughness and conductivity corrections and found that they have to be considered together. In the experimental setups the force between a sphere and a half-space was measured. According to the proximity force theorem [10] this force can be approximated with $2\pi R \times F(d)$, where R is the radius of the sphere, d is the closest separation, and F is the interaction energy (per unit area) between two half-spaces separated a distance d. This expression is a valid approximation if R is much larger than d. In this paper we intend to show that the finite conductivity has to be accounted for in order to find the correct temperature-dependent Casimir force. The inclusion of dissipation in the dielectric function of the metal will strongly influence the transverse electric part of the attraction. In particular when dissipation is accounted for this part of the interaction decreases with temperature, in contrast to the increase of the part of the interaction that originates from the transverse magnetic modes. The temperature dependence of real and ideal metals is very different. We will compare our theoretical results with the experimental result of Lamoreaux. The result will be discussed but clear-cut conclusions cannot be drawn from these data. We argue that further theoretical and experimental investigations of the temperature-dependent Casimir force are of great importance. We will also consider thermal effects on aluminum surfaces in the 0.1–2 μm separation range. We intend to show that thermal corrections already become important in this separation range. Since high accuracy measurements have been performed in agreement with zero temperature results at these separations, there is a disagreement between theory and experiment that needs to be resolved.

Lifshitz and co-workers [11,12] obtained an expression for the van der Waals force between two half-spaces that involved integration along the real frequency axis. By using analytical continuation, the integration path was

shifted to the imaginary frequency axis. Since the integrand has an infinite number of poles on the imaginary frequency axis, at $\omega_n = 2\pi k_B T n/\hbar$, the final result involves a summation over discrete frequencies. The free energy between two half-spaces, which can be obtained by an integration of the force, can, after a simple variable substitution, be rewritten in the following way:

$$F(d) = \frac{k_B T}{2\pi} {\sum_{n=0}^{\infty}}' \int_0^\infty dq\, q \{\ln[G^{\mathrm{TE}}(q, i\omega_n)] + \ln[G^{\mathrm{TM}}(q, i\omega_n)]\}, \quad (1)$$

$$G^{\mathrm{TE}} = 1 - \left(\frac{\gamma_1 - \gamma_0}{\gamma_1 + \gamma_0}\right)^2 e^{-2\gamma_0 d}, \quad (2)$$

$$G^{\mathrm{TM}} = 1 - \left(\frac{\epsilon_1\gamma_0 - \epsilon_0\gamma_1}{\epsilon_1\gamma_0 + \epsilon_0\gamma_1}\right)^2 e^{-2\gamma_0 d}, \quad (3)$$

$$\gamma_i = \sqrt{q^2 + \epsilon_i \omega^2/c^2}, \quad (4)$$

where the indices 0 and 1 represent the medium between the half-spaces and the half-spaces themselves, respectively. The energy has contributions from both the transverse electric (TE) and transverse magnetic (TM) modes. The prime on the frequency summation indicates that the $n = 0$ contribution should be multiplied with $1/2$. The $n = 0$ part of the TE contribution should be considered with some care:

$$F(d)^{\mathrm{TE}}_{n=0} = \lim_{\omega\to 0} \frac{k_B T}{4\pi} \int_0^\infty dq\, q \times \ln\left[1 - \left(\frac{\gamma_1 - \gamma_0}{\gamma_1 + \gamma_0}\right)^2\right] e^{-2\gamma_0 d}. \quad (5)$$

