

Precise Measurement of the b -Quark Fragmentation Function in Z^0 Boson Decays

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We have developed a new technique for inclusive reconstruction of the energy of B hadrons. The excellent efficiency and resolution of this technique allow us to make the most precise determination of the b -quark fragmentation function, using $e^+e^- \rightarrow Z^0$ decays recorded in the SLAC Large Detector

experiment. We compared our measurement with the predictions of a number of fragmentation models. We excluded several of these models and measured the average scaled energy of weakly decaying B hadrons to be $\langle x_B \rangle = 0.714 \pm 0.005(\text{stat}) \pm 0.007(\text{syst}) \pm 0.002$ (model dependence).

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In high-energy strong-interaction processes quarks and gluons are not observed directly, but appear as jets of colorless hadrons. This fundamental process of “jet fragmentation” affects all high-energy physics measurements involving strongly interacting particles, but is only poorly understood at a quantitative level. The fragmentation of heavy quarks is of particular experimental interest since many expected signatures of new heavy particles, such as Higgs and supersymmetric (SUSY) particles, involve decays to b quarks. It is hence vital to understand the production and properties of b jets. Here we present a significantly improved determination of the b -quark fragmentation function, $D(x_B) = (1/\sigma)d\sigma/dx_B$, measured using $Z^0 \rightarrow b\bar{b}$ decays, where $x_B = E_B/E_{\text{beam}}$ represents the fraction of the b -quark energy retained by the weakly decaying B hadron.

In quantum chromodynamics (QCD) the b -quark mass serves as a cutoff for collinear gluon radiation. The distribution of b -quark energies prior to hadronization can therefore be calculated perturbatively [1–5]. However, the additional effects which yield the experimentally accessible distribution $D(x_B)$ are nonperturbative, and have been studied phenomenologically via a number of different approaches [1–12]. Measurements of $D(x_B)$ provide direct tests of these perturbative QCD and model predictions.

$D(x_B)$ has been measured previously [13–16] by reconstructing the energies of B hadrons that decay semileptonically ($B \rightarrow l\nu DX$) in $Z^0 \rightarrow b\bar{b}$ events. In these studies the inclusive B selection efficiency was smaller than 1%, and much lower for $E_B < 20$ GeV. The resulting low-statistics samples, and limited energy resolutions, yielded relatively weak constraints on the shape of the distribution.

We have developed a new technique for measuring E_B using only kinematic information from charged tracks. Our 307 Mpixel charge-coupled-device (CCD)-based vertex detector, combined with the micron-sized SLC interaction point (IP), allows us to reconstruct B -decay vertices and the B -hadron flight direction very accurately. The method yields a significantly higher B -selection efficiency and superior energy resolution, both of which are almost independent of E_B , and has very low bias since it does not use a beam-energy constraint. This allows us to measure the shape of $D(x_B)$ with sufficient precision to make stringent tests of b -fragmentation model predictions, reduce model-dependent systematic errors, and therefore discriminate among these models for the first time. Furthermore, our technique for reconstructing B -hadron energies is of direct relevance to studies of other important properties of heavy-quark systems, such as B -hadron lifetimes and neutral B -meson mixing.

We used 150 000 hadronic Z^0 decays produced in e^+e^- annihilations at the SLAC Linear Collider (SLC) and

collected in the SLC Large Detector (SLD) between 1996 and 1997. A description of the SLD can be found elsewhere [17,18]. The trigger and selection criteria for $Z^0 \rightarrow$ hadrons events are described elsewhere [16]. This analysis used charged tracks measured in the Central Drift Chamber (CDC) [19] and in the upgraded CCD Vertex Detector (VXD) [20], with a momentum resolution of $\sigma_{p_\perp}/p_\perp = 0.01 \oplus 0.0026p_\perp$, where p_\perp is the track transverse momentum with respect to the beam line, in GeV/ c . The centroid of the SLC IP was reconstructed with a precision of 4.4 μm (30 μm) in the plane transverse to (containing) the beam line. Tracks from identified γ conversions and K^0 or Λ^0 decays were removed from consideration, and only well-reconstructed tracks [18] were used for B -hadron tagging and energy reconstruction.

Weakly decaying B hadrons were identified by exploiting their long lifetimes and large masses relative to light-flavor hadrons. Each hadronic event was divided into two hemispheres by the plane containing the IP and the normal to the thrust axis. A topological vertexing algorithm [21] was optimized for this analysis and applied to the set of well-reconstructed tracks in each hemisphere in an attempt to reconstruct a B -decay vertex. Vertices were required to be separated from the IP by at least 1 mm and to contain at least two tracks. A candidate vertex was found in 32 492 hemispheres [22].

