Method for Extracting the Quark Mixing Parameter $\cos\alpha$ via $B^{\pm} \to \pi^{\pm}e^{+}e^{-}$

Benjamín Grinstein,[1] Detlef R. Nolte,[1] and Ira Z. Rothstein[2]
[1]Department of Physics, University of California at San Diego, La Jolla, California 92093
[2]Physics Department, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213
(Received 7 October 1999; revised manuscript received 4 January 2000)

We show that it is possible to extract the weak mixing angle α via a measurement of the rate for $B^{\pm} \to \pi^{\pm}e^{+}e^{-}$. The sensitivity to $\cos\alpha$ results from the interference between the long and short distance contributions. The short distance contribution is given in terms of semileptonic form factors. The long distance contribution can be calculated using Ward identities and a short distance operator product expansion if the invariant mass of the lepton pair, q^2, is larger than $\Lambda^2_{\rm QCD}$. For $q^2 \geq 2$ GeV^2 the branching fraction is approximately $1 \times 10^{-8}|V_{td}/0.008|^2$. The shape of $d\Gamma/dq^2$ is very sensitive to the value of $\cos\alpha$ at small values of q^2 and varies by 50% when $-1 < \cos\alpha < 1$ at $q^2 = 2$ GeV^2.

PACS numbers: 12.15.Hh, 11.30.Er, 13.20.He

Great effort is presently being expended in attempting to understand the origin of *CP* violation. It is hoped that in the next generation of experiments we will be able to determine all the parameters in the Cabibbo-Kobayashi-Maskawa (CKM) matrix. However, the extraction of these parameters is, in general, hindered by our inability to perform first principle calculations of rates due to the nonperturbative nature of the long distance QCD effects. A particularly nettlesome extraction is that of the angle α in the unitarity triangle. The standard proposal for the extraction of α from $B \to \pi\pi$ is hindered by so called penguin pollution, which can be overcome only through a cumbersome SU(3) analysis. Here we propose to extract this angle via a measurement of the rate for the rare decay $B \to \pi e^{+}e^{-}$. (It is also possible to use the mode $B \to \rho e^{+}e^{-}$, which will be discussed in a separate publication [1].) It is usually assumed that the rate for this process is dominated by the short distance transition $b \to de^{+}e^{-}$ except when the invariant $e^{+}e^{-}$ mass, $q^2 = (p_{e^-} + p_{e^+})^2$, is of the order of charmonium resonances where long distance contributions are important (here and below by short distance transition we mean contributions to the amplitude that are effectively local at distances larger than the electroweak scale $1/M_W$). However, there is a long distance contribution, which arises through weak annihilation diagrams, like the one in Fig. 1, which can contribute significantly. The short distance amplitude is proportional to $V_{td}V_{tb}^{*}$ whereas the long distance annihilation graph is proportional to $V_{ud}V_{ub}^{*}$. Thus, the interference of these contributions leads to a rate which is sensitive to the value for $\cos\alpha$ where

$$\alpha = \arg\left[-\frac{V_{td}V_{tb}^{*}}{V_{ud}V_{ub}^{*}}\right]. \tag{1}$$

Even a crude measurement of $\cos\alpha$ would be of value since it would remove a twofold ambiguity in extractions of α from $\sin 2\alpha$. Naively, one would think that any hope of extracting α in this way is doomed by the fact that long distance contributions are notoriously intractable. However, in this paper we show that this weak annihilation can be calculated in an expansion in $1/m_b$ and α_s when the invariant mass of the electron pair q^2 is larger than $\Lambda^2_{\rm QCD}$. Moreover, as will be seen below, the rate is independent of the valence, as well as higher twist, wavefunctions of the pion, thereby reducing the uncertainty in the calculation.