For a perfect conductor the dielectric function should approach infinity before the frequency is allowed to go to zero. In this way Schwinger *et al.* [13] found a contribution from this part to the large separation asymptote. For a real metal the dielectric function at small momenta and low frequencies can be modeled with the simple Drude expression:

$$\epsilon(i\omega) \approx 1 + \omega_p^2/[\omega(\omega + \eta)], \quad (6)$$

where the presence of η is the result of dissipation. It is related to the finite conductivity of a metal and cannot be neglected. A real metal does not obtain any contribution from the $n = 0$ part of the TE modes since the dielectric function approaches infinity too slowly as ω goes to zero. Although this work presents calculations performed on the imaginary frequency axis, we would like to mention that we have also performed calculations on the real frequency axis. As expected the total result is the same regardless of the integration path. In the real frequency integration, thermal energy corrections originating from above and below the light line $\omega = cq$ may have different signs. The interaction between perfectly reflecting surfaces, which only receive contributions from above the light line, always increases with temperature. A real metal with dissipation also receives contributions from below the light line. The part of the free energy that originates from TE modes is interesting. We have found that the ratio between this quantity evaluated at 300 and 0 K to a good approximation decreases linearly with separation up to roughly 1 μm. For gold surfaces it is roughly $(0.9838 - 0.4337 \times d[\mu\mathrm{m}])$. Beyond 1 μm it decreases slower towards zero; there are also minor deviations from the linear decrease at the shortest separations. Thus, the common use of finite temperature results obtained from perfect conductors as comparison with experiment [5,6] is obviously not meaningful.

The imaginary part of the dielectric functions, ϵ_2, is obtained from tabulated optical data and the dielectric function on the imaginary axis, $\epsilon(i\omega)$, is obtained with the use of a Kramers-Kronig relation. For general separations one needs the full momentum dependence of the dielectric functions, which cannot be deduced from the optical data. For large separations only the small momentum range contributes, and in the separation range of interest here we do not need the momentum dependence. Lamoreaux [14] recently used optical data to calculate the zero temperature Casimir force between real metal half-spaces. A few errors have been pointed out [15,16] in the way interpolations and extrapolations were performed in Ref. [14]. The dielectric properties of copper, gold, and aluminum were taken from Ref. [17]. At high energies there are enough values available as input to the calculation of the dielectric response at imaginary frequencies but for small energies one has to resort to extrapolations. Lamoreaux used the optical data from Ref. [17] and came to the conclusion that one could use the asymptotic $1/\omega$ dependence of the imaginary part of the polarizability to extrapolate from the lowest experimental value. We find that this simple extrapolation is not valid for the metals that we are considering. The lowest energy data is not on the low energy asymptote. We use a slightly different extrapolation scheme in the low frequency range:

$$\epsilon_2(\omega) \approx \frac{4\pi}{\omega} \frac{\rho}{\rho^2 + (4\pi\omega/\eta^2)^2}. \quad (7)$$

This part of the dielectric function describes intraband transitions. It has a low frequency asymptote of the form $1/\omega$ and a high frequency asymptote of the form $1/\omega^3$. With $\eta = \omega_p$ this is the Drude expression with dissipation, which in the low energy limit gives the correct static resistivity (ρ). We use it in a slightly modified form and let η be an adjustable parameter chosen in such a way that we not only get the correct low energy limit but also produce good agreement with experimental data in an overlap region. The experimental static resistivities [18] of gold, copper, and aluminum are 2.611×10^{-18}, 1.855×10^{-18}, and 2.944×10^{-18} s, respectively. The corresponding values of η are chosen to be 1.245×10^{16}, 2.025×10^{16}, and 1.732×10^{16} s^{-1}, respectively. We will refer to these

dielectric functions as Au1, Cu1, and Al1. For copper, additional data from Ref. [19] reach into the region in which the imaginary part follows the $1/\omega$ asymptote. The transition region can be described well if we chose a resistivity of 1.053×10^{-17} s, and η is chosen to be 1.432×10^{16} s^{-1}. We refer to the function using this extrapolation as Cu2. The energy correction factor is defined as the ratio of the interaction energy of the metals at finite temperature to the corresponding energy of perfect conductors, at 0 K. The energy correction factor at 0 K and 0.5 μm for the models Au1, Cu1, Cu2, and Al1 are 0.83, 0.84, 0.82, and 0.87, respectively; at 300 K they are 0.74 for gold and copper and 0.79 for aluminum. Lambrecht and Reynaud [16] evaluated these quantities at zero temperature using a slightly different extrapolation procedure. For gold, copper, and aluminum they found the energy ratios 0.85, 0.85, and 0.88. If optical data from different samples are used the differences become even larger.