In each hemisphere the total energy, E_{ch} , momentum, \vec{P}_{ch} , and invariant mass, M_{ch} , of the vertex-associated tracks were calculated by assigning each track the charged-pion mass. Because of the missing particles, namely the set of neutral decay products and charged tracks missed from the vertex, \vec{P}_{ch} can be acollinear with the true B flight direction, which we estimated independently of the track momenta by the unit vector \vec{v} along the line joining the IP to the reconstructed vertex position. Our accurate knowledge of the IP and vertex positions yielded an excellent angular resolution of 26 mrad on \vec{v} . This allowed us to calculate the net transverse momentum relative to \vec{v} of the missing particles, $\vec{P}_t = \vec{P}_{\text{ch}} - (\vec{P}_{\text{ch}} \cdot \vec{v})\vec{v}$, and hence to improve our estimate of the mass of the decaying particle via $M_{P_t} \equiv \sqrt{M_{\text{ch}}^2 + P_t^2 + |P_t|}$; this quantity is a strong discriminator for selecting B hadrons [23]. We required $2.0 \text{ GeV}/c^2 < M_{P_t} < 2 \times M_{\text{ch}}$ to select 19 604 candidates, estimated to be 98.2% B hadrons, the main background (1.6%) being charmed hadrons in $Z^0 \rightarrow c\bar{c}$ events. The efficiency for selecting a true B hadron is 40.1%. For this sample, on average 92% of the reconstructed true B -decay tracks were associated with the vertex, and 98% of the vertex-associated tracks were from true B decays.

The reconstructed B -hadron energy, E_B^{rec} , can be expressed as $\sqrt{M_0^2 + P_t^2 + P_{0l}^2} + E_{\text{ch}}$, where the combined mass, M_0 , and the momentum along \vec{v} , P_{0l} , of the missing particles are the only unmeasured quantities. If we assume a B -hadron rest mass, M_B , we can eliminate one of the two unknowns and calculate an upper bound on M_0 :

$$M_0^2 \leq M_B^2 - 2M_B\sqrt{M_{\text{ch}}^2 + P_t^2} + M_{\text{ch}}^2 \equiv (M_0^2)_{\text{max}}, \quad (1)$$

where equality holds when $P_{0l} = 0$ in the B -hadron rest frame. Since, from phase-space requirements, small values of P_{0l} are the most probable, and the average B -decay multiplicity is high, the true M_0 tends to be close to $(M_0)_{\text{max}}$ [24]. We assumed $M_B = 5.28 \text{ GeV}/c^2$, equated M_0 with the measured $(M_0)_{\text{max}}$, and calculated E_B^{rec} . This estimate of the B energy is best when the B -decay kinematics are well constrained by the vertex-associated tracks, i.e., when $(M_0)_{\text{max}} \approx 0$ [25]. Hence, in order to improve the energy resolution we required $-1 < (M_0^2)_{\text{max}} < [1.1 + f(E_B^{\text{rec}})]^2$. The explicit E_B^{rec} dependence [26] was chosen to ensure that the B selection efficiency is only weakly energy dependent. The efficiency is above 3% for $E_B > 10 \text{ GeV}$; the average value is 3.9%. Since the small non- B background is concentrated at large $(M_0)_{\text{max}}$ this cut reduces it further. In total, 1920 candidates were selected, with an estimated B purity of 99.3%.

We generated events with our JETSET 7.4 [27]-based Monte Carlo simulation program [16,23] and examined the distribution of the normalized difference between the reconstructed and true B -hadron energies, $(E_B^{\text{rec}} - E_B^{\text{true}})/E_B^{\text{true}}$. This resolution can be characterized [24] by a double Gaussian function centered at zero. We found that the narrower Gaussian represents 83% of the population and has a width of 10.4%. This resolution depends only weakly on E_B^{true} ; in particular, it remains better than 15% even for B energies close to the mass threshold, which is a significant advantage of this technique. The simulated 0.7% non- B background was subtracted bin-by-bin from the measured distribution of the reconstructed scaled B -hadron energy, $D^{\text{rec}}(x_B^{\text{rec}})$, which is shown in Fig. 1.

We tested several models of b -quark fragmentation. These models are formulated in terms of experimentally inaccessible variables and must hence be implemented in an iterative fragmentation algorithm in order to derive the measurable quantity $D^{\text{rec}}(x_B^{\text{rec}})$. We employed our JETSET simulation program to generate $e^+e^- \rightarrow b\bar{b}$ events according to each model considered, with full simulation of the detector response. B -hadron energies were reconstructed, according to our algorithm, from the fully simulated event sample to derive $D^{\text{sim}}(x_B^{\text{rec}})$, which was compared with the data using a binned χ^2 . The χ^2 was minimized by repeating this procedure under variation of the input parameter(s) of the model. The fitted model predictions and best χ^2 values are shown in Fig. 1.