Let us naively estimate the relative importance of the short distance amplitude and of the long distance weak annihilation amplitude. The former must involve an electroweak loop, so it carries a factor of $1/16\pi^2$, while the latter should be suppressed by wave functions at the origin $f_B f_\pi/M_B^2$, where f_B and M_B are the B-meson decay constant and mass, respectively, and f_π is the π-meson decay constant. (At small q^2 the short distance amplitude is further suppressed by q^2/M_B^2.) In addition, the CKM factors are different. So the ratio of the long distance to short distance amplitudes is expected to be of the order of $|V_{ub}/V_{td}|16\pi^2 f_B f_\pi/M_B^2 \sim 0.07$. Here we have used $f_B = 170$ MeV, $f_\pi = 130$ MeV, and $|V_{ub}/V_{td}| = 0.5$. Thus, weak annihilation can easily give a correction of

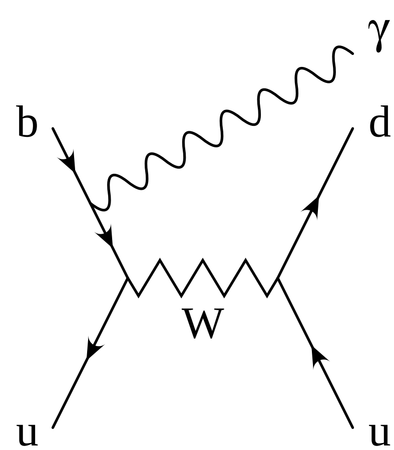

FIG. 1. Weak annihilation diagram underlying the decays $B \to \rho e^{+}e^{-}$ and $B \to \pi e^{+}e^{-}$. There are three other diagrams with the photon emitted from any of the three light quarks. Photon emission from the W-boson is suppressed by G_F.

10% to the rate, possibly larger. A more quantitative calculation is well motivated.

Let us now undertake a systematic calculation of the weak annihilation amplitude in $B^{\pm} \to \pi^{\pm} e^{+} e^{-}$, which will be the primary focus of this paper. The relevant effective Hamiltonian for the long distance, weak annihilation contribution to the rate for $B^{-} \to \pi^{-} e^{+} e^{-}$ is

$$\mathcal{H}'_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{ub} V_{ud}^{*} [c(\mu/M_W)\mathcal{O} + c'(\mu/M_W)\mathcal{O}'], \tag{2}$$

where

$$\mathcal{O} = \bar{u}\gamma^{\nu} P_{-} b \ \ \bar{d}\gamma_{\nu} P_{-} u, \tag{3}$$

and

$$\mathcal{O}' = \bar{u}\gamma^{\nu} P_{-} T^{a} b \ \ \bar{d}\gamma_{\nu} P_{-} T^{a} u, \tag{4}$$

$P_{\pm} \equiv (1 \pm \gamma_5)/2$ and T^a are the generators of color gauge symmetry. The dependence on the renormalization point μ of the short distance coefficients c and c' cancels the μ dependence of operators, so matrix elements of the effective Hamiltonian are μ independent. At next to leading log order, using $\Lambda_{\text{QCD}}^{(5)} = 225$ MeV, the coefficients at $\mu = 5$ GeV are [2] $c = 1.02$ and $c' = -0.34$.

The $B^{-} \to \pi^{-} e^{+} e^{-}$ decay rate is given by

$$\frac{d\Gamma}{dq^2 dt} = \frac{1}{2^8 \pi^3 M_B^3} \left| \frac{e}{q^2} \ell_{\mu} h^{\mu} \right|^2, \tag{5}$$

where $\ell^{\mu} = \bar{u}(p_{e^-})\gamma^{\mu} v(p_{e^+})$ is the leptons' electromagnetic current, $q \equiv p_B + p_{e^-}$ and $t \equiv (p_D + p_{e^+})^2 = (p_B - p_{e^-})^2$. A sum over final state lepton helicities is implicit. The nonlocal contribution to the hadronic current h is

$$h^{\mu} = \langle \pi | \int d^4x \ e^{iq\cdot x} \ T(j_{\text{em}}^{\mu}(x) \mathcal{H}'_{\text{eff}}(0)) |B\rangle. \tag{6}$$

The two body kinematics with an energetic massless final state hadron is known to factorize [3] in the sense that soft gluon exchange between initial and final state hadrons is suppressed by $1/m_b$ or $\alpha_s(m_b)$. This factorization results from the fact that the energetic outgoing light quarks form a small color singlet object which does not couple to leading order in the ratio k/E_q ("color transparency" [4]), where k is the soft gluon momentum and E_q is the energy of the outgoing quarks which scales with m_b provided q^2 is not close to $q_{\max}^2 = (M_B - m_{\pi})^2$. Furthermore, by the same reasoning, the color octet operator does not contribute to leading order in k/E_q once the final state is projected onto the color singlet channel. Notice that this is the *simplest* example of Bjorkens' color transparency argument, since there is only one hadron in the final state. There are no assumptions needed regarding the behavior of wave functions.

