Band Structure from Random Interactions

R. Bijker[1] and A. Frank[1,2]

[1]*Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, Apartado Postal 70-543, 04510 México, D.F., México*
[2]*Centro de Ciencias Físicas, Universidad Nacional Autónoma de México, Apartado Postal 139-B, Cuernavaca, Morelos, México*
(Received 10 September 1999)

The anharmonic vibrator and rotor regions in nuclei are investigated in the framework of the interacting boson model using an ensemble of random one- and two-body interactions. We find a predominance of $L^P = 0^+$ ground states, as well as strong evidence for the occurrence of both vibrational and rotational band structures. This remarkable result suggests that such band structures represent a far more general (robust) property of the collective model space than is generally thought.

PACS numbers: 21.10.Re, 21.60.Ev, 21.60.Fw, 24.60.Lz

A recent analysis of experimental energy systematics of medium and heavy even-even nuclei suggests a tripartite classification of nuclear structure into seniority, anharmonic vibrator, and rotor regions [1,2]. Plots of the excitation energies of the yrast states with $L^P = 4^+$ against $L^P = 2^+$ show a characteristic slope for each region: 1.00, 2.00, and 3.33, respectively. In each of these three regimes, the energy systematics is extremely robust. Moreover, the transitions between different regions occur very rapidly, typically with the addition or removal of only one or two pairs of nucleons. The transition between the seniority region (either semimagic or nearly semimagic nuclei) and the anharmonic vibrator regime (either vibrational or γ soft nuclei) was addressed in a simple schematic shell model calculation and attributed to the proton-neutron interaction [3]. The empirical characteristics of the collective regime which consists of the anharmonic vibrator and the rotor regions, as well as the transition between them, have been studied [4,5] in the framework of the interacting boson model (IBM) [6]. An analysis of phase transitions in the IBM [7,8] has shown that the collective region is characterized by two basic phases (spherical and deformed) with a sharp transition region, rather than a gradual softening which is traditionally associated with the onset of deformation in nuclei [9,10].

In a separate development, the characteristics of low-energy spectra of many-body even-even nuclear systems have been studied recently in the context of the nuclear shell model with random two-body interactions [11,12]. Despite the random nature of the interactions, the low-lying spectra still show surprisingly regular features, such as a predominance of $L^P = 0^+$ ground states separated by an energy gap from the excited states, and the evidence of phonon vibrations. The occurrence of these pairing effects cannot be explained by the time-reversal symmetry of the random interactions [13]. A subsequent analysis of the pair transfer amplitudes has shown that pairing is a robust feature of the general two-body nature of shell model interactions and the structure of the model space [14]. On the other hand, no evidence was found for rotational band structures.

The existence of robust features in the low-lying spectra of medium and heavy even-even nuclei [1,2] suggests that there exists an underlying simplicity of low-energy nuclear structure never before appreciated. In order to address this point, we carry out a study of the systematics of collective levels in the framework of the IBM with random interactions. In an analysis of energies and quadrupole transitions we show that, despite the random nature (both in size and sign) of the interaction terms, regular features characteristic of the anharmonic vibrator and rotor regions emerge. Our results imply that these features are, to a certain extent, independent of the specific character of the interaction, and probably arise from the two-body nature of the Hamiltonian and the structure of the collective model space.

In the IBM, collective nuclei are described as a system of N interacting monopole and quadrupole bosons. We consider the most general one- and two-body IBM Hamiltonian $H = H_1 + H_2$. The one- and two-body matrix elements are chosen independently using a Gaussian distribution of random numbers with zero mean and variances:

$$\begin{aligned} \langle H^2_{1,\alpha\alpha'}\rangle &= v^2\,(1 + \delta_{\alpha\alpha'})\,, \\ \langle H^2_{2,\beta\beta'}\rangle &= \frac{1}{(N-1)^2}\, v^2\,(1 + \delta_{\beta\beta'})\,. \end{aligned} \tag{1}$$

Since the matrix elements of H_1 and H_2 are proportional to N and $N(N-1)$, respectively, we have introduced a relative scaling between the one- and two-body interaction terms of $1/(N-1)$. The coefficient v^2 is independent of the angular momentum and represents an overall energy scale. The ensemble defined by Eq. (1) is similar, but not identical, to the two-body random ensemble of [15]. In all calculations we take $N = 16$ bosons and 1000 runs. For each set of randomly generated one- and two-body matrix elements, we calculate the entire energy spectrum and the $B(E2)$ values between the yrast states.

