Observation of the Continuous Stern-Gerlach Effect on an Electron Bound in an Atomic Ion

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We report on the first observation of the continuous Stern-Gerlach effect on an electron bound in an atomic ion. The measurement was performed on a single hydrogenlike ion ($^{12}C^{5+}$) in a Penning trap. The measured g factor of the bound electron, $g = 2.001\,042(2)$, is in excellent agreement with the theoretical value, confirming the relativistic correction at a level of 0.1%. This proves the possibility of g-factor determinations on atomic ions to high precision by using the continuous Stern-Gerlach effect. The result demonstrates the feasibility of conducting experiments on single heavy highly charged ions to test quantum electrodynamics in the strong electric field of the nucleus.

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The directional quantization of the quantum mechanical angular momentum was first experimentally observed in 1922 by Stern and Gerlach on a beam of neutral silver atoms [1]. In their famous experiment an inhomogeneous magnetic field with a linear gradient was employed to spatially separate atoms in different magnetic sublevels (spin up and spin down) and to measure the magnetic moment of the atoms. A Stern-Gerlach experiment on a beam of charged particles has never been attempted because the Lorentz force acting on the electric charge is in general much stronger than the force exerted on the magnetic moment of the particles in a magnetic gradient [2]. The first observation of directional quantization of the quantum mechanical angular momentum on a charged particle was made possible by Dehmelt and co-workers. In their electron g-2 experiment the orientation of the electron spin with respect to a magnetic field was measured via the so-called continuous Stern-Gerlach effect [3,4]. In this method, a single electron was confined in three dimensions in a Penning trap, and the direction of the electron spin was monitored in a quantum nondemolition measurement by means of a quadratic magnetic field component superimposed on the electric and magnetic fields of the Penning trap [5,6].

In this Letter we report on the first observation of the continuous Stern-Gerlach effect on an electron bound in an atomic ion. In our experiment, a single hydrogenlike carbon ion ($^{12}C^{5+}$) is stored in a Penning trap. The total magnetic moment of the $^{12}C^{5+}$ ion is purely due to the magnetic spin moment of the bound electron in the $1s_{1/2}$ ground state since the nucleus has $I = 0$. To measure the magnetic moment (or g factor) of the bound electron a microwave field at the electron's Larmor precession frequency is used to induce spin-flip transitions between the two spin states $m_s = \pm 1/2$. These transitions are detected via the continuous Stern-Gerlach effect [7]. An attractive feature of this novel method of atomic spectroscopy is its general applicability to hydrogenlike ions with any nuclear charge Z [8].

In a Penning trap a charged particle is stored in a combination of a homogeneous magnetic field B_0 and an electrostatic quadrupole potential [6,9]. The magnetic field confines the particle in the plane perpendicular to the magnetic field lines, and the electrostatic potential in the direction parallel to the magnetic field lines. The three eigenmotions that result are the trap-modified cyclotron motion (frequency ω_+), the magnetron motion (frequency ω_-), which is a circular $\mathbf{E} \times \mathbf{B}$ drift motion perpendicular to the magnetic field lines, and the axial motion (parallel to the magnetic field lines, frequency ω_z). The free-space cyclotron frequency $\omega_c = (Q/M)B_0$ of an ion with charge Q and mass M in a magnetic field B_0 can be determined from a combination of the trapped ion's three eigenfrequencies ω_+, ω_z, and ω_- with the formula [10]

$$\omega_c^2 = \omega_+^2 + \omega_z^2 + \omega_-^2. \tag{1}$$

The magnetron frequency ω_- can be calculated from the axial frequency ω_z and the trap-modified cyclotron frequency ω_+ through the relation $\omega_- = \omega_z^2/2\omega_+$. Thus, the free-space cyclotron frequency ω_c can be determined from a measurement of the axial frequency ω_z and the trap-modified cyclotron frequency ω_+.