Returning now to our result we find that factorization leads to

$$h^{(X)\mu} = \kappa \int d^4x e^{iq\cdot x} [\langle \pi | T(j_{\text{em}}^{\mu}(x) j_{\lambda}(0)) |0\rangle \frac{1}{2} f_B p_B^{\lambda} + \langle \pi | j_{\lambda}(0) |0\rangle \langle 0 | T(j_{\text{em}}^{\mu}(x) J^{\lambda}(0)) |B\rangle], \tag{7}$$

where $\kappa = 2\sqrt{2} G_F V_{ub} V_{ud}^{*} c$, $j_{\lambda} = \bar{d}\gamma_{\lambda} P_{-} u$, and $J^{\lambda} = \bar{u}\gamma^{\lambda} P_{-} b$.

The first line in Eq. (7) can be computed using an isospin Ward identity. The B momentum acts as a derivative on the T-ordered product which then gives the matrix element of a commutator. It makes a contribution to h^{μ} of $-e\kappa f_{\pi} f_B p_B^{\mu}$.

It is remarkable that this result is not sensitive to the internal structure of the pion. The first term in Eq. (7) may be written using a lightcone OPE, as a weighted integral over the pion valence wave function plus higher twist corrections [5]. If one were to perform this calculation, one would find that to all orders in α_s the Wilson coefficients are independent of the momentum fraction carried by the light quarks. The generality of our result allows one to see immediately that in addition it is independent of all higher twist wave functions as well.

The second line in Eq. (7) is also insensitive to the lightcone wave function of the B meson, in that it can be computed using a short distance OPE [6,7] and a heavy quark expansion (HQET) for the b quark, provided $q^2 \gg \Lambda_{\text{QCD}}^2$.

To leading order we find

$$h^{\mu} = -\tfrac{4}{3} e\kappa f_{\pi} f_B p_B^{\mu}. \tag{8}$$

The coefficient of the singlet operator (3) is a function of renormalization point μ. However, once we have matched to the HQET the μ dependence of the coefficient cancels that of the decay constant of the B meson in the HQET. This invariant combination is the physical decay constant f_B, or rather, the leading approximation to it in an expansion in $1/m_b$.

As we will see, the sensitivity to $\cos\alpha$ is greatest at small q^2 where the short distance contribution is suppressed due the rapidly falling form factors. Thus we would like to be able to trust our results to as low a value of q^2 as possible. If k denotes the momentum of the u quark in the B meson, the OPE is a double expansion in $qk/q^2 \sim \Lambda_{\text{QCD}} m_b/q^2$ and $k^2/q^2 \sim \Lambda_{\text{QCD}}^2/q^2$. The leading correction, of the order of qk/q^2 can be computed. The remaining corrections are of order 20% for $q^2 \approx 3.5$ GeV^2 and can be computed in terms of matrix elements of local operators. Including the leading correction we find

$$h^{\mu} = -\tfrac{4}{3} e\kappa f_{\pi} f_B p_B^{\mu} (1 + \tfrac{2}{3} \bar{\Lambda} m_b/q^2), \tag{9}$$

where $\bar{\Lambda} = M_B - m_b$ is the meson mass in HQET. A recent extraction [8] gives $\bar{\Lambda} \approx 330$ MeV.