Just as in the case of the nuclear shell model [11], we find a predominance (63.4%) of $L^P = 0^+$ ground states; in 13.8% of the cases the ground state has $L^P = 2^+$, and in 16.7% it has the maximum value of the angular momentum

 0031-9007/00/84(3)/420(3)$15.00

$L^P = 32^+$. For the cases with a $L^P = 0^+$ ground state, we have calculated the probability distribution of the energy ratio $R = [E(4^+) - E(0^+)]/[E(2^+) - E(0^+)]$. Figure 1 shows a remarkable result: the probability distribution $P(R)$ has two very pronounced peaks, one at $R \sim 1.95$ and a narrower one at $R \sim 3.35$. These values correspond almost exactly to the harmonic vibrator and rotor values [see the results for the U(5) and SU(3) limits in Table I]. No such peak is observed for the γ unstable or deformed oscillator case [SO(6) limit].

Energies by themselves are not sufficient to decide whether or not there exists band structure. Levels belonging to a collective band are connected by strong electromagnetic transitions. In Fig. 2 we show a correlation plot between the ratio of $B(E2)$ values for the $4^+ \rightarrow 2^+$ and $2^+ \rightarrow 0^+$ transitions and the energy ratio R. For the $B(E2)$ values we use the quadrupole operator

$$\hat{Q}_\mu(\chi) = (s^\dagger \tilde{d} + d^\dagger s)^{(2)}_\mu + \chi (d^\dagger \tilde{d})^{(2)}_\mu, \qquad (2)$$

with $\chi = -\sqrt{7}/2$. For completeness, in Table I we show the results for the three symmetry limits of the IBM [6]. In the large N limit, the ratio of $B(E2)$ values is 2 for the harmonic oscillator [U(5) limit] and $10/7$ both for the deformed oscillator [SO(6) limit] and the rotor [SU(3) limit]. There is a strong correlation between the first peak in the energy ratio and the vibrator value for the ratio of $B(E2)$ values (the concentration of points in this region corresponds to about 50% of all cases), as well as for the second peak and the rotor value (about 25% of all cases). For the region $2.3 \lesssim R \lesssim 3.0$, one can see a concentration of points around the value 1.4, which reflects the transition between the deformed oscillator and the rotor limits (see Table I). Calculations for different values of the number of bosons N show the same results.

Despite the randomness of the interactions these results constitute strong evidence for the occurrence of both vibrational and rotational band structures. We have repeated the calculations for different values of the number of bosons N and find the same results. Since the results presented in Figs. 1 and 2 were obtained with random interactions, with no restriction on the sign or size of the one- and two-body matrix elements, it is of interest to compare them with a calculation in which the parameters are restricted to the "physically" allowed region. To this end we consider the consistent Q formulation [16] which uses the same form for the quadrupole operator, Eq. (2), i.e., with the same value of χ, for the $E2$ operator and the Hamiltonian

$$H = \epsilon \hat{n}_d - \kappa \hat{Q}(\chi) \cdot \hat{Q}(\chi). \qquad (3)$$

The parameters ϵ and κ are restricted to be positive, whereas χ can be either positive or negative $-\sqrt{7}/2 \leq \chi \leq \sqrt{7}/2$. The properties of the Hamiltonian of Eq. (3) can be investigated by taking the scaled parameters $\eta = \epsilon/[\epsilon + 4\kappa(N-1)]$ and $\overline{\chi} = 2\chi/\sqrt{7}$ randomly on the intervals $0 \leq \eta \leq 1$ and $-1 \leq \overline{\chi} \leq 1$ (these coefficients have been used as control parameters in a study of phase transitions in the IBM [8,9]). In Figs. 3 and 4 we show the corresponding probability distribution and correlation plot for the consistent Q formulation of the IBM with realistic interactions. Although in this case the points are concentrated in a smaller region of the plot than before, the results

TABLE I. Energies of $B(E2)$ values in the dynamical symmetry limits of the IBM [6]. In the U(5) and SO(6) limits we show the result for the leading order contribution to the rotational spectra.