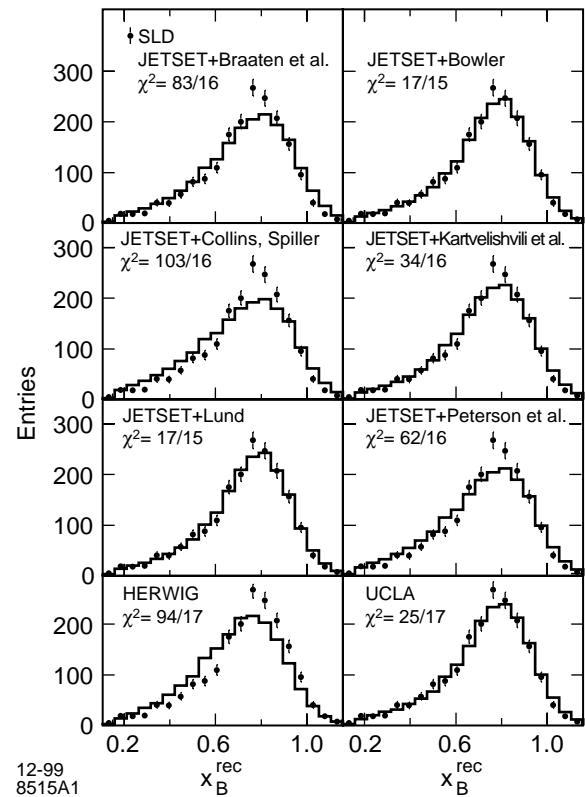


FIG. 1. Distribution of the reconstructed scaled energy of weakly decaying B hadrons (points); the errors are statistical. The predictions of eight models are shown as histograms.

Within this context the Bowler [9,27], Kartvelishvili *et al.* [8], and Lund [11] models reproduce the data; the models of Braaten *et al.* [4], Collins and Spiller [12], and Peterson *et al.* [10] have a χ^2 confidence level less than 0.1% and are not consistent with the data. We also tested the Monte Carlo models HERWIG 5.7 [28] and UCLA [29], which contain no explicit free parameters to control $D(x_B)$. The UCLA model is consistent with the data and the HERWIG model is not (Fig. 1).

In order to allow other models to be compared with our data, e.g., those of [1–3,5–7], we corrected for the effects of the selection and reconstruction procedures that were applied. We estimated the *true* weakly decaying B -hadron scaled-energy distribution, $D(x_B)$, from the (background-subtracted) reconstructed distribution, $D^{\text{rec}}(x_B^{\text{rec}})$; for each bin i : $D(x_B)_i = \sum_k M_{ik} D^{\text{rec}}(x_B^{\text{rec}})_k / \epsilon_i$. The selection efficiency $\tilde{\epsilon}$ and the unfolding matrix \mathbf{M} were calculated from the simulation using in turn each of the four fragmentation models that were consistent with the data, with the respective fitted parameter values. This unfolding procedure assumes a smooth underlying distribution and is explicitly model dependent, but we quantify this (below) using the variation among the four resulting distribution shapes.

This set of consistent models is small and does not appear to span the range of potentially acceptable shapes. We therefore considered a number of *ad hoc* functional forms for the true x_B distribution. In each case we produced in our simulated event sample a true x_B distribution of that

form, compared the corresponding x_B^{rec} distribution with the data, and optimized the respective parameter value(s) in a manner similar to that described above. This procedure is not equivalent to the model tests discussed previously, in which the predicted function was introduced into the iterative fragmentation mechanism at the end of the JETSET perturbative QCD parton shower. We found four functions that yielded a reconstructed distribution consistent with the data in Fig. 1: an eighth-order polynomial, the Peterson function, and two generalizations of the Peterson function [14]. Each of these four fitted functions was also used to calculate $\tilde{\epsilon}$ and \mathbf{M} from the simulation and to derive $D(x_B)$.

Each of the eight unfolded distributions was normalized to unit area, and in Fig. 2 we show their binwise average; the band includes the rms deviation. This represents our best estimate of the true B energy distribution $D(x_B)$. The corrected distribution is, by construction, smoother than the measured distribution, and the band provides an envelope within which acceptable predictions should fall. The constraint on the shape is much stronger than in any previous determination [13–16].