We now combine the long and short distance contributions to the amplitude for $B^{-} \to \pi^{-} e^{+} e^{-}$. At leading order the short distance contribution to the amplitude is obtained from the effective Hamiltonian [9]. (The penguin operators $\mathcal{O}_3$-$\mathcal{O}_6$ have small coefficients and we neglect them. The gluonic magnetic moment operator, $\mathcal{O}_8$, does not contribute to $B \to \pi \ell \ell$ at this order.)

$$\mathcal{H}_{\rm eff} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{td}^* \sum_{j=7,9,10} C_j(\mu) \mathcal{O}_j(\mu), \tag{10}$$

where

$$\mathcal{O}_7 = \frac{e}{16\pi^2} m_b (\bar{d} \sigma^{\mu\nu} P_+ b) F_{\mu\nu}, \tag{11}$$

$$\mathcal{O}_9 = \frac{e^2}{16\pi^2} (\bar{d} \gamma^\mu P_- b) \bar{e} \gamma_\mu e, \tag{12}$$

$$\mathcal{O}_{10} = \frac{e^2}{16\pi^2} (\bar{d} \gamma^\mu P_- b) \bar{e} \gamma_\mu \gamma_5 e. \tag{13}$$

At leading-log, with $\alpha_s(M_Z) = 0.12$ and $m_t = 175$ GeV, $C_7(m_b) = 0.33$ and $C_{10}(m_b) = 5.3$ [9]. At next to leading-log $C_9(m_b) = -4.3$ [10]. There are additional long distance contributions from the operators $\mathcal{O}$ and $\mathcal{O}'$, and from the corresponding operators with the u quark replaced by a c quark, in which a photon is emitted from a u- or c-quark loop. These contributions can be incorporated into a shift in C_9,

$$\tilde{C}_9 = C_9 + (c + \tfrac{4}{3}c') g(m_c/m_b, \hat{q}^2) + \frac{V_{ub} V_{ud}^*}{V_{tb} V_{td}^*} (c + \tfrac{4}{3}c') [g(m_c/m_b, \hat{q}^2) - g(0, \hat{q}^2)], \tag{14}$$

where $\hat{q}^2 = q^2/m_b^2$ and the function g is defined in Ref. [9]. The short distance amplitude is given in terms of form factors, $f_\pm$ and h, defined by

$$\langle \pi(p') | \bar{d} \gamma^\mu b | B(p) \rangle = (p + p')^\mu f_+ + (p - p')^\mu f_-, \tag{15}$$

$$\langle \pi(p') | \bar{d} \sigma^{\mu\nu} b | B(p) \rangle = 2ih(p^\nu p'^\mu - p^\mu p'^\nu). \tag{16}$$

Including the long distance contribution to the amplitude, as calculated above, and neglecting the mass of the electron, the rate for $B \to \pi e^+ e^-$ is

$$\frac{d\Gamma}{dq^2} = |V_{tb} V_{td}^*|^2 \frac{G_F^2 \alpha^2 m_B^3}{3 \times 2^9 \pi^5} \frac{(m_B^2 - q^2)^3}{m_B^6} \times \left[|C_{10} f_+|^2 + \left| \tilde{C}_9 f_+ + 2m_b C_7 h - \frac{16\pi^2}{3} \frac{V_{ub} V_{ud}^*}{V_{tb} V_{td}^*} \frac{c(m_b) f_\pi f_B}{q^2} \left(1 + \frac{2\bar{\Lambda} m_b}{3q^2}\right) \right|^2 \right]. \tag{17}$$

The contributions from the weak operator $\bar{d} \gamma^\mu b \bar{c} \gamma_\mu c$ will be poorly described by the function g when q^2 corresponds to the mass of a charmonium state, so we restrict our analysis to $q^2 < m_\psi^2$.

We see that the interference effect is largest at smaller values of q^2 where the form factors are suppressed. A part of the uncertainty in the extraction will depend on our knowledge of the form factors. The form factors f_+ and h may be extracted from measurements of the semileptonic B decays and $B \to K^* \gamma$, respectively. Alternatively, we may use heavy quark symmetry and use the relation

$$h(q^2) = \frac{[f_+(q^2) - f_-(q^2)]}{2m_b} + O(1/m_b). \tag{18}$$

Presently, there exist lattice QCD determinations at larger values of q^2, where there is little hadronic recoil.