Lamoreaux [5] measured the force between a sphere and a half-space. The major contribution to this force was an electric force that vanished as $1/d$. The data were fitted to the best $1/d$ dependence and that part of the interaction was subtracted off. In this way the coefficients α and β were determined for a force law in Ref. [5], $F(r) = \alpha/r^{\beta}$, with an estimated accuracy of 5%. This force law agreed rather well with the 0 K Casimir force between gold surfaces. Each data point, on the other hand, does not have an accuracy of 5%. When Lamoreaux kindly supplied us with the experimental data points he stressed that one should be careful to conclude anything from individual data points. We still consider it useful to compare these data points with theoretical calculations. The energy correction factor for gold surfaces will be investigated. In Fig. 1 the measured energy ratio is compared with four calculated energy ratios: the energy between Au surfaces evaluated at 0, 300, and 300 K with the static transverse electric part incorrectly treated as in the perfect conductor case, and finally the energy between perfect conductors evaluated at 300 K. The result would be more or less unchanged were the gold surfaces replaced with copper surfaces. The experimental data points between 0.63–1.26 μm agree very well with the result for gold surfaces at 300 K when the static transverse electric part is treated as in the perfect conductor case. This theoretical treatment roughly corresponds to totally neglecting any effects of dissipation on the dielectric function in the low frequency limit. In this separation range the relative difference between this theoretical and the experimental results is between 1% and 6%. The correct treatment gives a corresponding difference exceeding 17%. The experimental data points at 1.74 and 2.46 μm, on the other hand, agree better with the correct result for gold surfaces at 300 K. One should remember to be careful in the interpretation of the experimental data points. The data points at larger separations become less sensitive to material and surface properties. However, the highest accuracy is found in the lower part of the measuring range. Further experimental and theoretical investigations are required to determine the correct temperature dependence of the Casimir force between imperfect conductors.

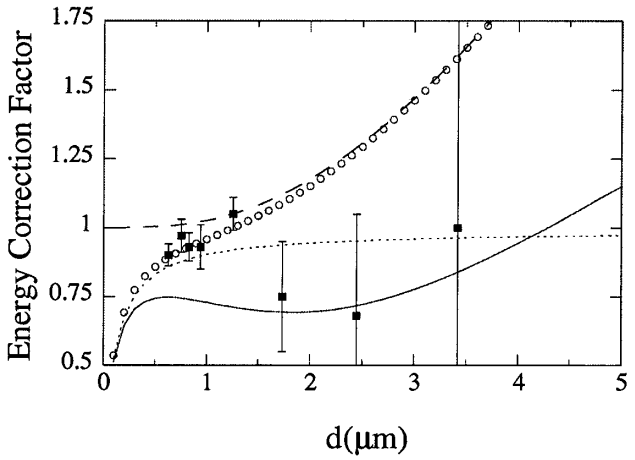

FIG. 1. The energy correction factor for Au at 0 K (dotted line), Au at 300 K (solid line), Au at 300 K with the static transverse electric part incorrectly treated as in the perfect conductor case (circles), and finally the energy between perfect conductors evaluated at 300 K (dashed line). We have, as a comparison, also plotted the experimental energy of Ref. [5] (squares).