	$\frac{E(4^+)-E(0^+)}{E(2^+)-E(0^+)}$	$\frac{B(E2;4^+ \rightarrow 2^+)}{B(E2;2^+ \rightarrow 0^+)}$
U(5)	2	$\frac{2(N-1)}{N}$
SO(6)	$\frac{5}{2}$	$\frac{10(N-1)(N+5)}{7N(N+4)}$
SU(3)	$\frac{10}{3}$	$\frac{10(N-1)(2N+5)}{7N(2N+3)}$

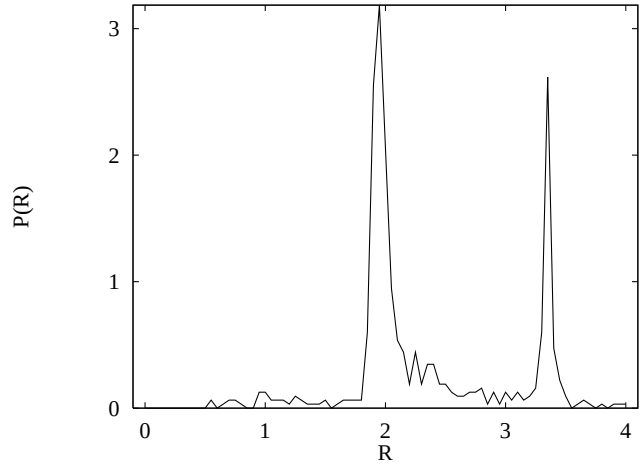

FIG. 1. Probability distribution $P(R)$ of the energy ratio $R = [E(4^+) - E(0^+)]/[E(2^+) - E(0^+)]$ with $\int P(R)\, dR = 1$ in the IBM with random one- and two-body interactions.

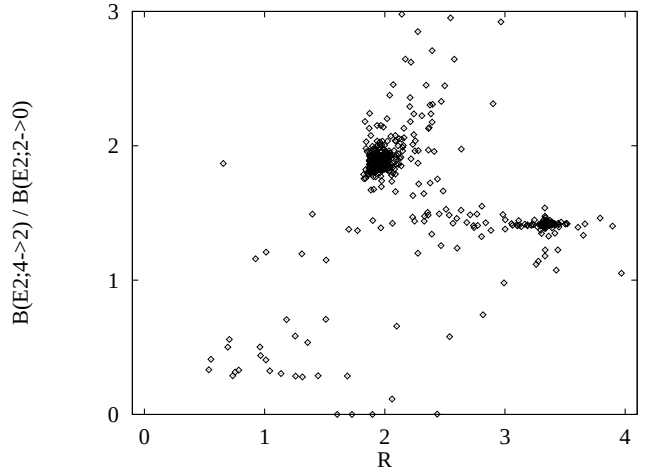

FIG. 2. Correlation between ratios of $B(E2)$ values and energies in the IBM with random one- and two-body interactions.

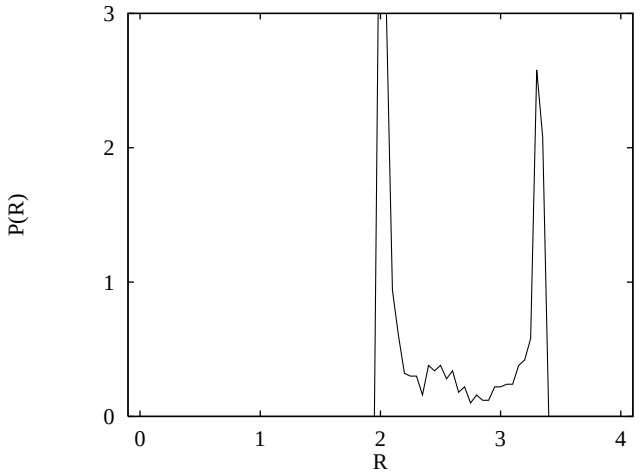

FIG. 3. As Fig. 1, but in the consistent Q formulation of the IBM.

show the same qualitative behavior as for the IBM with random one- and two-body interactions. In Fig. 4 we have identified each of the dynamical symmetries of the IBM (and the transitions between them). There is a large overlap between the regions with the highest concentration of points in Figs. 2 and 4.

In conclusion, we have studied the IBM using random ensembles of one- and two-body Hamiltonians. It was found that despite the randomness of the interactions the ground state has $L^P = 0^+$ in 63.4% of the cases. For this subset, the analysis of both energies and quadrupole transitions shows strong evidence for the occurrence of both vibrational and rotational band structure. These features arise from a much wider class of Hamiltonians than are generally considered to be "realistic." This suggests that these band structures arise, at least in part, as a consequence of the one- and two-body nature of the interactions and the structure of the collective model space, and, hence, represent a far more general and robust property of collective Hamiltonians than is commonly thought. This is in qualitative agreement with the empirical observations of robust features in the low-lying spectra of medium and heavy even-even nuclei [1,2].