Given that presently we do not know the form factors away from largest values of q^2, for illustration purposes we will use the BK model (of Becirevic and Kaidalov [11]) which satisfies the unitarity sum rules bounds [12] and fits lattice determinations [13] at large q^2:

$$f_+^{(\rm BK)} = \frac{N(1-\beta)}{(1-\tilde{q}^2)(1-\beta\tilde{q}^2)}, \qquad f_0^{(\rm BK)} = \frac{N(1-\beta)}{1-\gamma\tilde{q}^2}, \tag{19}$$

where $f_0 \equiv \frac{q^2}{m_B^2 - m_\pi^2} f_- + f_+$, $\tilde{q}^2 = q^2/M_{B^*}^2$, and the parameters are $\beta = 0.54$, $\gamma = 0.8$, $N = 0.6$, and $M_{B^*} = 5.325$ GeV. The value of γ is fixed by the Callan-Treiman relation $f_0(m_B^2) = f_B/f_\pi$. The BK model does not give the form factor h. We calculate h using the heavy quark spin symmetry relation (18).

In Fig. 2 we plot the rate of Eq. (17) as a fraction of the total width $\Gamma = \tau_B^{-1}$ as a function of q^2. We have used the coefficients c, c', and C_9 at next to leading order, and the rest at leading log order. We have restricted the plot to $q^2 \geq 1.5$ GeV^2 for our approximations to remain valid, and to $q^2 \leq 8.5$ GeV^2 to avoid contributions from charmonium resonances. For illustration we have used $|V_{ub} V_{ud}^*| = 0.004$, $|V_{tb} V_{td}^*| = 0.008$, and $f_B = 0.17$ GeV. The solid, dashed, and dotted lines correspond to $\cos\alpha = 0$, -1, and 1, respectively, and the shaded region represents an uncertainty $\pm(\bar{\Lambda} m_b/q^2)^2$ to the correction in Eq. (9) plus a correction to factorization of order Λ/E_q. The values used for $|V_{ub} V_{ud}^*/V_{tb} V_{td}^*|$ and f_B are uncertain. The long distance correction could be even more pronounced if they happened to be larger.

The two limitations of our proposal are the size of the branching fraction and the unknown form factors. We see from (17) the branching fraction is sensitive to V_{td} which is presently constrained to be in the range 0.004–0.012. Thus, the total branching fraction will vary between 10^{-7}–10^{-9}. Such branching ratios are most probably out of the reach of e^+e^- machines, but well within the reach of hadronic machines given the mode. To extract $\cos\alpha$ we should look at the partially integrated rate for $1.5 \leq q^2 \leq 8.5$ GeV^2 which will further reduce the rate

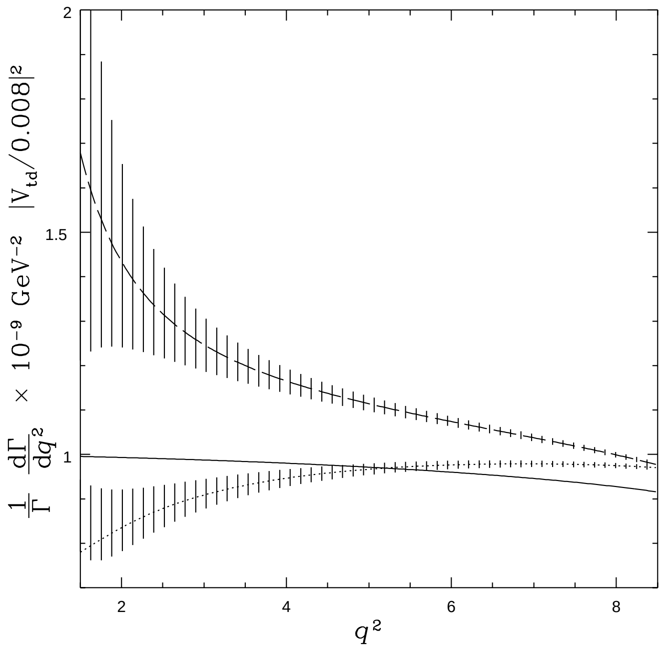

FIG. 2. Differential branching fraction, $\tau_B \, d\Gamma/dq^2$ for $B \to \pi e^+ e^-$. The solid, dashed, and dotted lines correspond to $\cos\alpha = 0, -1$, and 1, respectively. The shaded region shows the uncertainty in our calculation (see text). For the short distance contribution the BK form factors of Ref. [11] have been used.