In our experiment, the magnetic field ($B_0 = 3.8$ T) is provided by a superconducting solenoid. The electrostatic quadrupole potential is produced by a stack of five cylindrical electrodes with the same inner diameter (7 mm): the so-called ring electrode at the center and the compensation electrodes and end caps placed on either side of the ring electrode [11,12]. With a positive voltage U_0 applied between the two end caps and the ring electrode, a potential well $V_{el}(z) = QU_0 z^2/d^2$ is created along the magnetic field lines for a positively charged ion, where d is a characteristic size of the trap electrodes. The axial motion of the trapped ion is a harmonic oscillation with frequency

$$\omega_z = \sqrt{\frac{Q}{M}\,\frac{U_0}{d^2}}\,. \tag{2}$$

The principle of the continuous Stern-Gerlach effect is based on a coupling of the magnetic moment μ of the particle to its axial oscillation frequency ω_z in the Penning trap. This coupling is achieved by a quadratic magnetic field component ("magnetic bottle") superimposed on the homogeneous magnetic field B_0 of the Penning trap

$$B(z) = B_0 + \beta_2 z^2. \qquad (3)$$

Because of the interaction of the z component μ_z of the magnetic moment with the magnetic bottle term, the trapped ion possesses a position-dependent potential energy $V_m = -\mu_z(B_0 + \beta_2 z^2)$, which adds to the potential energy V_{el} of the ion in the electrostatic well. Therefore, the effective trapping force is modified by the magnetic interaction, and the axial frequency of the trapped ion is shifted upwards or downwards, depending on the sign of the z component μ_z of the magnetic moment. This axial frequency shift is given by

$$\delta\omega_z = \frac{\beta_2 \mu_z}{M\omega_z}, \qquad (4)$$

where ω_z is the unshifted axial frequency.

In our Penning trap apparatus, which has been described in [12], there are three positions in the stack of cylindrical electrodes where ions can be trapped (as shown in Fig. 1). (i) In the reservoir trap, which consists of three cylindrical electrodes, ions can be "parked" for later use. (ii) In the preparation trap ions are created in different charge states by electron impact ionization of neutral atoms. An electron beam with an energy of 2 keV and a current of 5 nA is emitted from a tungsten field emission point close to the trap electrodes. In the preparation trap unwanted ion species are removed by selectively heating their axial motion; consecutively, the number of $^{12}C^{5+}$ ions is reduced to one particle by lowering the electrostatic potential well. The single $^{12}C^{5+}$ ion is then transported to the (iii) spin-flip trap. The ring electrode of this trap is made out of ferromagnetic material (nickel) to produce the quadratic component of the magnetic field ($\beta_2 = 1\ \mathrm{T/cm^2}$) which is necessary to observe the continuous Stern-Gerlach effect. The trap electrodes are housed in a sealed ultrahigh vacuum chamber which is kept at a temperature of 4 K. With the effect of cryopumping a vacuum pressure better than 10^{-16} mbar is reached.

The axial oscillation frequency ω_z of the single $^{12}C^{5+}$ ion in the spin-flip trap is measured nondestructively with an electronic detection method through the image currents which are induced in the trap electrodes by the particle motion. A LC circuit resonant at $\omega_z = 2\pi \times 360$ kHz (with quality factor $Q = 2500$) is attached to one of the trap electrodes to increase the detection sensitivity. Through the image currents dissipated in the LC circuit the trapped ion is cooled to the ambient temperature of 4 K with a time constant of $\tau_z = 100$ ms (resistive cooling). The ion