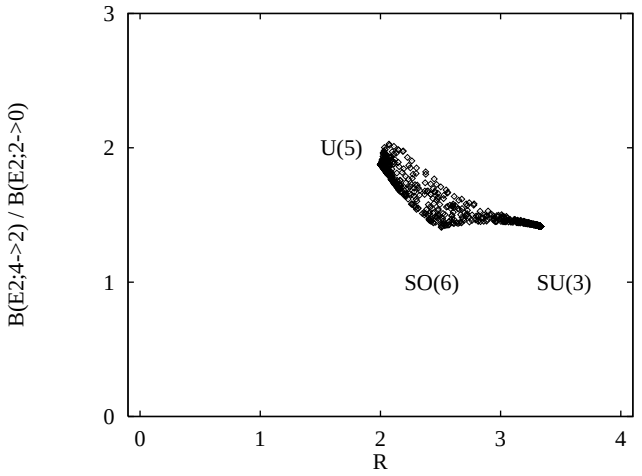

FIG. 4. As Fig. 2, but in the consistent Q formulation of the IBM.

A similar situation has been observed in the context of the nuclear shell model with respect to the pairing properties [11,14] which were formerly exclusively attributed to the particular form of the nucleon-nucleon force. On the other hand, the random IBM Hamiltonians studied in this Letter display not only vibrational-like phonon collectivity but, in contrast to the results in [11,14], also imply the emergence of rotational bands. The IBM is based on the assumption that low-lying collective excitations in nuclei can be described as a system of interacting monopole and quadrupole bosons, which in turn are associated with generalized pairs of like nucleons with angular momentum $L = 0$ and $L = 2$. It would be very interesting to establish whether rotational features can also arise from ensembles of random interactions in the nuclear shell model, if appropriate (minimal) restrictions are imposed on the parameter space.

It is a pleasure to thank Stuart Pittel and Rick Casten for interesting discussions. This work was supported in part by DGAPA-UNAM under Project No. IN101997.

[1] R. F. Casten, N. V. Zamfir, and D. S. Brenner, Phys. Rev. Lett. **71**, 227 (1993).
[2] N. V. Zamfir, R. F. Casten, and D. S. Brenner, Phys. Rev. Lett. **72**, 3480 (1994).
[3] R. Bijker, A. Frank, and S. Pittel, Phys. Rev. C **55**, R585 (1997).
[4] N. V. Zamfir and R. F. Casten, Phys. Lett. B **341**, 1 (1994).
[5] R. V. Jolos, P. von Brentano, R. F. Casten, and N. V. Zamfir, Phys. Rev. C **51**, R2298 (1995).
[6] F. Iachello and A. Arima, *The Interacting Boson Model* (Cambridge University, Cambridge, England, 1987).
[7] J. N. Ginocchio and M. Kirson, Phys. Rev. Lett. **44**, 1744 (1980).
[8] A. E. L. Dieperink, O. Scholten, and F. Iachello, Phys. Rev. Lett. **44**, 1747 (1980).
[9] F. Iachello, N. V. Zamfir, and R. F. Casten, Phys. Rev. Lett. **81**, 1191 (1998).
[10] R. F. Casten, D. Kusnezov, and N. V. Zamfir, Phys. Rev. Lett. **82**, 5000 (1999).
[11] C. W. Johnson, G. F. Bertsch, and D. J. Dean, Phys. Rev. Lett. **80**, 2749 (1998).
[12] C. W. Johnson, Rev. Mex. Fís. **45 S2**, 25 (1999).
[13] R. Bijker, A. Frank, and S. Pittel, Phys. Rev. C **60**, 021302 (1999).
[14] C. W. Johnson, G. F. Bertsch, D. J. Dean, and I. Talmi, nucl-th/9908037.
[15] J. B. French and S. S. M. Wong, Phys. Lett. **33B**, 449 (1970); O. Bohigas and J. Flores, Phys. Lett. **34B**, 261 (1971).
[16] D. D. Warner and R. F. Casten, Phys. Rev. Lett. **48**, 1385 (1982).