by a factor of 2 or so. The accuracy with which we may extract the *CP* violating parameters from $B^\pm \to \pi^\pm l^+ l^-$ is limited by how well we know the form factors. This analysis is useful only if the form factors are known to an accuracy significantly better than the size of the dependence on $\cos\alpha$; that is, they have to be known much better than 30%. Fortunately, it is expected that the relevant form factors will be determined with percent accuracy in the near future. Note that the overall normalization is irrelevant for our analysis.

We have shown that the rate for $B^\pm \to \pi^\pm e^+ e^-$ is sensitive to $\frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*}$. If the magnitude of this ratio is known, then the rate and particularly the shape of the spectrum depend sensitively on $\cos\alpha$. Even a crude measurement of the shape would almost certainly determine the sign of $\cos\alpha$ and remove a twofold ambiguity from $\sin 2\alpha$. If the magnitude is not known, then a measurement of the rate and spectrum would constrain the unitarity triangle, e.g., a region of the (ρ, η) plane [1]. The analysis requires knowledge of decay form factors. Semileptonic decay spectra will determine the combination $|V_{ub} f_+|$. Such a measurement can be incorporated in our analysis [1] and constrain the unitarity triangle in a meaningful way even if separate knowledge of $|V_{ub}|$ and f_+ is lacking.

The authors benefited from conversations with Jim Russ. B. G. and D. R. N. are supported by the Department of Energy under Contract No. DOE-FG03-97ER40546. I. R. is supported by the Department of Energy under Contract No. DOE-ER-40682-143.

[1] B. Grinstein, A. Leibovich, I. Low, D. R. Nolte, and I. Z. Rothstein (to be published).
[2] G. Buchalla, A. J. Buras, and M. E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996).
[3] M. J. Dugan and B. Grinstein, Phys. Lett. B **255**, 583 (1991).
[4] J. Bjorken, Nucl. Phys. (Proc. Suppl.) **11**, 325 (1989).
[5] I. I. Balitsky, V. M. Braun, and A. V. Kolesnichenko, Nucl. Phys. **B312**, 509 (1989); V. M. Braun and I. B. Filyanov, Z. Phys. C **44**, 157 (1989); **48**, 239 (1990); V. L. Chernyak and I. R. Zhitnitsky, Nucl. Phys. **B345**, 137 (1990); P. Ball, V. M. Braun, and H. G. Dosch, Phys. Rev. D **44**, 3567 (1991); V. M. Belyaev, A. Khodjamirian, and R. Rückl, Z. Phys. C **60**, 349 (1993); V. L. Chernyak and A. R. Zhitntsky, Phys. Rep. **112**, 173 (1984).
[6] D. H. Evans, B. Grinstein, and D. R. Nolte, Phys. Rev. Lett. **83**, 4957 (1999).
[7] D. H. Evans, B. Grinstein, and D. R. Nolte, Report No. UCSD-PTH-99-09 (unpublished) [e-print hep-ph/9906528].
[8] J. Bartelt *et al.*, CLEO Collaboration, Report No. CLEO CONF 98-21.
[9] B. Grinstein, M. J. Savage, and M. B. Wise, Nucl. Phys. **B319**, 271 (1989).
[10] A. J. Buras and M. Munz, Phys. Rev. D **52**, 186 (1995).
[11] D. Becirevic and A. B. Kaidalov, Report No. LPT-ORSAY-99-32 (unpublished) [e-print hep-ph/9904490].
[12] G. Boyd and I. Z. Rothstein, Phys. Lett. B **395**, 96 (1997); **420**, 350 (1997).
[13] APE Collaboration, C. R. Allton *et al.*, Phys. Lett. B **345**, 513 (1995); UKQCD Collaboration, C. M. Maynard *et al.*, Report No. EDINBURGH-99-12 (unpublished) [e-print hep-lat/9909100].