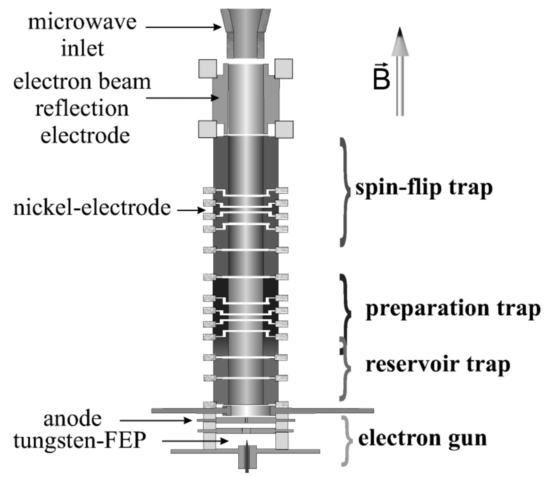

FIG. 1. Schematic of the stack of cylindrical electrodes forming the electric trapping potential. Trapping is possible at three different positions.

is detected in thermal equilibrium without any external excitation. Its axial frequency is observed as a minimum in a frequency analysis of the LC circuit's thermal noise voltage [13]. The elegance of the described measurement technique lies in the fact that it allows for the detection of a single trapped ion while it is at a temperature of 4 K. At this low temperature its oscillation amplitudes in the trap are strongly reduced ($<50\ \mu$m), and systematic errors due to inhomogeneities of the magnetic field or anharmonicities of the electrostatic trapping field are minimized. The main systematic error left is due to possible small changes in the magnetron radius which limits the accuracy to 1×10^{-6}.

The quantum state of the $^{12}C^{5+}$ ion, i.e., the magnetic quantum number m_s of the bound electron in the $1s_{1/2}$ ground state, can be monitored nondestructively in the spin-flip trap in repeated measurements of the axial frequency of the trapped ion [see Eq. (4)]. Transitions between the two spin states $m_s = \pm 1/2$ are induced by a microwave field (at 104 GHz) resonant with the Larmor precession frequency ω_L of the bound electron

$$\hbar\omega_L = g\,\frac{e\hbar}{2m_e}\,B = g\mu_B B. \qquad (5)$$

Here, g is the g factor of the bound electron and $\mu_B = e\hbar/2m_e$ is the Bohr magneton. The spin-flip transitions are observed as discrete changes of the ion's axial frequency. Figure 2 shows a clear demonstration of such quantum jumps observed via the continuous Stern-Gerlach effect. The measured axial frequency shift for a transition between the two quantum levels is $\omega_z(\uparrow) - \omega_z(\downarrow) = 2\pi \times 0.7$ Hz, in excellent agreement with the expected value calculated from Eq. (4).

The observation of the continuous Stern-Gerlach effect in the hydrogenlike carbon ion $^{12}C^{5+}$ makes it possible to measure its electronic g factor to high accuracy. Using

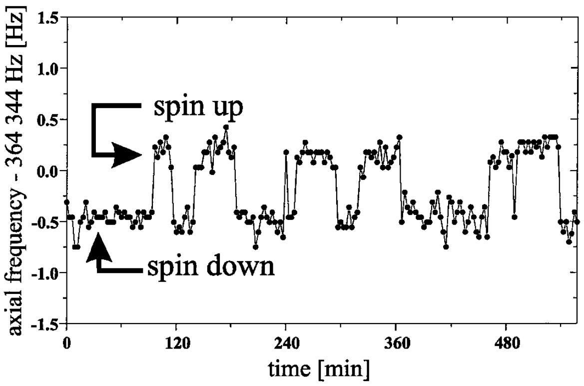

FIG. 2. Quantum nondemolition measurement of the spin state: the spin-flip transitions are observed as small discrete changes of the axial frequency of the stored $^{12}C^{5+}$ ion.

the cyclotron frequency ω_c of the ion [Eq. (1)] for the calibration of the magnetic field B, the g factor of the bound electron can be expressed as the ratio of the Larmor precession frequency ω_L of the electron and the cyclotron frequency ω_c of the $^{12}C^{5+}$ ion via

$$g = 2\,\frac{\omega_L}{\omega_c}\,\frac{Q/M}{e/m_e}\,. \tag{6}$$

The ratio of charge-to-mass ratios of the carbon ion (Q/M) and of the electron (e/m_e) was measured in a Penning trap to an accuracy of 2×10^{-9} [14].