In Fig. 2 we investigate the energy correction ratio between aluminum surfaces in the 0.1–2 μm range. The choice of material and separation range is related to the experiment of Mohideen and Roy [6]. Besides our numerical results for Al at 0, 300, and 300 K, with the static transverse electric part incorrectly treated as in the perfect conductor case, we show the thermal correction, conductivity correction, and the combination of these two corrections as presented in Eqs. (2) and (4) of Ref. [6]. The thermal

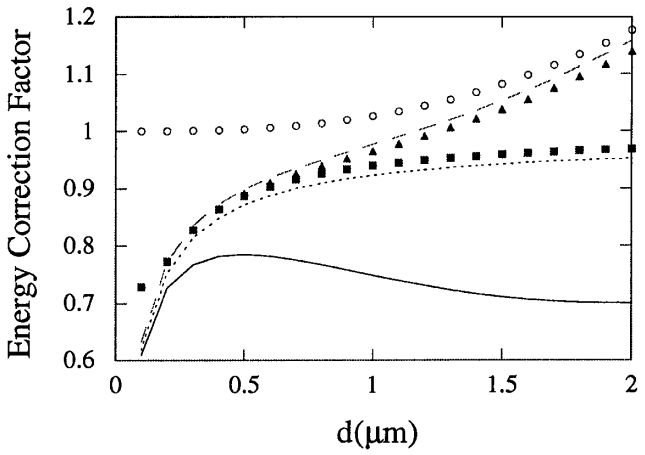

FIG. 2. The energy correction factor for Al at 0 K (dotted line), Al at 300 K (solid line) and Al at 300 K with the static transverse electric part incorrectly treated as in the perfect conductor case (dashed line). We have further added the corrections used in Ref. [6]. These corrections are the thermal correction (circles), the conductivity correction (squares), and a combination of these two corrections (triangles).

correction is based on a series expansion valid for perfect conductors not too far apart. The combination of the two corrections agrees nicely with the result when the $n = 0$ part of the interaction is treated as in the perfect conductor case. At 0.1 μm there is 13% deviation, otherwise the difference is less than 2%. The effect of surface roughness has been argued to be substantial in this case. Since we do not take this or the semitransparent Au-Pd layers on top of the Al surfaces into account we cannot say anything conclusive about the agreement with experiment. However, according to our numerical calculations thermal effects already become important at these separations. Measurements with the same level of accuracy as in Ref. [6], but between much smoother aluminum surfaces, have recently been reported [7]. Unfortunately, the data points presented in Fig. 4 of Ref. [7] appear to be insufficiently accurate for observing any thermal effects. At 0.1 μm, where a 1% level of accuracy has been claimed, thermal effects are very small. At larger separations (0.2–0.5 μm) the spread in experimental data points appears to exceed the thermal corrections. It is therefore unlikely that this experimental result can be used to investigate thermal effects on the Casimir force.

Since the early work of Mehra [20], thermal effects on the Casimir force between plates of perfect conductors are known to be substantial in the separation range of interest. In this paper we have stressed the importance of considering corrections to the Casimir attraction due to finite temperature and finite conductivity together. The Casimir interaction between perfect conductors always increases with temperature. Our main result is the insight that the temperature dependence of the attraction between real metal surfaces can only be found if the finite conductivity is accounted for. The low temperature limit, when dissipation is accounted for, is quite different than previously reported. At intermediate separations the attraction decreases with increasing temperature. The high temperature limit for the interaction between dielectric surfaces given by Lifshitz (Eq. [5.5] of Ref. [11]) is valid also for metals. The discrepancy between the Lifshitz theory and experiment demonstrated in this paper deserves further investigation. At the small separation end of the experimental range, where the experimental accuracy is best, the deviations are clearly visible. The cluster of data points around 1 μm are close to the results for both the perfect metal at room temperature and the real metal at zero temperature. This is clearly against the theoretical prediction. On the other hand in this end of the measuring range, effects of surface roughness on the experimental results and effects of theoretical idealizations, such as a perfectly sharp metal-vacuum interface, are most important. At the large separation end of the experimental range, experiments and theory seem to agree, but here the experimental accuracy is poorer and the agreement therefore not conclusive. We hope that the discrepancy between theory and experiments will be resolved in the near future.

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