An important advantage of our method is that it is not biased by tracks that were not reconstructed or attached to the vertex [24]. We considered other potential sources of systematic uncertainty which derive from the modeling of our detector response. In each case the simulated events were reweighted or adjusted in order to reproduce the variation in question, and the entire analysis was repeated. Variation of quantities such as the track reconstruction efficiency, the point resolution of the vertex detector, and the momentum and dip angle resolutions of the tracking system within their uncertainties [23] affects the B selection efficiency and, in some cases, the

energy resolution. However, there is little effect on the shape of the distribution or on the mean value, $\langle x_B \rangle$. In no case was the χ^2 of a model or function test changed significantly. The largest effect on $\langle x_B \rangle$, ± 0.005 , arose from the uncertainty in the momentum resolution, which was measured in the data using $e^+e^- \rightarrow \mu^+\mu^-$ events. An *ad hoc* correction was applied to the simulation to reproduce the measurement, and the full effect of this correction was considered as a symmetric systematic uncertainty. As a cross check we changed the upper cut on $(M_0^2)_{\text{max}}$ to fixed values between 1 and 5 $(\text{GeV}/c^2)^2$; the change in $\langle x_B \rangle$ was smaller than the statistical error.

We also considered the uncertainties on a large number of measured quantities [23] related to the production and decay of B and charmed hadrons. These are used in the simulation and potentially affect the values of $\tilde{\epsilon}$ and \mathbf{M} used to unfold the data. We varied each quantity by the error on its measured value; none of these variations affects our conclusions. The production of primary excited B hadrons, collectively denoted B^* and B^{**} , which decay into the weakly decaying B hadrons that we measure, was investigated in more detail, and we varied independently the fraction of primary B^* (B^{**}) from zero to unity (0.5). No significant effect on the shape of $D(x_B)$, or on the χ^2 values for the model comparisons, was observed. The largest effect on $\langle x_B \rangle$, of ± 0.002 , was due to variation of the number of charged tracks and K_s^0 produced per B -hadron decay. In each x_B bin the sum in quadrature of the detector- and physics-related systematic uncertainties on $D(x_B)$ is much smaller than the statistical error; they are not shown in Fig. 2.

From the eight distribution shapes that are consistent with our data we extract the mean value of the scaled energy of weakly decaying B hadrons in Z^0 decays: $\langle x_B \rangle = 0.714 \pm 0.005(\text{stat}) \pm 0.007(\text{syst}) \pm 0.002(\text{model})$. This is the most precise of the world's measurements that take the $D(x_B)$ shape dependence into account, and the uncertainty is relatively small since we have excluded a wide range of shapes. Our result is consistent with a recent average over many Z^0 measurements of $\langle x_B \rangle = 0.702 \pm 0.008$ [30]. We also calculated the second and third moments to be $\langle x_B^2 \rangle = 0.537 \pm 0.011(\text{expt}) \pm 0.003(\text{model})$ and $\langle x_B^3 \rangle = 0.417 \pm 0.012(\text{expt}) \pm 0.004(\text{model})$.

In order to derive results for the inclusive sample of primary B hadrons, one must assume values for the production fractions of B^* and B^{**} mesons. Postulating a B^* production fraction of 0.75, expected from naive spin counting, and zero production of B^{**} mesons, leads to $\langle x_B \rangle^{\text{primary}} = 0.718$. Postulating independently a B^{**} production fraction of 0.25 [31], and zero B^* production, yields $\langle x_B \rangle^{\text{primary}} = 0.728$.

In summary, we have developed a new, inclusive technique for reconstructing the energies of B hadrons. It has substantially higher efficiency and better energy resolution than previous methods. We have employed this technique to measure the scaled-energy distribution of weakly decaying B hadrons produced in $e^+e^- \rightarrow Z^0$ decays with

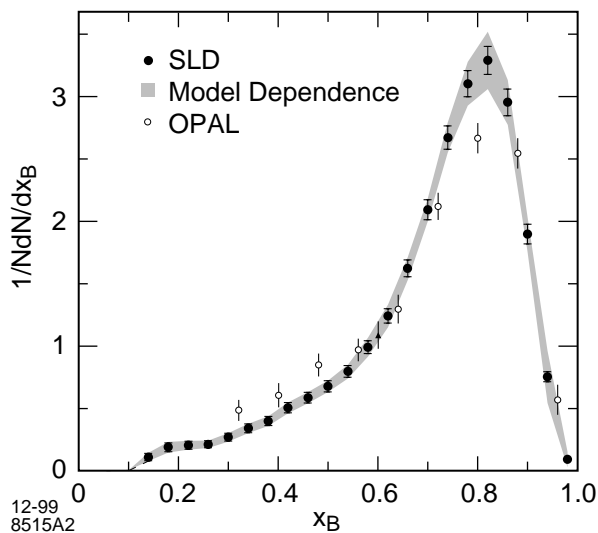


FIG. 2. Unfolded distribution of weakly decaying scaled B -hadron energy (points). The errors are statistical only and do not include point-to-point correlations. The band represents the envelope of acceptable functions (see text). Also shown (circles) is the best previous measurement [15].

unprecedented precision over the entire kinematic range from the B mass to the beam energy. As a result we are able to exclude several models of b -quark fragmentation, including the widely used JETSET+Peterson model.

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