A resonance spectrum of the Larmor precession frequency ω_L of the bound electron is obtained in the following measurement procedure. A microwave field around ω_L is applied, and the spin state m_s is analyzed in a subsequent quantum nondemolition measurement by determining the axial frequency of the $^{12}C^{5+}$ ion. The number of spin-flip transitions is then counted for a fixed time interval (of about five hours). Near resonance one quantum jump is observed about every seven minutes on the average. Then the microwave frequency is varied and the measurement is repeated at about 20 different excitation frequencies. Finally, the plot of the quantum jump rate versus excitation frequency yields the resonance spectrum of the Larmor precession frequency (Fig. 3). The asymmetry of the line shape of the resonance curve is due to the thermal Boltzmann distribution of the ion's axial oscillation amplitude. The Larmor precession frequency depends on the ion's oscillation amplitude due to the presence of the magnetic field inhomogeneity $\beta_2 z^2$. The experimental data points are fitted to a theoretical line shape [15]. In the theoretical fit (solid line) axial-motion sidebands have been taken into account. This way the Larmor precession frequency can be determined to within 10^{-7}.

The trap-modified cyclotron frequency ω_+ of the stored $^{12}C^{5+}$ ion is determined by exciting the cyclotron motion with a drive frequency which is applied to a segment of one of the trap electrodes and swept through the cyclotron

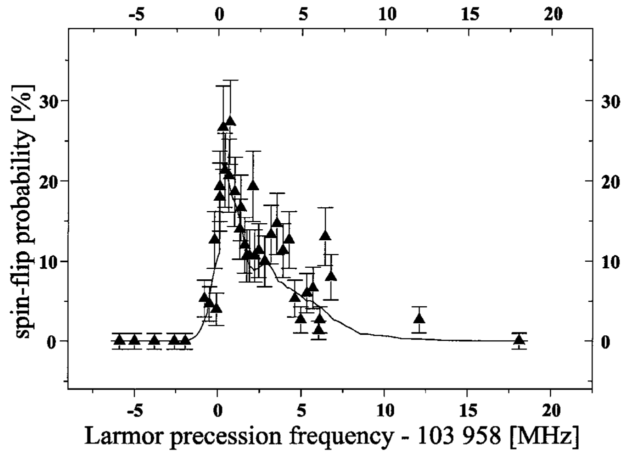

FIG. 3. The Larmor precession frequency ω_L is measured by resonant excitation of the transition between the two spin states of the bound electron in the magnetic field of the Penning trap.

resonance. The excitation of the cyclotron energy is detected via a shift of the axial frequency of the trapped ion (Fig. 4). The cyclotron motion is a circular one with a classical orbital magnetic moment, which modifies the effective trapping force in the axial direction due to the presence of the magnetic bottle—just as in the case of the magnetic spin moment of the bound electron as shown in Eq. (4). Typically, the cyclotron energy during such a measurement is increased up to a few meV.

The measurements are performed on the same single $^{12}C^{5+}$ ion in the trap. The measurement time to record a full set of data for the determination of the g factor of the bound electron is about four days. During this time the measured cyclotron frequency does not change within 10^{-6}.

With the measured values of $\omega_L = 2\pi \times 103\,958.105(10)$ MHz for the Larmor precession frequency of the bound electron and $\omega_c = 2\pi \times 23.755\,285(20)$ MHz for the free-space cyclotron

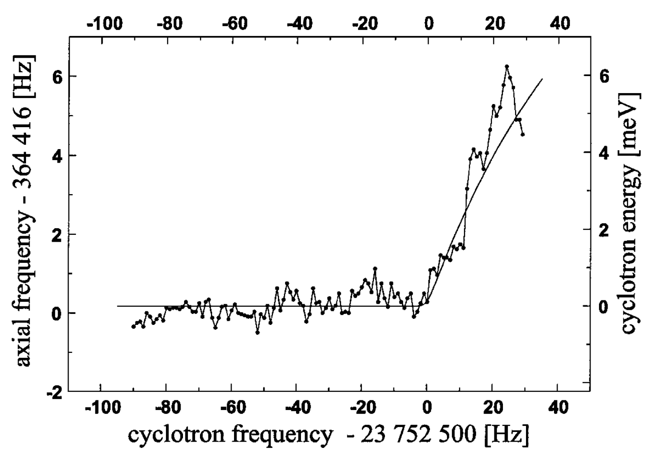

FIG. 4. Measurement of the trap-modified cyclotron frequency ω_+. Such a single measurement yields the resonance frequency typically to within 20 Hz.

frequency of the $^{12}C^{5+}$ ion the g factor of the bound electron in $^{12}C^{5+}$ is calculated using Eq. (6) to

$$g_e(C^{5+}) = 2.001\,042(2)\,. \tag{7}$$

This experimental result is the first direct measurement of the g factor of the bound electron in a highly charged ion and nicely demonstrates the great potential of this new method of microwave spectroscopy on atomic ions in high charge states.

The theoretical g factor can be written as $g = g_e^{\mathrm{free}} + \Delta g_{\mathrm{rel}} + \Delta g_{\mathrm{BS}}$. Compared to the g factor of the free electron $g_e^{\mathrm{free}} = 2.002\,319\,304\,377(9)$ [5], the g factor of the bound electron in the Coulomb field of a nucleus with charge Z is reduced due to the relativistic motion of the electron in the $1s_{1/2}$ state. This relativistic correction was first calculated by Breit who solved the Dirac equation for the bound system giving a correction of [16]

$$\Delta g_{\mathrm{rel}} = \frac{4}{3}\,[\sqrt{1 - (Z\alpha)^2} - 1]\,. \tag{8}$$

In the case of the hydrogenlike carbon ion ($Z = 6$) the relativistic term is $\Delta g_{\mathrm{rel}} = -0.001\,278\,646$. Other corrections to the g factor of the bound electron are the bound-state QED terms. The sum of all bound-state QED terms on the one-photon level gives a contribution of $\Delta g_{\mathrm{BS}} = +0.84 \times 10^{-6}$. The accuracy of our measurement (1×10^{-6}) is not yet sufficient to verify the bound-state QED terms. The theoretical prediction, including relativistic, bound-state QED and nuclear corrections, for the bound-state g factor in $^{12}C^{5+}$ is $g_e(C^{5+}) = 2.001\,041\,59$ [17]. Our experimental result is in excellent agreement with this theoretical value, confirming the relativistic correction at a level of 0.1%. Taken together with the result obtained on atomic hydrogen [18] the Z dependence of this correction is confirmed.

Presently, our measurement accuracy of 1×10^{-6} is mainly limited by the magnetic field inhomogeneity which is needed to detect the spin flips. We plan to improve the relative accuracy of the measurement to below 10^{-8} by spatially separating the functions of inducing and detecting the spin-flip transitions. This will be accomplished by transporting the ion back and forth between two adjacent potential minima forming two traps (distance 20 mm). The first trap is equipped with a nickel ring for spin-flip detection, and in the second trap, where the spin flips will be induced, the magnetic field is more homogeneous to guarantee a narrow linewidth of the cyclotron as well as the Larmor frequency spectrum. Furthermore, we plan to extend the g factor measurements to heavier hydrogenlike systems, up to hydrogenlike uranium U^{91+} [7,19]. The measurements on heavy highly charged ions are of particular interest since the relativistic as well as the bound-state QED correction terms scale as Z^2 [17,20]. In summary, we have developed a novel method to determine g factors of hydrogenlike ions with any nuclear charge Z. This will provide a sensitive test of QED corrections in strong electric fields.

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